

β^+ decay $^{203,205}\text{At} \rightarrow ^{203,205}\text{Po}$ in the framework of the interacting boson fermion model

Falih H. Al-Khudair  and Ashwaq F. Jaafer 

Department of Physics, College of Education for Pure Sciences, *University of Basrah*, Basrah, Iraq



(Received 18 November 2024; revised 16 July 2025; accepted 3 November 2025; published 17 December 2025)

The energy states and electromagnetic transitions of even-even $^{202-206}\text{Po}$ and odd-mass $^{203,205}\text{Po}$ and $^{203,205}\text{At}$ isotopes within the framework of the proton-neutron interacting boson model (IBM-2) and its extension (IBFM-2) are calculated. For odd- A nuclei, the $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ and $0i_{13/2}$ single-particle orbits are considered as model space. The obtained results are used to assign spins to some uncertain spin states. The $\log ft$ values of the allowed β decay are calculated using the model wave function and compared with available experimental values.

DOI: [10.1103/PhysRevC.112.064322](https://doi.org/10.1103/PhysRevC.112.064322)

I. INTRODUCTION

The nuclear structure of heavy nuclei ($Z > 82$) has not been investigated enough compared to that of other heavy nuclei. Oros *et al.* [1] studied the shape coexistence in the $^{192-210}\text{Po}$ isotopes. The $^{202-210}\text{Po}$ isotopes were described within the parameters of the particle-core model (PCM). In these nuclei, regular protons and neutrons (one proton boson and neutron-hole boson number according to the 126 closed shell) play an important role in determining excitation energies. In the light Po isotopes, the 0_2^+ states were identified by the assignment of a 4p-2h structure. For $^{192-200}\text{Po}$ isotopes, the low-lying energy levels were calculated by a 2p-2h excitation (three proton bosons) using the interacting boson model with mixing Hamiltonian.

The shape coexistence, intruder $J^\pi = 0^+$ states and the collective band structures of the light Po isotopes which are associated with the oblate proton 4p-2h intruder were investigated in Ref. [2]. Caurier *et al.* [3] have implemented the large-scale shell model space $\pi(0h_{9/2}, 1f_{7/2}, 0i_{13/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2})$ to study the structure of $N = 126$ Po-Pu isotones. For $^{208,210}\text{Rn}$ and $^{206,208}\text{Po}$ isotopes, shell model calculations are performed in Ref. [4]. The results show that the valence neutron excitation is crucial for identifying the structure of the low-lying levels, and the $0h_{9/2}$ proton pair is required for the excited states $J^+ > 8^+$.

The low-lying states and electromagnetic transitions of $^{196-202}\text{Po}$ were measured using the Coulomb excitation [5]. The measured data were compared with three models: the calculation beyond-mean-field model, the interacting boson model, and the general Bohr-Hamiltonian model. With regard to $^{200,202}\text{Po}$ isotopes, the transitional and diagonal matrix elements of the first $J^\pi = 2^+$ level were calculated. Large-scale

shell-model calculations with six single-particle orbitals between 82 and 126 closed shells are performed. For even-even, odd-mass, and odd-odd nuclei of Pb, Bi, Po, At, Rn, and Fr isotopes, many isomeric levels are investigated in terms of the model configurations [6]. The lifetimes of $5/2_1^-$, $9/2_1^-$, and $11/2_1^-$ states, as well as the electromagnetic transitions in ^{209}Po isotope, were measured in the β decay of ^{209}At [7]. The lifetimes of the first excited $J^\pi = 2^+$ and 4^+ states of ^{208}Po isotope were measured in the α -transfer $^{204}\text{Pb}(^{12}\text{C}, ^8\text{Be})^{208}\text{Po}$ reaction [8]. Recently, Shukla *et al.* [9] performed a systematic large-scale shell-model calculation using a KHH7B interaction in a model space around ^{208}Pb . The energy spectra, electromagnetic properties, wave-function configurations, and isomeric states were studied in detail.

In this paper, the basic concepts, Hamiltonian, and transition operators of the interacting boson models IBM-2 and IBFM-2 are explained in Sec. II. Low-lying energy states of the even $^{202-206}\text{Po}$ and odd- A $^{203,205}\text{Po}$ and $^{203,205}\text{At}$ isotopes are presented in Sec. III. In Sec. IV, the transition probabilities and branching ratios are calculated using the wave function of initial- and final-energy states. In Sec. V, the $\log ft$ values of the allowed β decay from states of odd- Z At isotopes to states in odd- N Po isotopes are calculated.

