

Rotational energy of multi-quasiparticle states of the superheavy nuclei ^{256}Rf , ^{258}Sg and ^{260}Hs ($N = 152$)

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The rotational properties of some even–even superheavy nuclei (SHN) have been investigated via the projected shell model (PSM). The yrast band energy levels and band diagram of ^{256}Rf , ^{258}Sg , and ^{260}Hs isotones are calculated. The kinematic ($j^{(1)}$) and dynamic ($j^{(2)}$) moments of inertia of the yrast band in these nuclei are analyzed. For ^{256}Rf isotope, the $j^{(1)}$ and $j^{(2)}$ results agree well with the available experimental data. The energy of Nilsson single-particle levels for proton and neutron with $N = 152$ is investigated.

Keywords: Energy levels; projected shell model; rotational band; superheavy nuclei.

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1. Introduction

The study of properties of the nuclear excited states is very important to create a major development for our understanding of the structure of nuclear system. As well as, the availability of experimental techniques allows us to conduct further investigations into these excited states at high angular momenta. Numerous theoretical predictions in the structure and stability of SHN have been predicted by different models.^{1–9}

The stability of superheavy nuclei has been studied well by self-consistent theory and microscopic models.^{10–21} The ground-state properties for nuclei with $Z \geq 100$ change in a quite radical way when considering the self-consistent calculations which achieved satisfying success in describing the properties of the SHN.^{22–25} Further theoretical investigation about nuclear shell structure of superheavy nuclei has been achieved to explore the shell effects drastically influencing

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the fission barriers, spontaneous fission half-lives, and α -decay.^{26–30} Systematically, significant discrepancies in the single-particle (SP) structure of proton and neutron in SHN arising from the decreased ground-state binding energy of the orbits due to the Coulomb repulsion represent the main prediction of the macroscopic liquid drop model about nonexistence of SHN. On the other hand, the Coulomb interactions make the proton radii to be larger, the consequences are a change in the density distribution and a complicated potential single-particle for protons and neutrons which represent fundamental quantities in the mean-field models. Asai *et al.*³¹ presented significant experimental results to study the nuclei with atomic number $100 < Z < 109$ and odd neutrons number $N = 151–155$ by applying spectroscopic investigations through α and $\alpha - \gamma$ decay modes and internal-conversion-electron coincidence spectroscopy.

The study by Xu *et al.*³² focused on investigating multi-quasiparticle states in even–even SHN using configuration-constrained potential energy surface (PES) calculations. The researchers aimed to understand the properties of low-energy high- K deformed configurations. Stone *et al.*³³ showed advanced evidence about the effects of $N = 152$ and 162 subshell gaps on the ground-state binding energies, moment of inertia, pairing correlations, stability, and so on to form a more complete picture of SHN. The energies of the first rotational $J^\pi = 2^+$ states of SHN ($108 \leq Z \leq 126$) and ($148 \leq N \leq 180$) have been reported using the Skyrme–Hartree–Fock–Bogoliubov calculations.³⁴ Around the region of $Z = 102$ and $N = 152$, multi- K isomeric states can be observed with a large value of K projection.³⁵ The modified two-center shell model has been used to describe structure and configurations of high- K isomeric states in several even $Z \geq 100$ nuclei.³⁶

Starting from the self-consistent approach, the total-Routhian-surface method expected the existence of many properties of the moment of inertia at high angular momentum of even–even ^{254–258}Rf studied in multi-dimensional space (β_2, γ, β_4).³⁷ Zhang *et al.*³⁸ performed calculations using CSM with the pairing correlations treated by a particle-number conserving (PNC), their calculations strongly suggest the presence rotational properties of ²⁵⁶Rf and its neighboring even–even nuclei. The authors made an investigation into the impact of the high-order deformation parameter ε_6 and pairing correlations on the $j^{(1)}$ and $j^{(2)}$ results. Greenlees *et al.*³⁹ observed the energy levels of rotational band of ²⁵⁶Rf using state-of-the-art γ -ray spectroscopic techniques. Robinson *et al.*⁴⁰ discussed the existence of 2-qp states with high- K values in ²⁵⁶Rf. The energies of these states have been calculated. In this paper, we investigate the superheavy nuclei structure in the shell model by allowing for the artificial variation of parameter strengths from self-consistent values (in the case of the quadrupole–quadrupole interaction) or systematic values (Nilsson mean-field deformation and pairing).⁴¹

The aim of this study is to perform a detailed investigation of the rotational properties of some even–even superheavy nuclei SHN via the projected shell model. The yrast band energy levels and band diagram of ²⁵⁶Rf, ²⁵⁸Sg, and ²⁶⁰Hs isotones are calculated. The ($j^{(1)}$) and ($j^{(2)}$) of the yrast band in these nuclei are analyzed. As per

PSM results shown for ^{256}Rf isotope, the results for $j^{(1)}$ and $j^{(2)}$ agree well with the corresponding data. The energy of Nilsson single-particle levels for proton and neutron is described using PSM predications.

In Sec. 2, we briefly review the theory of PSM. The energy levels of yrast band and band diagram are presented in Sec. 3. A discussion of moment of inertia follows in Sec. 4. In Sec. 5, the calculated Nilsson levels with different values of ε_2 are investigated. The $B(E2)$ transitions and g -factor are studied in Sec. 6. Finally, in Sec. 7, we summarize our results.

2. The Theory of PSM

The first subject to be encountered in the study of the nuclear shell model theory is how to deal with a large number of valence nucleons which are coupled with core to form the relevant system. As is well known, most of the nuclei are deformed, one should find a suitable method to solve the above issue. The Hartree–Fock–Bogolyubov method solved the issue of deformed many particle systems (heavier systems)⁴² but this method could not conserve the symmetry of the rotating deformed nuclei. Hara and Sun⁴³ introduced a way of doing feasible calculation for the shell model by the Projected Shell Model. They used the deformed intrinsic state to achieve a naturally and efficiently classification for the shell model basis. The intrinsic deformed basis of the single-particle model plays a major role in the compiling of the PSM calculations. The model uses the Nilsson + BCS quasiparticle scheme to take into account the strong pairing correlations. The projected shell model more effectively depicted the physical problems of the heavy, superheavy deformed nuclei with various truncated schemes from its configurations space. This allows for the easy inclusion of a large model space as well as the construction of many different nuclear correlations before configuration mixing. This model can be used as a unified way to investigate even–even, odd–odd, and odd mass heavy nuclei. The PSM can be described with different quasiparticles systems with axial symmetry, triaxiality and both quasiparticles, phonon excitations.⁴⁴ In this work, we use the theory of the PSM for the axial symmetry basis.

The method of angular momentum projection can be considered as an effective way to truncate the too-large basis space of shell model. It is popular that the broken-rotational symmetry of the deformed bases system that is processed microscopically should be restored. The angular momentum projection method fundamentally restores the broken-rotational symmetry. We will brief the important properties of the projection techniques and are also concerned in the rotational symmetry concept (in the theory of group representations). For a state $|\mu IM\rangle$ which denotes a complete set of quantum numbers in a Hilbert space, the following closure holds⁴³

$$\sum_{\mu IM} |\mu IM\rangle \langle \mu IM| = 1. \quad (1)$$

The explicit form of the rotational operator is $\hat{R}(\Omega) = e^{-i\alpha J_x} \cdot e^{-i\beta J_y} \cdot e^{-i\gamma J_z}$, from unitary representation and closure holds (Eq. (1)), we get the following⁴³:

$$\hat{R}(\Omega)|\nu IK\rangle = \sum_M |\nu IM\rangle D_{MK}^{I*}(\Omega), \quad (2)$$

where $\hat{R}(\Omega)$ is the Rotational operator, Ω is group parameter (a set of Euler angles). $D_{MK}^{I*}(\Omega)$ is D -function of the associated unitary group which serves as matrix element for the operators that generated finite unitary transformations. Using the orthogonality of D -function and Eq. (2), we obtain

