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RESEARCH ARTICLE

Inference on the Process Capability Index $C_{N\,pmk}$ Under Modified Multiple Interval Type-I Censoring Using the Fréchet Distribution

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ABSTRACT

This study is devoted to the estimation of the process capability index (PCI), C_{Npmk}, within the framework of the Fréchet distribution in the purview of modified version of a multiple interval type-I censoring scheme. The estimation of C_{Npmk} is approached through three distinct methodologies: maximum likelihood estimation, maximum product spacing estimation, and a Bayesian framework. In the Bayesian paradigm, independent gamma priors are specified, and Bayes estimators of C_{Npmk} are derived under the squared error loss, general entropy loss and the LINEX loss functions. Inference is further augmented by constructing asymptotic confidence intervals and highest posterior density intervals. A comprehensive simulation study is undertaken to evaluate the finite-sample performance of the proposed estimators in terms of mean squared error. Moreover, an optimal censoring scheme from a collection of potential schemes under various censoring scenarios has been investigated. The practical efficacy of the proposed methodologies is illustrated through the analysis of a real-world data set.

1 | Introduction

The Fréchet distribution (FD), named in honor of the French mathematician Maurice René Fréchet, who introduced it in the 1920s, arises naturally in the statistical modeling of extreme maximum values. Classified as the type-II extreme value distribution, it occupies a central place within the broader framework of extreme value theory. Its theoretical foundations and extensive applicability have been thoroughly documented, most notably in the work of Kotz and Nadarajah [1], who provide a comprehensive treatment of its properties and diverse domains of application. These include, but are not limited to, accelerated life testing, hydrology, meteorology, environmental risk assessment, horse racing analytics, queueing systems in retail environments, sea current modeling, wind speed evaluation, and performance analysis in athletic events.

The two-parameter FD constitutes a particular case of the generalized extreme value family and is especially well-suited for modeling heavy-tailed phenomena associated with extreme events, such as catastrophic floods or high-intensity wind occurrences. It is also frequently referred to as the inverse Weibull distribution in reliability and survival analysis contexts. The form of its probability density function (PDF) is governed by the shape parameter, allowing for either unimodal or monotonically decreasing behavior. Regardless of the PDF's modality, the associated hazard function invariably exhibits a unimodal structure. This characteristic aligns the FD closely with the log-normal distribution in terms of hazard rate behavior.

Let $Z \sim \mathrm{FD}(\alpha, \beta)$, where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. The probability density function (PDF) is given by

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