II. THE INTERACTING BOSON-FERMION MODEL

The spectrum of the odd- A At and Po ($A = 203$ and 205) isotopes can be investigated by the Hamiltonian of the interacting boson fermion model [10]

$$H = H_B + H_F + V_{BF}, \quad (1)$$

where H_B denotes the IBM-2 Hamiltonian for the even-even core nuclei. The term H_F is the odd-nucleon Hamiltonian. The third term is boson-fermion interaction.

The IBM-2 Hamiltonian can be written as

$$H_B = \varepsilon_d(\hat{n}_{d\pi} + \hat{n}_{dv}) + \kappa_{\pi\nu}\hat{Q}_\pi \cdot \hat{Q}_\nu + \sum_{\rho=\pi,\nu} \hat{V}_{\rho\rho} + \hat{M}_{\pi\nu}, \quad (2)$$

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

where ε_d and $n_{d\rho}$ ($\rho = \pi$ or ν) are the d -boson energy and the number d -boson operator. $\kappa_{\pi\nu} Q_\pi \cdot Q_\nu$ implies the neutron and proton quadrupole interaction. The boson quadrupole operator Q_ρ is given by

$$\hat{Q}_\rho = d_\rho^\dagger s_\rho + s_\rho^\dagger \tilde{d}_\rho + \chi_\rho [d_\rho^\dagger \tilde{d}_\rho]^{(2)}. \quad (3)$$

The $\hat{V}_{\rho\rho}$ represents the interaction between like bosons, and it is given by

$$\hat{V}_{\rho\rho} = 1/2 \sum_{L=0,2,4} [2L+1] C_\rho^{(L)} ([d_\rho^\dagger d_\rho]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]^{(L)}). \quad (4)$$

The last term in Eq. (2) is the Majorana operator. It affects the energy of the mixed symmetry states [11, 12], usually it is

$$\begin{aligned} \hat{M}_{\pi\nu} = & \xi_2 [(d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger) \cdot (\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)]^{(2)} \\ & + \frac{1}{2} \sum_{k=1,3} \xi_k ([d_\pi^\dagger d_\pi]^{(k)} \cdot [\tilde{d}_\pi \tilde{d}_\pi]^{(k)}). \end{aligned} \quad (5)$$

The nucleon Hamiltonian is

$$H_F = \sum_i \epsilon_i n_i, \quad (6)$$

where ϵ_i is the quasiparticle energy of the odd nucleon, can be calculated by solving the BCS equations as [13]

$$\epsilon_i = \sqrt{(E_i - \lambda)^2 + \Delta^2}, \quad (7)$$

where E_i represents the energy of the i th orbital, λ is the Fermi energy and $\Delta = 12/\sqrt{A}$ (in MeV) is the pairing gap. The occupation probabilities are determined by the following relation:

$$v_i = \left[\frac{1}{2} \left(1 - \frac{E_i - \lambda}{\epsilon_i} \right) \right]^{1/2}, \quad u_i = (1 - v_i^2)^{1/2}. \quad (8)$$

The last term in Eq. (1) is boson-fermion interaction, which contains three interaction terms as follows [14]:

$$\begin{aligned} V_{BF} = & \sum_{i,j} \Gamma_{ij} ([a_i^\dagger \tilde{a}_j]^{(2)} \cdot Q_\rho^B) + A \sum_i n_i n_{d\rho} \\ & + \sum_{i,j} \Lambda_{ki}^j \{ [[d_\rho^\dagger \tilde{a}_j]^{(k)} a_i^\dagger s_\rho]^{(2)} : [s_\rho^\dagger \tilde{d}_\rho]^{(2)} + \text{H.c.} \}, \end{aligned} \quad (9)$$

where $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$, $\tilde{a}_{j\mu} = (-1)^{j-\mu} a_{j-\mu}$. The symbols ρ and $\dot{\rho}$ denote $\pi(\nu)$ and $\nu(\pi)$ if the odd fermion is a proton (a neutron). The first term represents the direct part of the quadrupole interaction between a single fermion and bosons. The second term is a monopole interaction that depends on the number of d bosons and influences the spectrum by compressing or expanding it without altering the order of states. The last term is the exchange interaction between the quasiparticle and one of two fermions that form a boson, which incorporates the Pauli exclusion principle and causes splitting and shifting of the multiplets. The interaction parameters are estimated as

$$\Gamma_{i,j} = (u_i u_j - v_i v_j) Q_{i,j} \Gamma, \quad (10)$$

$$\Lambda_{k,i}^j = -\beta_{k,i} \beta_{j,k} \left(\frac{10}{N_\rho (2j_k + 1)} \right)^{1/2} \Lambda, \quad (11)$$

TABLE I. The parameters of the IBM-2 Hamiltonian for core nuclei. For all isotopes, $\chi_\pi = -0.15$ and $\chi_\nu = -0.12$ are chosen. All the parameters are in MeV unit.