$$\int d\Omega D_{MK}^{I*}(\Omega) D_{M'K'}^I(\Omega) = \frac{8\pi^2}{2I+1} \delta_{II'} \delta_{MM'} \delta_{KK'}, \quad (3)$$

$$\hat{P}_{MK}^I = |\nu I' K'\rangle = \delta_{II'} \delta_{KK'} |\nu IM\rangle, \quad (4)$$

where $\hat{P}_{MK}^I$ is operator of angular momentum projection and given by

$$\hat{P}_{MK}^I = \frac{2I+1}{8\pi^2} \int d\Omega D_{MK}^I(\Omega) \hat{R}(\Omega), \quad (5)$$

where M denotes the projections of total angular momentum I in the laboratory frame. Owing to the completeness of the D -functions, a trial function $F(\Omega)$ and a system of laboratory frame $|\psi\rangle$ can be written as⁴³

$$F(\Omega) = \sum_{IMK} \frac{2I+1}{8\pi^2} F_{MK}^I D_{MK}^I(\Omega), \quad (6)$$

$$|\psi\rangle = \sum_{IMK} F_{MK}^I \hat{P}_{MK}^I |\phi\rangle, \quad (7)$$

where $|\phi\rangle$ is the intrinsic function of multi-quasiparticle vacuum. The coefficient F_{MK}^I acts as the variational parameters instead of using the variational function $F(\Omega)$. Executing the variation with $|\psi\rangle$ (Eq. (7)), the summation over M and I vanishes due to the orthogonality relations of $\hat{P}_{MK}^I$ and the rotational invariance of the Hamiltonian. As a result, performing the variational calculations with the function $|\psi\rangle$ will give

$$|\psi\rangle = \sum_K F_K^I \hat{P}_{MK}^I |\phi\rangle, \quad (8)$$

when M and I are neglected from the summation, $|\psi\rangle$ will represent eigenstate of angular momentum I . There are several ways to generate the state $|\phi\rangle$ used in the above debate. It is possible to construct the rotational symmetry violating in the intrinsic frame using the deformed Nilsson⁴⁵ (or Hartree–Fock) single-particle basis or, if there is a strong pairing interaction, the Nilsson+BCS (Hartree–Fock–Bogolyubov) quasi-particle basis. To restore rotational symmetries, the transformation of the intrinsic function (body-fixed) $|\phi\rangle$ to a system of laboratory (space-fixed) frame $|\psi\rangle$ is done via the intricate angular momentum projection method.^{46,47}

The rotational symmetry that is violated in original state $|\phi\rangle$ is thus restored in the new state $|\psi\rangle$ by angular momentum projection method, that is later referred to as “space-fixed”, and it is an approximate possible solution of Hamiltonian.

One has to choose a set of multi-quasiparticle state $|\phi\rangle$ that we want to consider in the configuration space via projecting them into good angular momentum I . As soon as the quasiparticle basis is formulated, the Hamiltonian will be diagonalized in the shell model space, which is spanned by $\hat{P}_{MK}^I|\phi_k\rangle$. The resulting equations are eigenvalue equations of the PSM for each I which is independent of M and a normalization condition, written as

$$\sum_{K'} [H_{\kappa K \kappa' K'}^I - E N_{\kappa K \kappa' K'}^I] F_{\kappa' K'}^I = 0, \quad \sum_{\kappa K \kappa' K'} F_{\kappa K}^I N_{\kappa K \kappa' K'}^I F_{\kappa' K'}^I = 1. \quad (9)$$

The Hamiltonian and norm matrix elements for multi-quasiparticle configuration are defined by

$$H_{\kappa K \kappa' K'}^I = \langle \phi_\kappa | \hat{H} \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle, \quad N_{\kappa K \kappa' K'}^I = \langle \phi_\kappa | \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle. \quad (10)$$

When Eq. (9) is solved, the normalized eigenstate has the following form:

$$|\psi_{IM}\rangle = \sum_{\kappa K} F_{\kappa K}^I \hat{P}_{MK}^I | \phi_\kappa \rangle. \quad (11)$$

Equation (11) constitutes the interesting wave function (final states for projection) and it is the generalization of Eq. (8). The wave function of PSM is demonstrated as a linear combination of the Slater determinants constructed from quasiparticle vacuums with good angular momentum projection. For simplicity, one can eliminate writing K and K' in Eq. (9), since only one particular K contributes to the sum for a given k , the summations over K may thus be deleted. Equation (10) that defines the matrix elements can be rewritten as

$$H_{\kappa \kappa'}^I = \langle \phi_\kappa | \hat{H} \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle, \quad N_{\kappa \kappa'}^I = \langle \phi_\kappa | \hat{P}_{KK'}^I | \phi_{\kappa'} \rangle. \quad (12)$$

To complement a model, it is necessary to select a proper Hamiltonian and determine the deformed basis. Let us use the standard Hamiltonian of the projected shell model. It has the single-particle term $\hat{H}_o$ and the quadrupole pairing term $\hat{H}_{QP}$ ⁴³

$$\hat{H} = \hat{H}_o + \hat{H}_{QP}. \quad (13)$$

The first term of Eq. (13) is a spherical single-particle Hamiltonian with the proper $\vec{l} \cdot \vec{s}$ force

$$\hat{H}_o = \sum_{\alpha} c_{\alpha}^{\dagger} \epsilon_{\alpha} c_{\alpha}, \quad (14)$$

where ϵ_{α} denotes the energies of the Nilsson single-particle states given as⁴⁸

$$\epsilon_{\alpha} = \hbar\omega \{ N - \kappa [2l \cdot s + \mu (\vec{l}^2 - \langle \vec{l}^2 \rangle_N)] \}, \quad (15)$$

where (κ, μ) are Nilsson parameters of oscillator shells, the second term of Eq. (13) is the pairing + quadrupole forces. Then, the corresponding Hamiltonian is⁴³

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_{\mu} \hat{Q}_{\mu}^{\dagger} \hat{Q}_{\mu} - G_M \hat{P}^{\dagger} \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^{\dagger} \hat{P}_{\mu}, \quad (16)$$

where χ is the strength of the quadrupole–quadrupole force and it is a function of the deformed constant ε . The operators in Eq. (16) are defined as follows:

$$\hat{Q}_{\mu} = \sum_{\alpha\beta} Q_{\mu\alpha\beta} c_{\alpha}^{\dagger} c_{\beta}, \quad \hat{P}^{\dagger} = \frac{1}{2} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\bar{\alpha}}^{\dagger}, \quad \hat{P}_{\mu}^{\dagger} = \frac{1}{2} \sum_{\alpha\beta} Q_{\mu\alpha\beta} c_{\alpha}^{\dagger} c_{\bar{\beta}}^{\dagger}, \quad (17)$$

where $Q_{\mu\alpha\beta}$ is the expectation value $\langle \alpha | \rho^2 Y_{\mu} | \beta \rangle$, where $\rho = \sqrt{m\omega_o/\hbar r}$. The $\hat{Q}_{\mu}$, $\hat{P}^{\dagger}$, and $\hat{P}_{\mu}^{\dagger}$ represent quadrupole–quadrupole force, monopole pairing, and quadrupole pairing operators, respectively. $\bar{\alpha}$ is the time reversal of α where $c_{\bar{\alpha}} = \hat{T} c_{\alpha} \hat{T}^{\dagger} = (-1)^{j-m} c_{N-j-m}$. The strength χ can be obtained self-consistently from the potentials of the Hartree–Nilsson model.⁴¹ G_M and G_Q are the monopole and quadrupole pairing strengths that represent a residual interaction in the BCS form. The residual interactions of the monopole and quadrupole pairing act only in the nucleon valence space in the same major shell that is located near the Fermi surface.⁴⁹ The monopole interactions contribute to the actual amount of energy gap Δ . Although, the quadrupole force can reproduce most efficiently the crossing between neutron and proton bands,⁵⁰ it has been constituted that the pairing strengths of the neutron and proton are associated with a model space. The monopole pairing strength can be defined in terms of two adjustable parameters G_1 and G_2 for the neutron and proton, respectively^{51,52}

$$G_M = \frac{G_1 \mp G_2 \frac{N-Z}{A}}{A}, \quad (18)$$

the sign $- (+)$ is for the neutron (proton). The quadrupole pairing strength G_Q is assumed to be proportional to G_M by proportional constant γ ($G_Q = \gamma G_M$). The G_1 and G_2 are selected as 21.24 and 13.24, respectively, and are appropriate to the region of superheavy nuclei around $Z = 100$, $N = 152$.