Nucleus	ε_d	$\kappa_{\pi\nu}$	$\xi_1 = \xi_2 = \xi_3$	$C_v^L (L = 0, 2, 4)$
^{202}Po	0.677	-0.005	0.600	0.050, -0.200, -0.150
^{204}Po	0.684	-0.005	0.800	0.050, -0.300, -0.250
^{206}Po	0.700	-0.005	0.850	0.050, -0.600, -0.500

$$\beta_{i,j} = (u_i v_j + v_i u_j) Q_{i,j}, \quad (12)$$

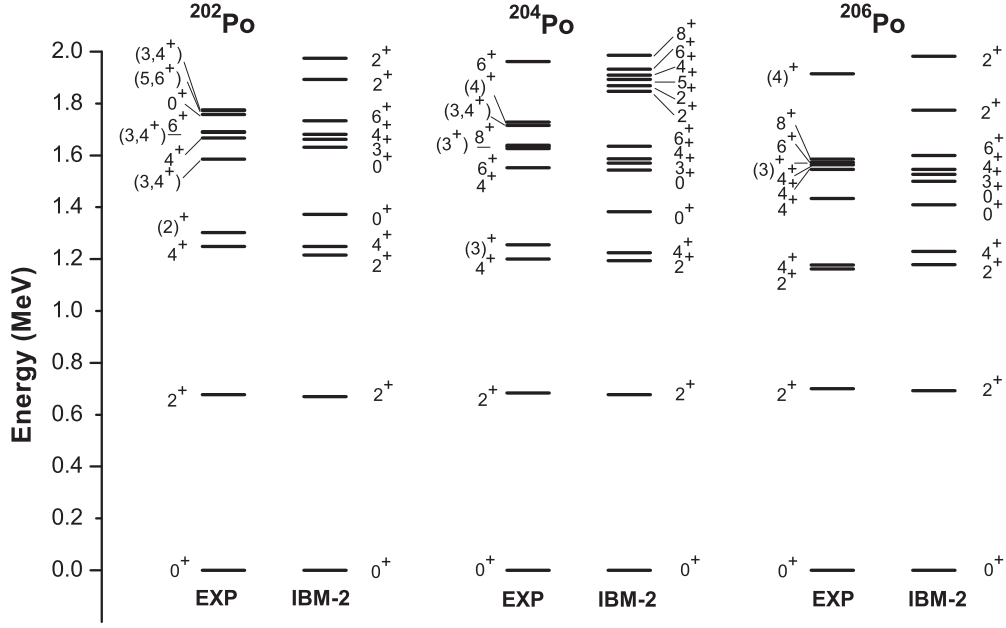
$$Q_{i,j} = \left\langle l_i, \frac{1}{2}, j_i \left\| Y^{(2)} \right\| l_j, \frac{1}{2}, j_j \right\rangle. \quad (13)$$

In the model space, the spectrum of odd- A nucleus is described as a coupled system of a boson core and an odd fermion. The basis state is written as $|L_\pi(L_\nu), I, j, J\rangle$, where $L_\pi(L_\nu)$ is proton (neutron) boson angular momentum, and I is total boson angular momentum, which, together with the fermion angular momentum j , constructs the total angular momentum J . Using the standard core-basis states $|n_d \nu I\rangle$, where n_d is the d -boson number and ν is the d -boson seniority (number of d bosons not coupled to angular momentum zero). The IBFM-2 wave functions are expressed as

$$|J_r\rangle = \sum \zeta_{j,n_d \nu I}^{(J_r)} |j, n_d \nu I; J\rangle. \quad (14)$$

III. ENERGY STATES

The even-even $^{202-206}\text{Po}$ isotopes with $Z = 84$ and $N = 118-122$ are studied, taking $Z = 82$ and $N = 126$ as closed shells. We obtain a particle-proton boson type and hole-neutron boson type. The calculated excitation energies are obtained by diagonalizing the Hamiltonian in Eq. (2), usually using the NPBOS code [15]. The Hamiltonian parameters are determined as to fit as closely as possible the energies of the lowest-lying core states. ε_d values are determined to be close to the energy of the first excited 2^+ state. A small value of the $\kappa_{\pi\nu}$ value is constant for all nuclei. The Majorana parameters are chosen to push the first mixed-symmetry state $J^\pi = 2_{MS}^+$ (one d boson) near the 2 MeV and $J^\pi = 1^+$ scissor state above 2.5 MeV. The best values of Hamiltonian parameters are listed in Table I. Reproduction of the experimental-spectrum structure [16] can be seen in Fig. 1. The calculated 0_2^+ states are predicted in energy at 1.372, 1.383, and 1.410 MeV for $^{202,204,206}\text{Po}$ isotopes, respectively. It is consistent with the recent shell-model calculations [9]. For three isotopes, the model calculation points out that the position of 3_1^+ lies above 1.5 MeV, and close to uncertain spin states $(3, 4^+)$ 1.585, $(3)^+$ 1.634, and $(3)^+$ 1.564 MeV, respectively. For the ^{204}Po isotope, there is no calculated $J^\pi = 3^+$ state close to the uncertain spin state $(3)^+$ (1.255 MeV) and it is in a good agreement with the shell-model results. On the other hand, the second $J^\pi = 2^+$ state appears at energy 1.194 MeV. An important advantage of the present calculation is the reproduction of an energy gap in the spectrum. This agreement contributes to reproducing the energy levels of the odd nuclei under study. The ^{202}Po , ^{204}Po , and ^{206}Po are considered cores of the ^{203}At , ^{203}Po , ^{205}At , and ^{205}Po , respectively.

FIG. 1. Comparison of calculated states of $^{202-206}\text{Po}$ isotopes with available experimental data [16].

For odd nuclei, the single-particle energies are taken from Ref. [17]. The quasiparticle energies and occupation probabilities are calculated by solving the BCS equations, their values are listed in Table II. The exchange boson-fermion interaction parameter is kept constant for each element, while the monopole interaction parameter is increased to raise energy levels. The comparison between the calculated and available experimental-energy states for $^{203,205}\text{Po}$ isotopes up to 1.5 MeV are shown in Figs. 2 and 3.

For ^{203}Po isotope, the ground state $5/2^-$ as $(96\%f_{5/2})$ configuration is coupled to the $0^+_{g.s.}$ of the core. The energies of the first two excited states $3/2^-$ ($93\%p_{3/2}$) and $1/2^-$ ($99\%p_{1/2}$) are calculated accurately. The calculated $3/2^-$ at 0.530 MeV comes after the energy gap; its structure is $(79\%f_{5/2} + 17\%p_{3/2})$. The IBFM-2 calculation predicts the $7/2^-$ ($88\%f_{5/2} + 9\%p_{3/2}$) state at 0.629 MeV close to the experimental one at 0.639 MeV, which is consistent with

the shell-model calculation at 0.755 MeV [18]. All of the above configurations agree with those presented in Ref. [16]. In the model calculation, the $5/2^-$ and $5/2^-$ states are predicted at 0.656 MeV and 0.719 MeV, which are higher than the experimental ones by 0.125 and 0.033 MeV, respectively. Their configurations are $45\%f_{5/2} + 48\%p_{3/2} + 5\%p_{1/2}$ and $28\%f_{5/2} + 43\%p_{3/2} + 26\%p_{1/2}$, respectively. The calculated energy of the experimental $(7/2, 9/2)^-$ state at 1.056 MeV agrees with the shell-model calculation [18] as it is a $9/2^-$ ($79\%h_{9/2} + 21\%f_{5/2}$) at 1.061 MeV. On the other hand, the nearest $7/2^-$ ($79\%f_{7/2} + 18\%p_{3/2}$) at 1.053 MeV is close to the experimental state at 1.029 MeV. The first negative parity states $J^\pi = 11/2^- - 21/2^-$ are dominated by the $f_{5/2}$ quasiparticle state.

Theoretical positive-parity states are calculated in the one single-particle orbital ($i_{13/2}$) using the same interaction parameters of negative-states calculation. The $J^\pi = 13/2^+$ is

TABLE II. The single-particle and quasiparticle energies in MeV and occupation probabilities for $^{203,205}\text{Po}$ and $^{203,205}\text{At}$ isotopes. The boson-fermion interaction parameters Γ (quadrupole), A (monopole), and Λ (exchange) are listed in the last three rows in MeV.