In this work, the proportional parameter is chosen as 0.1. After the Hamiltonian has been fixed, it is diagonalized within the deformed shell model space that is spanned by a specified type of multi-quasiparticle states $\hat{P}_{MK}^{IN} |\phi_{\kappa}\rangle$. The wave function $|\phi_{\kappa}\rangle$ is the corresponding quasiparticle vacuum state. In heavy nuclei, the configuration space of the PSM is $N = 4, 5$, and 6 for the proton and $N = 5, 6$, and 7 for the neutron.^{48,53} The exact solution of the Nilsson + BCS provides multi-quasiparticles states in different single-particle orbits. The truncation of the deformed basis is an essential tool in PSM applied to the large number of quasiparticle levels around the Fermi surface.^{43,54} The truncated space is defined by the set of quasiparticle (qp) states $|\phi_{\kappa}\rangle$. For different types of axial symmetry nuclei, we can consider the following multi-quasiparticle configurations for, even–even, odd–odd, odd–neutron, and odd–proton nuclei, respectively, and all are represented by $|\phi_{\kappa}\rangle$

$$\begin{aligned}
 |\phi_\kappa\rangle &= |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger |0\rangle, \quad a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\nu_i}^\dagger a_{\nu_j}^\dagger |0\rangle, \quad a_{\pi_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \dots \\
 &= a_{\nu_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \dots \\
 &= a_{\nu_i}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\nu_i}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\nu_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger |0\rangle, \quad \dots \\
 &= a_{\pi_i}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\pi_i}^\dagger |0\rangle, \quad a_{\pi_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger |0\rangle, \quad a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\nu_i}^\dagger a_{\pi_j}^\dagger a_{\pi_i}^\dagger |0\rangle, \quad \dots,
 \end{aligned}$$

where $a^\dagger$ is the quasiparticle creation operator, ν_i and ν_j (π_i and π_j) denote the Nilsson quantum numbers for $\nu(i_{13/2}, j_{15/2})$ ($\pi(h_{11/2}, i_{13/2})$) orbits which are labeled in each deformed states in Nilsson diagram. $|0\rangle$ is the BCS ground state. It is crucial to keep in mind that in the triaxial case, each configuration state (already indicated for various types of nuclei) corresponds to a combination of many K -states rather than one.

The band diagram can introduce a helpful concept through which the numerical results can be interpreted. In this work, the basis of PSM for even-even nuclei comprises the projected 0-, 2-, and 4-quasiparticle states that are a mixture of 2-qp proton and 2-qp neutrons states:

$$\hat{P}_{MK}^I |0\rangle, \quad \hat{P}_{MK}^I a_{\nu_i}^\dagger a_{\nu_j}^\dagger |0\rangle, \quad \hat{P}_{MK}^I a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle, \quad \hat{P}_{MK}^I a_{\nu_i}^\dagger a_{\nu_j}^\dagger a_{\pi_i}^\dagger a_{\pi_j}^\dagger |0\rangle. \quad (19)$$

In the above equation, each projected state could be considered as a rotational band. For example, the $\hat{P}_{MK}^I |0\rangle$ basis, where all the particles are paired, represent ground-state energy band. The remaining basis states represent various energy bands constructed from multi-quasiparticle. The quasiparticle bands become dominant at high angular momenta, which is clearly explained by the band diagram as well as by the amplitudes of their wave function. We have to define the rotational energy band by considering a various configurations in the vicinity of Fermi level. Calling Eq. (12) and defining relation of expectation value

$$E_\kappa(I) = \frac{\langle \phi_\kappa | \hat{H} \hat{P}_{KK}^I | \phi_\kappa \rangle}{\langle \phi_\kappa | \hat{P}_{KK}^I | \phi_\kappa \rangle}. \quad (20)$$

The cranking model⁵⁵ has been used to calculate the moments of inertia that can be described from the rotational energy levels sequences of relevant nuclei, for which the investigation of nucleons motion is achieved in a potential driven round with a fixed angular velocity. When pairing correlation force is taken into account, the values of moment of inertia are in reasonable agreement with the data. It is possible to deduce information about the moment of inertia and quadrupole moment of deformed nuclei from nuclear rotation which is a form of the collective excitation, and this information can be employed to test the general features of theoretical models. As a rule, the transfermium region nuclei are deformed, according to the single-particle energy spectra with rotational structures. The rotational frequency can be defined by analogies to classical mechanics relation $\omega = \frac{dE(I)}{dI}$ as⁵⁶

$$\omega = \frac{E(I) - E(I - \Delta I)}{\Delta I} = \frac{E(I) - E(I - 2)}{2}, \quad (21)$$

and it can be defined experimentally as

$$\hbar\omega \approx \frac{\Delta E}{\Delta I} = \frac{E_\gamma}{2}. \quad (22)$$

For a deformed nuclei, the rotational frequency is related to the γ -ray energy which is therefore linked to the moment of inertia by the relative relation

$$E_\gamma(I \rightarrow I - 2) = E(I) - E(I - 2) = \frac{\hbar^2}{\mathfrak{I}}(2I - 1), \quad (23)$$

where $\mathfrak{I}$ is moment of inertia. When the angular momentum I is determined, the kinematic moment of inertia $j^{(1)}$ is then calculated as

$$j^{(1)} = \frac{2I - 1}{E(I) - E(I - 2)}. \quad (24)$$

The dynamic moment of inertia $j^{(2)}$ is calculated using Eq. (23), it is found that the difference between transitions γ -ray energy for two consecutive levels is constant $\Delta E_\gamma = \frac{4}{j^{(2)}}$. Hence, it explains why the electromagnetic transitions in spectra occur at regular spacing. $j^{(2)}$ is defined by

$$j^{(2)} = \frac{4}{(E(I) - E(I - 2)) - (E(I - 2) - E(I - 4))}, \quad (25)$$

these quantities are expressed both in $\hbar^2\text{MeV}^{-1}$.

It is possible to compare the theoretical predictions of nuclear models with experimentally observed γ -ray reduced transition probabilities between nuclear states. Theoretical predictions for electromagnetic transitions, which use the wave function obtained from the angular momentum projected method, are incredibly accurate compared to other models. The emission of a γ -ray photon removes energy and allows the nucleus to transition to a lower energy state, which is (often) a more stable state. In the PSM, $B(E2)$ from $|\psi^I\rangle$ state to the $|\psi^{I-2}\rangle$ state can be computed by the expression⁴³

$$B(E2, I \rightarrow I - 2) = \frac{1}{2I + 1} |\langle \psi^{I-2} | \hat{Q}_2 | \psi^I \rangle|^2, \quad (26)$$

where ψ^{I-2} and ψ^I are the wave functions of a PSM system for final and initial states, respectively. The effective charges (e_{eff}) used to calculate the $B(E2)$ are taken to be $e^\pi = 1.5\text{ e}$ and $e^\nu = 0.5\text{ e}$. Because we have enough configuration space, we do not use any core contributions.