	^{203}Po			^{205}Po			^{203}At			^{205}At		
	E_i	ϵ_i	v_i^2	E_i	ϵ_i	v_i^2	E_i	ϵ_i	v_i^2	E_i	ϵ_i	v_i^2
$h_{9/2}$	0.008	2.132	0.959	0.004	2.380	0.968	0.012	1.087	0.184	0.007	1.080	0.185
$f_{7/2}$	-0.005	2.144	0.960	-0.003	2.386	0.968	0.698	1.611	0.074	0.699	1.608	0.073
$f_{5/2}$	1.788	0.861	0.603	1.779	0.953	0.738	2.839	3.614	0.014	2.856	3.628	0.014
$p_{3/2}$	1.457	0.984	0.758	1.453	1.143	0.840	3.128	3.895	0.012	3.118	3.883	0.012
$p_{1/2}$	2.359	0.929	0.288	2.349	0.846	0.431	3.679	4.435	0.009	3.665	4.419	0.009
$i_{13/2}$	0.896	1.361	0.893	0.898	1.575	0.923	1.643	2.466	0.030	1.635	2.456	0.030
Γ	1.450			1.750			1.050			1.050		
A	-0.050			-0.250			-0.250			-0.350		
Λ	0.950			0.950			0.700			0.700		

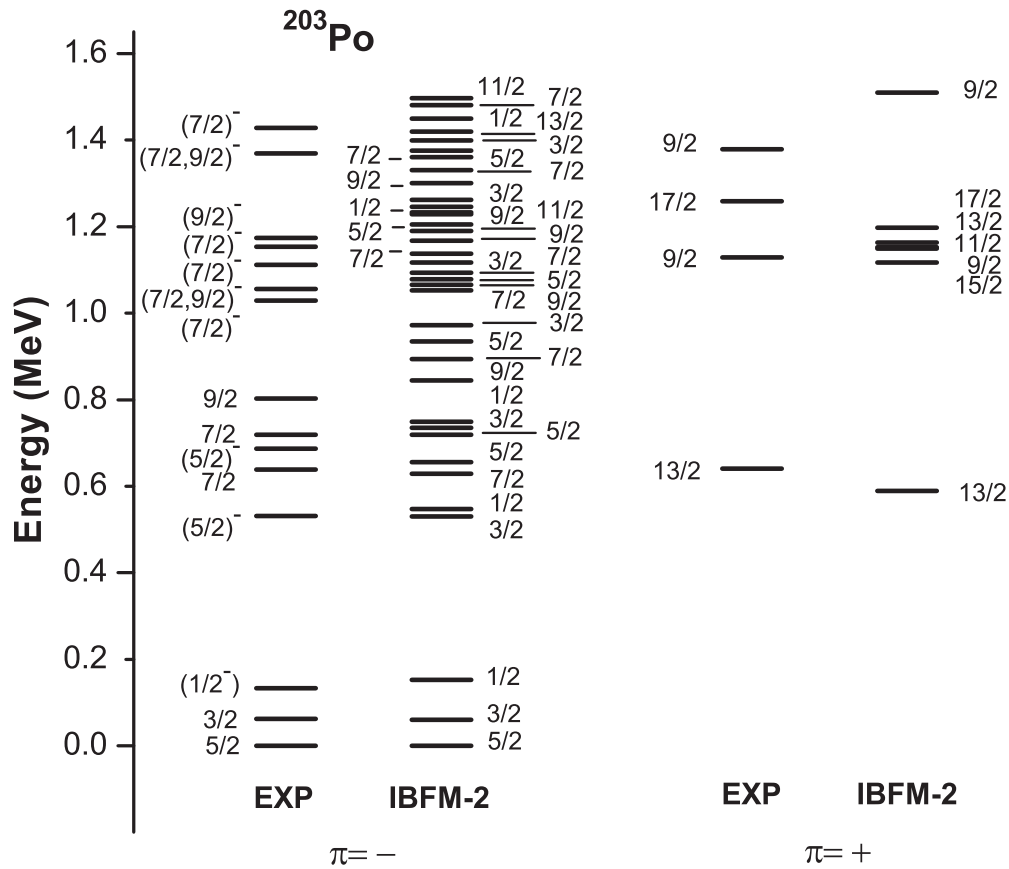


FIG. 2. Comparison of calculated negative- and positive-parity states of ^{203}Po isotope with available experimental data [16]; the experimental states with uncertain spin and parity are plotted with parentheses.