The magnitude of g -factors of the excited states in rotating nuclei allows one to distinguish between the behavior of the proton and neutron. The magnetic properties of the yrast states are identifiable by measuring the effective gyromagnetic factor (g -factor which is a quantity provide essential information about the state occupancy) of the angular momentum (I) using the following equations:

$$g(I) = \frac{\mu(I)}{\mu_N I} = \frac{1}{\mu_N I} [\mu_\pi(I) + \mu_\nu(I)], \quad (27)$$

and

$$\begin{aligned}
 \mu_\tau(I) &= \langle \psi^I | \hat{\mu}_z^\tau | \psi^I \rangle \\
 &= \frac{I}{\sqrt{I(I+1)}} \langle \psi^I | \hat{\mu}^\tau | \psi^I \rangle \\
 &= \frac{I}{\sqrt{I(I+1)}} [g_l^\tau \langle \psi^I | \hat{j}^\tau | \psi^I \rangle + (g_s^\tau - g_l^\tau) \langle \psi^I | \hat{s}^\tau | \psi^I \rangle],
 \end{aligned}$$

where $\tau = \pi$ or ν , $\hat{\mu}_z^\tau$ is magnetic moment operator around z -axis, and $\mu_\pi(I)$ ($\mu_\nu(I)$) is the proton (neutron) magnetic moment of a state $|\psi^I\rangle$ which is given in units of nuclear magneton. g_l and g_s are the orbital and spin gyromagnetic factors, they are damped by a usual 0.75 factor from the free-nucleon values to account for the core-polarization and meson-exchange current corrections.⁵⁷

3. Energy Levels and Band Diagram

The yrast band states are obtained exactly from the final diagonalization processes. The multi-quasiparticle bands with higher- K region have little effect on the yrast state. The yrast band states are pushed down from the lowest band as the spin increases, which indicates that band decoupling has a greater impact. We concentrate on axially deformed $N = 152$ isotones with even $Z = 104$ – 108 which are influenced by spherical orbits (subshells) due to deformation effects. We can obtain a piece of significant information about deformed single-particle orbits in the vicinity of the Fermi surface from high-spin states of SHN. More of the nuclear properties of these isotones have not been seen yet. For ^{256}Rf , the observed value of 2_1^+ state and $j^{(1)}$, $j^{(2)}$ MOI of the yrast band have been employed to estimate the deformation parameters in the Hamiltonian. Whereas the parameters of the ^{258}Sg and ^{260}Hs are estimated using the predictions of Sobiczewski *et al.*¹⁵ for 2_1^+ state and moment of inertia for ^{258}Sg and the nearest isotope to ^{260}Hs nuclei.

The values of the monopole pairing interaction parameters are taken as 21.24 and 13.86 MeV, respectively, and are appropriate to this mass region.^{16–18} To study the effect of proton shells ($Z = 104, 106, 108$) and also neutron shell $N = 152$ on the rotational properties, an extensive discussion of ^{256}Rf , ^{258}Sg , and ^{260}Hs isotones has been conducted. In Table 1, the quadrupole and hexadecapole (ε_2 , ε_4) deformation parameters are listed. The deformation parameter used in the PSM calculation is the stretched deformation ($\varepsilon_2, \varepsilon_4$), not (β_2, β_4), although the difference is not of much

Table 1. Values of deformation parameter used in the PSM calculations for the present isotones.

	^{256}Rf	^{258}Sg	^{260}Hs
ε_2	0.2770	0.2520	0.2430
ε_4	0.0100	-0.0160	-0.0200

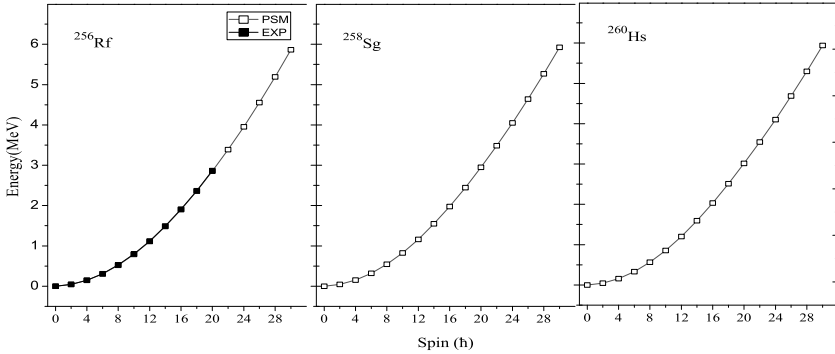


Fig. 1. Comparison of calculated positive-parity energy levels of yrast band for ^{256}Rf , ^{258}Sg , and ^{260}Hs nuclei with available experimental data.⁵⁸

relevance.⁴³ As it is known, the deformation of the nuclei depends on the concepts of the model. Therefore, our used deformation parameters are not required to be the same as the number in the other models. However, it was found that our axial deformation parameters were overused close to those calculated in Refs. 15, 59 and 60 and the same direction of variation along increasing mass number. In addition, the change in the sign of the hexadecapole parameter from the $Z = 104$ isotope to the $Z = 106$ isotope agrees with the prediction of the calculations in Refs. 15 and 27. The yrast bands of the three isotones are shown in Fig. 1 formed by 0-qp band at low spin region and by 2-qp neutron and proton bands at high spin, then by a combination of 2-qp neutron and 2-qp proton (4-qp) at very high-spin region with multi- K projection value.⁴⁷

Depending on the numerical results, the band's diagram is composed of odd and even K^π configurations of the different excited state bands originating from the intruder $j_{15/2}$ and $i_{13/2}$ shells for neutron and proton, respectively. The band with $K = 0$ represents a first rotational band and corresponds to the 0-band containing projection energies from 0-qp (all particles are paired). However, the wave functions of the yrast band at low spins are built by $K = 0$ configuration only, and hence the 0-band will be the dominant component. While the remaining rotational bands where proximate to the Fermi surface be dominant component at high spins. From Fig. 2, we show the projected energy from 0-qp, 2-qp, and 4-qp with there configurations. The $K^\pi = 0$ configuration is belonging to 0-qp only because of the requirement of vacuum configuration symmetry. We also mention that many other bands have been obtained before configurations mixing, but do not cross g -band over the calculated range of spins (I). It was found that $9/2$ states are the closest states to Fermi level for both neutron and proton.

The lowest 2-qp band has $K^\pi = 5^+$ (the highest- K value) and projected state $\nu^2[1/2, 9/2]$, the energy of $K^\pi = 5^+$ band-head is 1.4983 MeV at $I = 6$. The first crossing occurs by a $K^\pi = 1^+$ neutron band with configuration $\nu^2[-7/2, 9/2]$ at $I = 22$. In fact, the energies of $K^\pi = 1^+$ neutron band push down slightly then cross

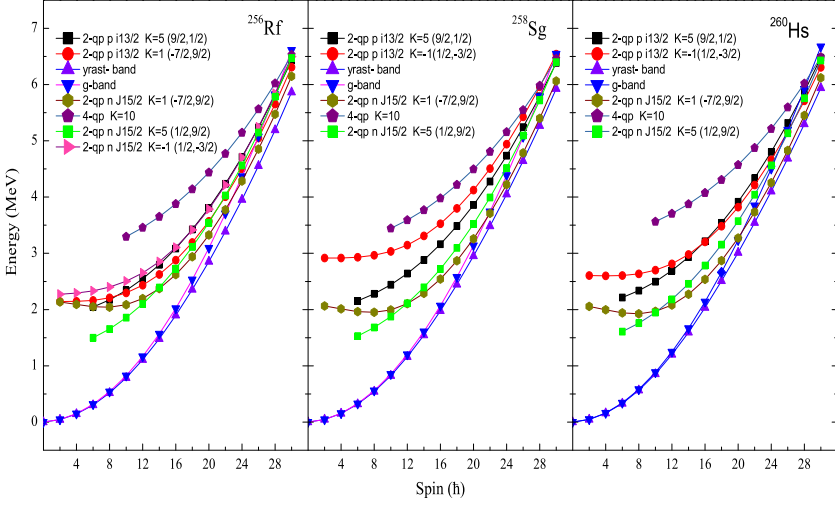


Fig. 2. Band diagram for the low-lying states of ^{256}Rf , ^{258}Sg , and ^{260}Hs isotopes, only the bands that can cross ground-state band are shown.