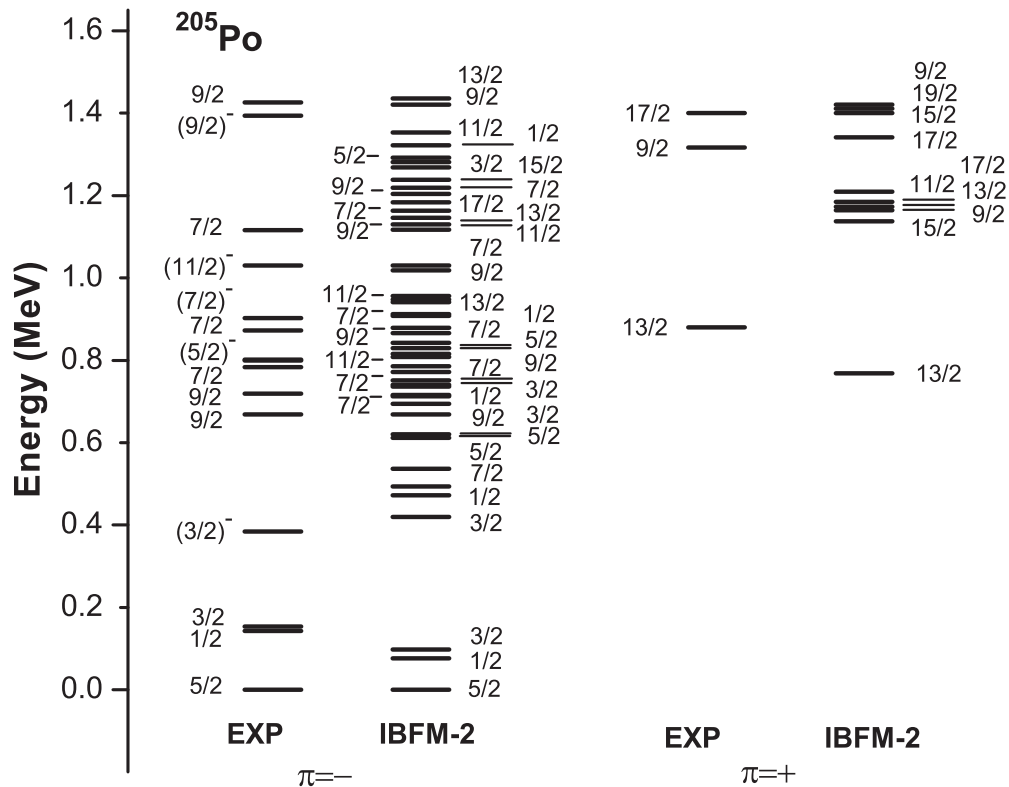


FIG. 3. . Same as Fig. 2 but for ^{205}Po isotope.

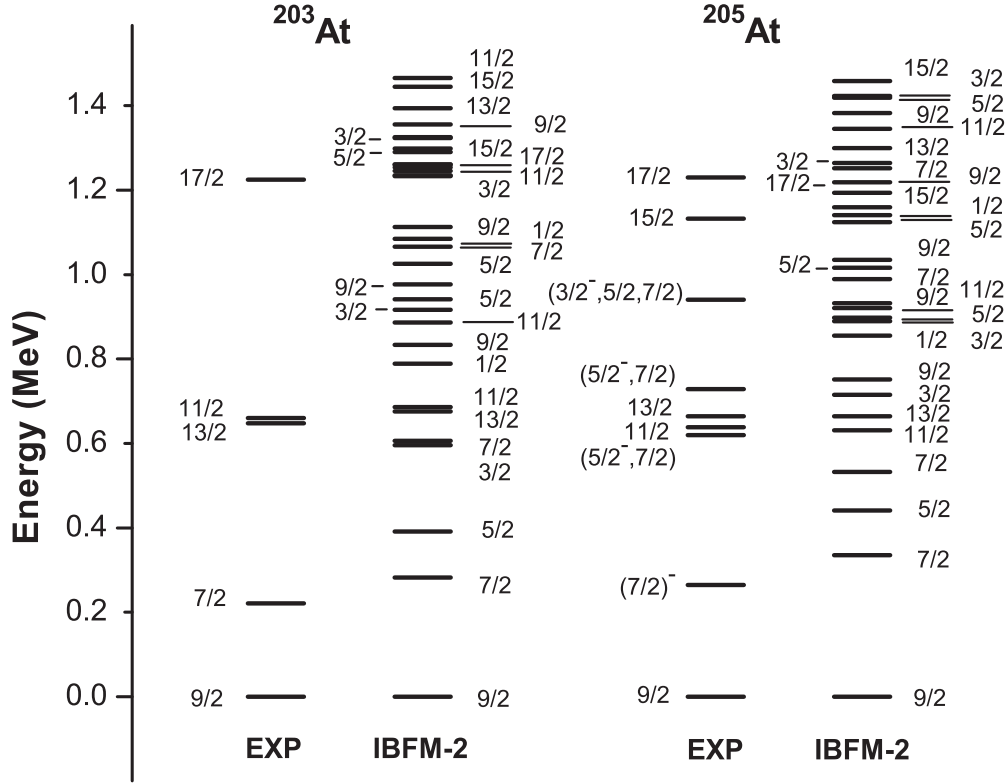


FIG. 4. Comparison of calculated negative-parity states of $^{203,205}\text{At}$ isotopes with available experimental data [16]; the experimental states with uncertain spin and parity are plotted with parentheses.

the lowest positive state; its energy equals 0.641 and 0.589 in the experimental and model results, respectively. The second state is $15/2^+$ at 1.117 MeV, which is not yet seen in the experiment. The next excited positive-parity state $J^\pi = 9/2^+$ at 1.258 MeV is coming from $89\%i_{13/2}$ and gets close to the experimental one at 1.129 MeV. The model calculation predicts the $9/2_2^+$ state at 1.513 MeV, while the observed one is at 1.379 MeV.

The sequence of the $3/2_{1,2}^-$ states of ^{205}Po isotope is achieved. The $3/2_2^-$ state at 0.420 MeV can be interpreted as the $f_{5/2}$ single particle coupled with one phonon 2_1^+ state. It is close to the uncertain spin ($3/2_2^-$) at 0.384 MeV. The $7/2_1^-$ and $9/2_1^-$ state components of the $\nu f_{5/2}$ appeared at 0.494 and 0.694 MeV, respectively, while the $9/2_1^-$ is observed at 0.669 MeV. The calculated $7/2_2^-$ (0.716 MeV) state is close to the experimental one at 0.873 MeV. Experimentally, the $13/2^+$ (0.880), $9/2^+$ (1.400), and $17/2^+$ (1.516 MeV) were known to be the lowest states [16]. The model calculation predicts $13/2_1^+$, $15/2_1^+$, $9/2_1^+$ and $17/2_1^+$ states at 0.768, 1.137, 1.164, 1.184 MeV, respectively.

For $^{203,205}\text{At}$ isotopes, the main advantage of the energy scheme is that all available experimental states are produced well. Only five observed negative-parity states of ^{203}At up to 1.5 MeV are plotted in Fig. 4. The $9/2_{gs}^-$ state has a structure dominated by the $h_{9/2}$ single-particle orbital (approximately $93\%h_{9/2} + 5\%f_{5/2}$). The IBFM calculation reproduced successfully the experimental order inversion of $J = 11/2^- - 13/2^-$. According to the model and experimental results, their

energies are equal to (0.676, 0.686) and (0.648, 0.660) MeV, respectively. The two states are calculated with the structure $90\%h_{9/2} + 7\%f_{5/2}$ and $95\%h_{9/2} + 3\%f_{5/2}$, respectively. The calculated energy of the $17/2_1^-(90\%h_{9/2})$ state is 1.246 MeV, which is close to the experimental value of 1.225 MeV. The experimental results showed that all states $J^+ < 13/2^+$ below 1.5 MeV are not generated from the $1i_{13/2}$ single-particle orbit [16]. So these states are out of the present model space.

In ^{205}At isotope, the first excited state has $J^\pi = (7/2)^-$ assignments, and it is close to calculated $7/2_1^-$ state at 0.334 MeV. It is coming from $80\%f_{7/2} + 17\%f_{7/2}$. The state $(5/2^-, 7/2)$ is observed at 0.620 MeV, whereas theoretically only the $5/2_1^-$ (0.441) and $7/2_2^-$ (0.533 MeV) are found in this energy region. Here, the order of $J = 11/2^- - 13/2^-$ is achieved in the model. The calculated energies of the $15/2_1^-$ and $17/2_1^-$ states are in good agreement with the experimental ones as shown in Fig. 3. Further, the main component in their wave function is $\pi h_{9/2}$.

IV. ELECTROMAGNETIC TRANSITIONS

In the IBFM-2, the electric-quadrupole operation $E2$ and magnetic-dipole operator $M1$, which contain a boson part and fermion part, are expressed as [10]

$$T^{E2} = e_\pi^B Q_\pi^B + e_\nu^B Q_\nu^B + \sum_{i,j} e_{i,j}^F [a_i^\dagger \tilde{a}_j]^{(2)}, \quad (15)$$

TABLE III. A comparison between theoretical and available experimental data of $B(E2)$ (e^2b^2) for core nuclei. Available experimental value is given in the second row [16,19,20].