the $K^\pi = 5^+$ band and $K^\pi = 0$ (g -band) at $I = 14$ and 22 , respectively. It is clear from the figure that the proton and neutron $K = 1^+$ bands started from the same excitation energy value. At high spins, the $K^\pi = 1^+$ proton band becomes nearly degenerate with the $K^\pi = 5^+$ neutron band showing crossing at $I = 26$. Likewise, one can note from the figure, that the proton band $\pi^2[9/2, 1/2]$, $K^\pi = 5^+$ appears to match with neutron band $\nu^2[1/2, -3/2]$, $K^\pi = 1^-$ along high-spin region then they cross g -band together with $\nu^2[1/2, 9/2]$, $K^\pi = 5^+$ band at $I = 28$. The combination of two neutron and proton ($K^\pi = 5^+$, $K^\pi = 5^+$) 2-qp bands gives a 4-qp band with high- K value ($K^\pi = 10^+$), it crosses g -band at $I = 30$. Furthermore, the neutron 2-qp ($K^\pi = 1^+$) and proton 2-qp ($K^\pi = 1^+$) configurations are the most important because they cross the neutron and proton 2-qp bands ($K^\pi = 5^+$) before the first crossing and make them less favorable in energy.

For ^{258}Sg nucleus, one can see in Fig. 2, there are two 2-qp proton bands and two 2-qp neutron bands belonging to $N_\pi = 6$ and $N_\nu = 7$ shells, respectively. The configurations of 2-qp neutron bands in ^{258}Sg are similar to that of ^{256}Rf . The first crossing between neutron $K^\pi = 1^+$ band and g -band in ^{256}Rf and ^{258}Sg nuclei occurs at $I = 22$. The second crossing is observed at $I = 26$ by $K^\pi = 5^+$ neutron band. At $I = 28$, the g -band is crossed by two 2-qp bands simultaneously, having the same K^π values and their configurations are $\nu^2[1/2, 9/2]$ and $\pi^2[9/2, 1/2]$ from the $j_{15/2}$ and $i_{13/2}$, respectively. The $K^\pi = 1^-$ proton 2-qp head-band is predicted at 2.9172 MeV, a high energy, because the two proton states $[1/2, -3/2]$ are above the Fermi level. As indicated on the band diagram of Fig. 2, there are two single-proton states ($1/2[651]$, $-3/2[642]$) which occupy 2-qp proton band ($K^\pi = 1^-$) and have large excitation energies compared to its isotone ^{256}Rf . It crossed g -band with 4qp at

$I = 30$. The structure of 4-qp band has the configurations $\nu^2[1/2, 9/2] \otimes \pi^2[9/2, 1/2]$, $K^\pi = 10^+$.

For ^{260}Hs nucleus, the neutron $K^\pi = 1^+$ band crosses g -band at $I = 20$. It consists of two quasineutrons with angular momentum projections $-7/2$ and $9/2$ as shown in Fig. 2. In addition, the $\nu^2[9/2, 1/2]$, $K^\pi = 5^+$ neutron band is crossed by the $K^\pi = 1^+$ neutron band at $I = 10$. However, the $K^\pi = 5^+$ neutron band crosses g -band at $I = 24$ and 26 , as well as there is the crossing of the g -band by $K^\pi = 1^-$ proton band at $I = 26$. Hence, the spin $I = 26$ is formed by the $K^\pi = 5^+$ neutron and $K^\pi = 1^-$ proton bands. On the other hand, there is crossing between the 2-qp proton bands at $I = 16$ then immediately they became nearly degenerate, the proton pair $[1/2, -3/2]$ originates from $i_{13/2}$ orbit which is the dominant contribution to build yrast band rather than in ^{258}Sg , with emphasis on the effective contribution of neutron pair $[-7/2, 9/2]$ originating from $j_{15/2}$ orbit. The band-head of 2-qp proton band ($K^\pi = 1^-$) of ^{260}Hs becomes slightly lower than that of ^{258}Sg . Because the unprojected $1/2$ state from $i_{13/2}$ is the nearest to the Fermi level than that in ^{258}Sg . As seen in ^{256}Rf and ^{258}Sg , the most favored configuration for a 4-qp band in ^{260}Hs is the $K^\pi = 10^+$ which crosses g -band at around $I = 28$.

4. Moment of Inertia

In general, the MOI represents the frequency-dependent characteristics of the yrast band, as usual at $\omega = 0$, $j^{(2)}$ (dynamic) moment of inertia is identical to the $j^{(1)}$.^{15,61} In even-even nuclei, the properties of the bands are the implication of the 2-quasiparticle excitations that occupy these bands. And it is well known that the MOI is the most sensitive quantity to the changes in nuclear deformation. When the system rotates, band structure, pairing correlations, and the occupation of time-reversed pairs closed to Fermi level are affected by the Coriolis force.⁶²

For the ^{256}Rf , a good agreement between the calculated and available experimental results for the kinematic and dynamic moment of inertia has been found as shown in Figs. 3 and 4. In the experimental data region, the kinematic MOI $j^{(1)}$

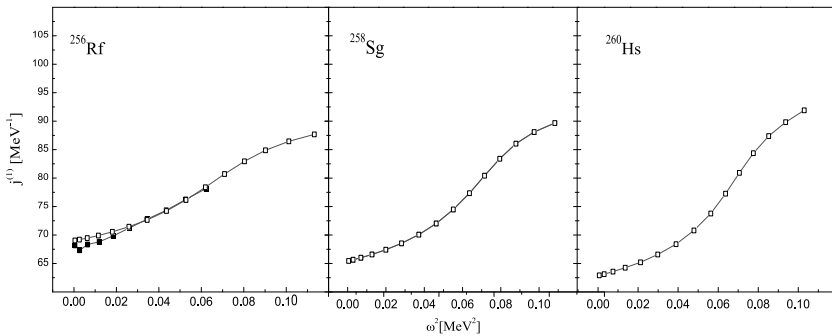


Fig. 3. Calculations for ^{256}Rf , ^{258}Sg , and ^{260}Hs isotones. The calculated kinematic moments of inertia are indicated by open symbols, compared to available data indicated by solid symbols.

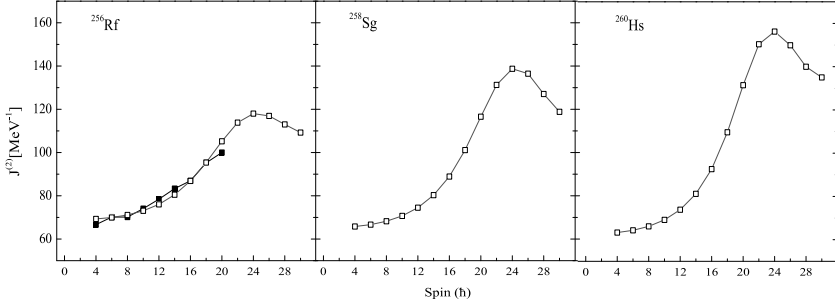


Fig. 4. Calculated dynamic moment of inertia (open symbols) compared with available data (solid symbols) for ^{256}Rf , ^{258}Sg , and ^{260}Hs isotones.

depicts a smooth increase as functions of ω , then it just slightly increases due to the first crossing (at $I = 22$) performed by a neutron pair $-7/2$ [743] and $9/2$ [734] originating from the intruder $j_{15/2}$ orbit. The calculated and available experimental $j^{(2)}$ values for ^{256}Rf have been displayed in Fig. 4, and a good agreement has been found. The PSM predicted rather constant value in $j^{(2)}$ up to the $I = 8$ then a slower alignment has been shown gradually in the range $I = 16$ to 22 (the rotational frequency at $I = 16$ is 0.2088 MeV). We cannot attribute a small increase in calculated $j^{(2)}$ for ^{256}Rf to the alignment of two-quasiparticle broken pairs, but we can interpret that as a result of the diminishing effect of the closure $Z = 100$ on the $Z = 104$ subshell gap.³⁹

As seen in Fig. 4, the calculated $j^{(2)}$ shows upturning at $I = 24$ corresponding to rotational frequency $\omega = 0.2833$ MeV, which is due to the alignment of a $\nu^2 j_{15/2}$ neutron pair which has the highest- j value $[-7/2, 9/2]$, this makes it more susceptible to Coriolis force.⁶² The calculated results indicate that there is a competition between 2-qp neutron band with configurations $(\nu^2[1/2, 9/2], K^\pi = 5^+)$ and 2-qp proton band with configurations $(\pi^2[-7/2, 9/2], K^\pi = 1^+)$ at $I = 26$, another competition occurs between two 2-qp neutron bands with configurations $(\nu^2[1/2, 9/2], K^\pi = 5^+)$, $\nu^2[1/2, -3/2], K^\pi = 1^-)$ and one 2-qp proton with configurations $(\pi^2[9/2, 1/2], K^\pi = 5^+)$ is resulting in downsloping $j^{(2)}$ curve at $I = 28$. The simultaneous alignment processes between 2-qp proton and neutron of $Z = 104$ nucleus are corresponding to rotational frequency $\omega = 0.3003$ and 0.3181 MeV, respectively, which has been predicted in the work of Herzberg *et al.*⁶³ In ^{258}Sg nucleus, there are two more protons than ^{256}Rf . An increase slightly in calculated $j^{(1)}$ is predicted by present calculation at high-spin states which can attribute to reduced pairing correlation as well as it does not seem to be the reason for band crossing. The calculated $j^{(2)}$ does show a much gradual rise through the spin range $I = 16$ –22 corresponding to a rotational frequency range $\omega = 0.2151$ –0.2674 MeV. The pronounced increase in the dynamic moment of inertia with spin ($I = 16$ –22) may explain in terms of the alignment of a high- $j_{15/2}$ neutron pair $\nu^2[-7/2, 9/2]$ and a high- $i_{11/2}$ proton pair $\pi^2[1/2, -3/2]$ with its Nilsson quantum numbers $9/2[734]$, $-7/2[743]$ for neutron and

$1/2[640]$, $-3/2[631]$ for proton at low spin region. On the other hand, we can see from diagram of Nilsson single-particle neutron state that the $N = 152$ subshell gap has expanded significantly with increasing protons number, this could contribute to increasing dynamic $j^{(2)}$ MOI.⁶⁴ The upturning in the calculated $j^{(2)}$ has been shown at $I = 24$ corresponding to $\omega = 0.2785$ MeV. The competition between two high- K configurations for neutron ($K^\pi = 5^+$) and proton ($K^\pi = 5^+$) has been seen at $I = 28$.

For ^{260}Hs , a slightly quicker growth seen on the $j^{(1)}$ curve at high-spin region does not represent an anomaly in the values of kinematic moment of inertia that occurs due to the crossing between g -band and 2-qp band. The maximum calculated value of the $j^{(1)}$ for ^{260}Hs is 91.9146 MeV^{-1} at rotational frequency $\omega = 0.3209$ MeV and spin $I = 30$. Similar rises of $j^{(2)}$ MOI in PSM calculations have also been noticed in ^{260}Hs as the ^{258}Sg . From the $j^{(2)}$ values beyond $I = 14$ and (at the intermediate between low- and high-spin) regions, it is noticeable that there is a sharp rise in dynamical $j^{(2)}$ MOI that corresponds to a rotational alignment $\omega = 0.1974\text{--}0.2657$ MeV. We suggest that due to the strong Coriolis effect, states of two-quasiparticle protons $[1/2, 3/2]$ near the Fermi surface are dominant rather than other states. As well as the expanding of subshell gap $N = 152$ as increasing proton number all that contributed to rapid rise in dynamic moment of inertia. The upturning for $j^{(2)}$ has been obtained at spin $I = 24$ and a rotational frequency $\omega = 0.321$ MeV. The high- K configuration competition has been obtained at $I = 26$.

Figure 5 displays a calculated $j^{(1)}$ as a function rotational frequency for ^{256}Rf and compared with ^{258}Sg and ^{260}Hs nuclei. Note that in ^{256}Rf , the calculated first excited rotational state 2_1^+ increases with increasing proton number³⁹ for present isotones and the consequences decrease in $j^{(1)}$ at low-spin region, but in general, their $j^{(1)}$ moments are shown a similar behavior as rotational frequency. At $I = 22$ and $\omega = 0.2662$ MeV, the curves of ^{256}Rf , ^{258}Sg and ^{260}Hs have the same $j^{(1)}$ value around

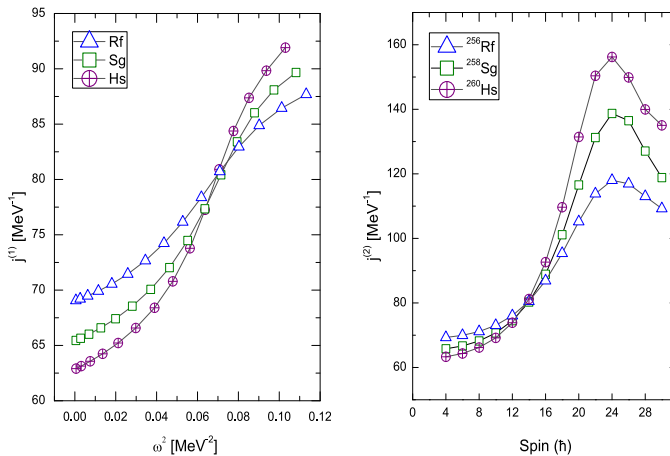


Fig. 5. Calculated kinematic and dynamic moments of inertia for ^{256}Rf , ^{258}Sg , and ^{260}Hs nuclei.

80 MeV^{-1} . As we mentioned before, the 2-qp neutron band structure $[-7/2, 9/2]$ crosses g -band at $I = 22$ in all three nuclei. It is likely that at this spin, the shell effects of $N = 152$ and $Z = 104$ gaps and hence strong Coriolis force are equal in all isotones. Interestingly, it can be considered as a turning point where the $j^{(1)}$ in ^{260}Hs at high spins is the highest whereas for ^{256}Rf , is the lower trend. A comparison between $j^{(2)}$ values as a function of spin of ^{256}Rf , ^{258}Sg and ^{260}Hs is shown in Fig. 5.

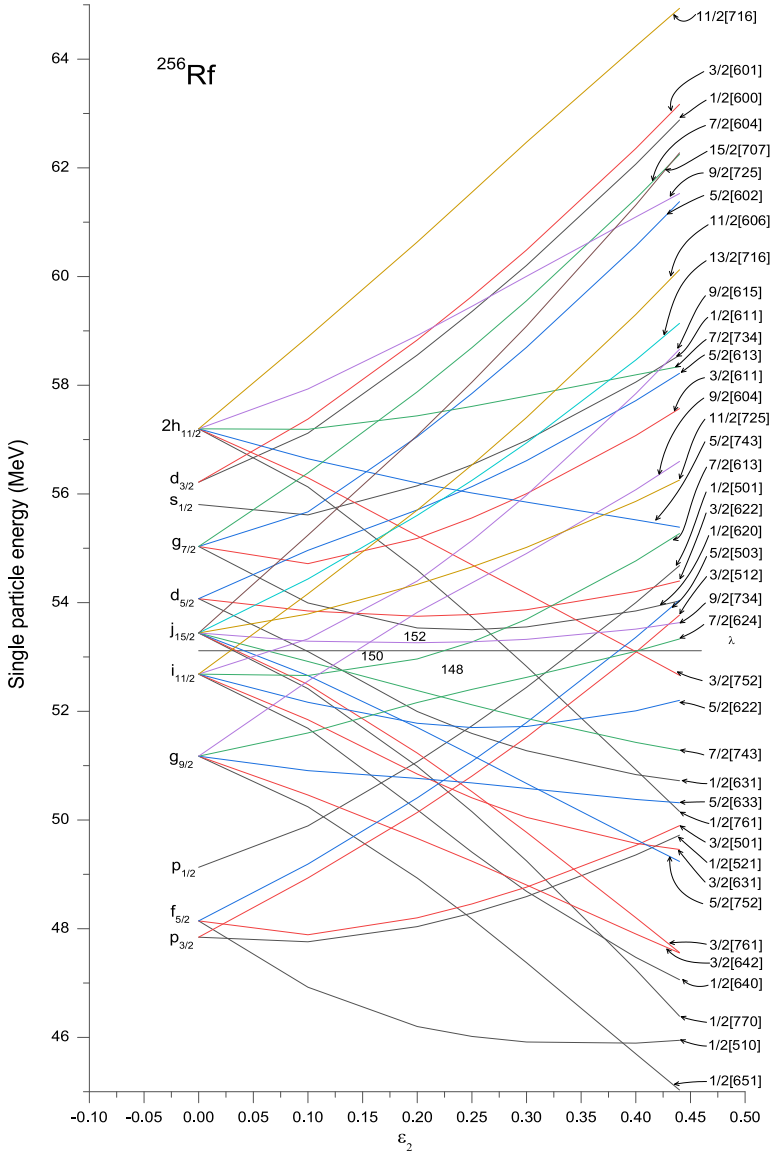


Fig. 6. Neutron single-particle spectra of ^{256}Rf isotope, the Fermi level has been emphasized.

5. Nilsson Orbits Around $Z = 104$ and $N = 152$

Here, the Nilsson parameters for proton and neutron have been taken from Ref. 45. The diagrams of the Nilsson single-particle energy for neutron and proton have been presented in Figs. 6 and 7 for ^{256}Rf within PSM calculations by diagonalizing the Nilsson Hamiltonian. To compare the locations of the gaps in the three isotones diagram, it should be noted in this section that the single-particle energies are

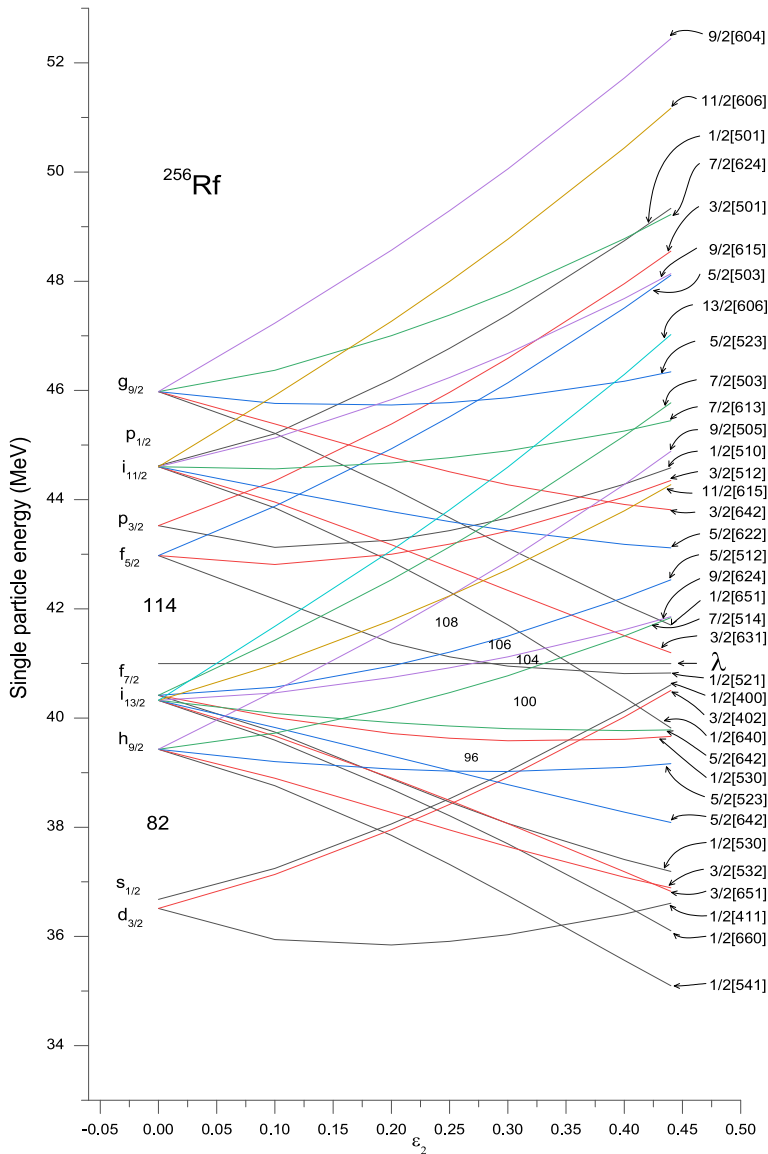


Fig. 7. Proton single-particle spectra of ^{256}Rf isotope, the Fermi level has been emphasized.

calculated using deformation parameters different from that listed in Table 1. We selected the $\varepsilon_4 = 0.01$ value to be for all three isotones.

For Fig. 6, the overall positions of the Nilsson neutrons single-particle are reproduced the predication of Patyk and Sobiczewski.⁶⁵ The spacing between $g_{9/2}$ and $i_{11/2}$ is rather good due to the potential of modifying oscillator used in PSM Hamiltonian. The position of $N = 152$ gap is located between the deformation range (0.15–0.277). We can observe that the $9/2[734]$ level seems above the Fermi surface and its occupation probability indicates that it is partially occupied. On the other hand, the $7/2[743]$ level is being fully occupied, hence we shall get earlier neutron alignment from this broken pair.¹⁰ Similarly, we can observe from Fig. 7, the proton Fermi level is between $\pi - 7/2[514]$ and $9/2[624]$ levels which constitute the $Z = 104$ subshell and originate from $h_{9/2}$ and $i_{13/2}$, respectively.

The PSM predictions of deformation shell gaps are $N = 150$ for neutrons and $Z = 96, 100, 108$ for protons (other subshell gaps are considered to be small). However, the larger energy gap near the Fermi surface is the $Z = 100$ proton and $N = 148$ neutron. The $\nu 9/2[734]$ configuration originates from $j_{15/2}$ which is assigned to the neutron-ground-state of ^{256}Rf , ^{258}Sg , and ^{260}Hs .

It is worthy to note that the $\nu 1/2[620]$ state originates from $g_{7/2}$ intruder orbitals that penetrate the deformed shell gap $N = 152$ by shifting down with respect to the $\nu 9/2[734]$ and keep it opened. Because of the properties of neutron single-particle being similar in the three isotones, we will only concentrate, in Fig. 8, on the interesting proton single-particle levels in the ^{256}Rf , ^{258}Sg , and ^{260}Hs . We can also see from Fig. 9, the $\pi 9/2[624]$ level of ^{256}Rf is the nearest level to Fermi level than other isotones.

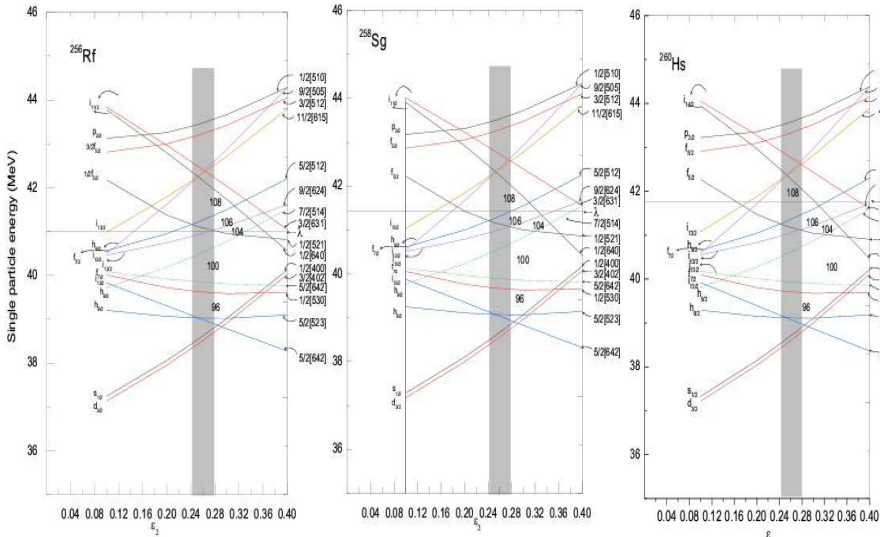


Fig. 8. Nilsson diagram for some proton single-particle states near the Fermi level. The vertical gray bar depicts the estimated range of deformation parameters of ^{256}Rf , ^{258}Sg , and ^{260}Hs nuclei.

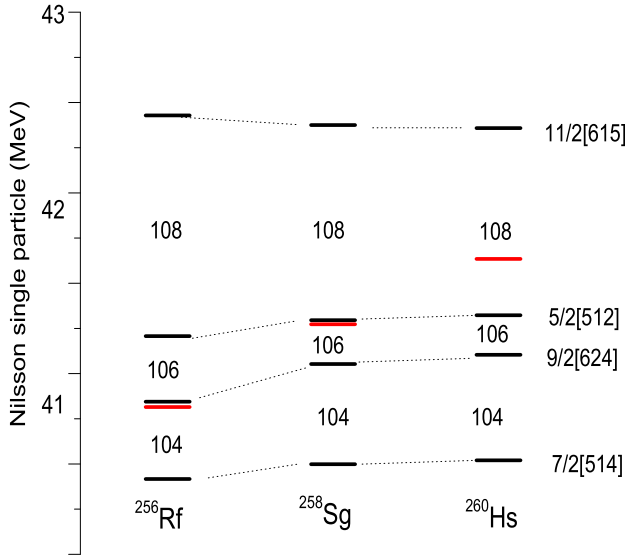


Fig. 9. (Color online) Comparison of the gaps between single-particle states for ^{256}Rf , ^{258}Sg , and ^{260}Hs , the Fermi levels are denoted by red lines.

The ground-state band of the $Z = 104$ nucleus is based on $\pi - 1/2[521]$ together with $\nu 9/2[734]$ in which their occupancy is partial. However, the PSM results for ^{258}Sg at quadrupole deformation parameter $\varepsilon_2 = 0.2520$ show the $Z = 106$ deformed shell gap is constructed from $\pi 9/2[624]$ and $\pi - 5/2[512]$ originates from $i_{13/2}$ and $2f_{7/2}$, respectively. Furthermore, the $5/2[512]$ level lies close to (above to) Fermi level of proton $Z = 106$. In the range of deformation parameters, the proton Fermi level in ^{260}Hs is just between $\pi 5/2[512]$ and $\pi 11/2[615]$ giving $Z = 108$ shell gap, it creates a deformation parameter $\varepsilon_2 = 0.2430$. Both the ^{258}Sg and ^{260}Hs show similar pattern in its proton shell gaps compared with that in ^{256}Rf , indicating that these gaps are more stable when Z changes. Moreover, an investigation of the neutron and proton single-particle diagrams emphasized that the $Z = 100$ and $N = 148$ deformed shell gaps constitute the most pronounced energy gaps among the proton and neutron gaps near the Fermi level for ^{256}Rf , ^{258}Sg , and ^{260}Hs . The theoretical analysis carried out up to now enhances our predictions about shell structure of the three $N = 152$ isotones.

6. Electric Transitions Probability $B(E2)$ and g -Factor

The projected wave functions of the PSM are used to calculate $B(E2)$ transition probabilities (Eq. (26)) which contain correlations that go beyond the mean field. Now the electrical intra-band transition of the observed yrast band in ^{256}Rf , ^{258}Sg , and ^{260}Hs is evaluated and displayed in Fig. 10. These nuclei have not detailed experimental data including $B(E2)$ transition probabilities. The PSM results showed

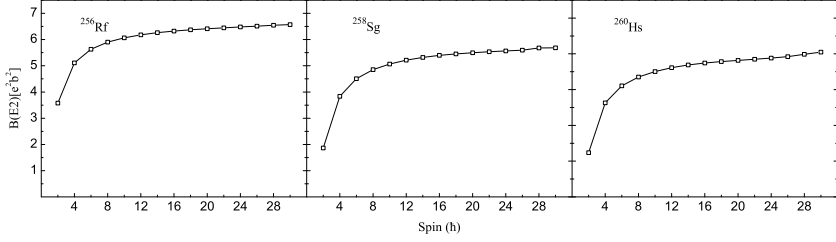


Fig. 10. Calculated electric quadrupole transition probability ($B(E2)$) for ^{256}Rf , ^{258}Sg and ^{260}Hs nuclei.

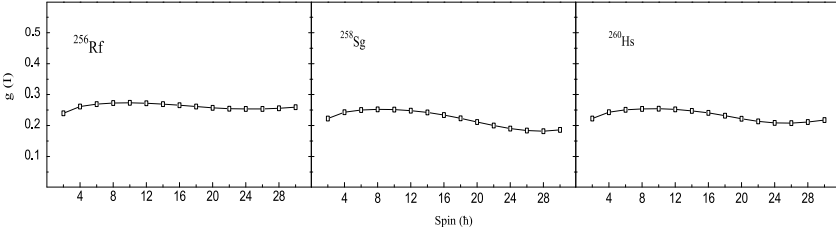


Fig. 11. Calculated g -factor for ^{256}Rf , ^{258}Sg and ^{260}Hs nuclei.

a discernible increase in $B(E2)$ from spin $I = 2$ –6. The $B(E2)$ value of each nucleus reaches up to the maximum value at spin $I = 6$ and then stabilizes at this relatively high value along high-spin region. Further, the calculated $B(E2)$ transition after $I = 6$ demonstrates a smooth behavior in the present isotones, indicating that these nuclei have almost similar yrast band structure.

The determining of the g -factor for the yrast band can be used to examine the proton and neutron identity. Figure 11 presents the calculation g -factor values for the ^{256}Rf , ^{258}Sg , and ^{260}Hs yrast band. In either quantity, we discovered that the band crossing does not result in significant changes, leading to a flat behavior with respect to spin.

7. Conclusion

The nuclear structure of the Rf, Sg and Hs nuclei ($N = 152$) has been studied using the projected shell model. A systematic investigation has been conducted into the rotational properties at normal deformation. The $j_{15/2}$ for neutron and $i_{13/2}$ for proton have been found as intruder orbits. The nuclei's band diagram reveals that a 2-qp neutron from orbit $j_{15/2}$ causes the first crossing at spin $I = 22$ in ^{256}Rf , ^{258}Sg nuclei and at $I = 20$ in ^{260}Hs nucleus. It is interesting to note that the alignment occurs simultaneously in these isotopes at some angular momentum region. Because the $9/2[734]_{\nu}$ state is partially occupied, the $N = 152$ nuclei are regarded as open shells. We found that when two broken pairs originate from different orbits and one from the same orbit, the later pair aligns more slowly than the other. For three

isotones, the main contribution of high- j orbit comes from the $9/2[734]_{\nu}$ state. The PSM results of Nilsson single-particle states in ^{256}Rf , ^{258}Sg , and ^{260}Hs suggest that the downsloping of $1/2[761]_{\nu}$ and $1/2[770]_{\pi}$ single-particle states lead to lower excitation energy of two-quasiparticle which occupied these levels. The calculated kinematic and dynamic MOIs of ^{256}Rf reproduced properly the observed data.

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