$J_i^+ \rightarrow J_f^+$	^{202}Po	^{204}Po	^{206}Po
$2_1 \rightarrow 0_1$	0.2231	0.0683	0.0302
	0.2238 $^{+682}_{-520}$	≥ 0.0620	
$4_1 \rightarrow 2_1$	0.3553	0.1016	0.0376
		0.0941 $^{+320}_{-192}$	0.0359(28)
$6_1 \rightarrow 4_1$	0.3974	0.1003	0.0286
$2_2 \rightarrow 2_1$	0.3557	0.1013	0.0376
$3_1 \rightarrow 2_2$	0.2843	0.0713	0.0201
$3_1 \rightarrow 4_1$	0.1139	0.0284	0.0080
$4_2 \rightarrow 4_1$	0.1896	0.0474	0.0134
$4_2 \rightarrow 2_2$	0.2083	0.0524	0.0148
$0_2 \rightarrow 2_1$	0.3514	0.1007	0.0394

where the quadrupole operator Q_ρ ($\rho = \pi, \nu$) is defined in Eq. (3).

$$e_{i,j}^F = -\frac{1}{\sqrt{5}}(u_i u_j - v_i v_j) \left\langle \ell_i, \frac{1}{2}, j_i \left\| r^2 Y^{(2)} \right\| \ell_j, \frac{1}{2}, j_j \right\rangle. \quad (16)$$

The $M1$ transition operator is

$$T^{M1} = \sqrt{\frac{3}{4\pi}} \left(g_\pi^B L_\pi^B + g_\nu^B L_\nu^B + \sum_{i,j} g_{i,j}^F [a_i^\dagger \tilde{a}_j]^{(1)} \right), \quad (17)$$

where L_ρ^B is the angular momentum operator

$$L_\rho^B = \sqrt{10} [d_\rho^\dagger \tilde{d}_\rho]^{(1)}, \quad (18)$$

and the coefficient

$$g_{i,j}^F = -\frac{1}{\sqrt{3}}(u_i u_j + v_i v_j) \left\langle \ell_i, \frac{1}{2}, j_i \left\| (g_\ell \ell + g_s s) \right\| \ell_j, \frac{1}{2}, j_j \right\rangle. \quad (19)$$

In the present calculation, the adopted values of the proton and neutron effective boson charge in the $E2$ operator are adjusted to fit the available experimental data, where $e_\pi = e_\nu = 0.21, 0.13$, and 0.10 eb for $^{202-206}\text{Po}$ isotopes, respectively. The calculated and variable experimental data [16] are listed in Table III. The value of $B(E2; J \rightarrow J-2)$ in the ground-state band decreases with increasing mass number. This feature is found in the experimental value of $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 4_1^+ \rightarrow 2_1^+)$ [16].

The structure of the wave functions that are generated during the energy-level calculation is tested and the electromagnetic properties of the odd- A nuclei are calculated. The effective charges $e_\pi^F = 1.5$ e and $e_\nu^F = 0.5$ e are used. For gyromagnetic ratios, the fermion $g_{l,\pi}^F = 1$ μ_N and $g_{l,\nu}^F = 0$ μ_N are used. The g_s factors lie $g_s^F/g_s^{\text{free}} \approx 0.4-0.7$ [21–26], whereas $g_{s,\pi}^F = 3.91$ μ_N and $g_{s,\nu}^F = -1.91$ μ_N are taken. The magnetic moments of the $^{203,205}\text{Po}$ ground state are calculated to be $\mu(5/2_{gs}^-) = 0.7801$ μ_N and 0.7802 μ_N , respectively, so they are in good agreement with the experimental values of $0.74(5)$ μ_N and $0.76(6)$ μ_N , respectively [16]. The ground-state electric-quadrupole moment is found to be 0.0473 eb and

0.1182 eb compared with $0.17(10)$ eb and 0.17 eb of the experimental data [16] for ^{203}Po and ^{205}Po isotopes, respectively.

For $E2$ transitions, the boson-transition collective part is dominant. Hence, the $J \rightarrow J \pm 2$ transition reflects the quadrupole nature of the boson. Accordingly, the ratio $B(E2; 9/2_1^- \rightarrow 5/2_1^-)/B(E2; 9/2_1^- \rightarrow 5/2_2^-) = 632$, where $9/2_1^-$ and $5/2_1^-$ are dominated by $f_{5/2}$ single particle orbital. The largest calculated $B(M1)$ value is found for the $1/2_1^- \rightarrow 3/2_1^-$ transition and is equal to 0.5890 μ_N^2 and 0.6558 μ_N^2 for $^{203,205}\text{Po}$ isotopes, respectively. These values lead to the single-particle states $p_{1/2}$ and $p_{3/2}$, being the initial and final states, respectively. The experimental [16] and calculated branching ratios presented in Table IV are in agreement, even in the few transitions where the strongest branch is inaccurately predicted. For ^{203}Po isotope, the decay of the 0.639 MeV, $7/2_1^-$ level is very well produced. The decay of $7/2_1^-$ state at 0.783 in ^{205}Po requires an additional clarification regarding the strongest calculated branch in comparison to the experiment. The decay to the $5/2_{gs}^-$ is stronger than the other calculated transitions, but no experimental data is available for this transition.

V. β DECAY

Using the calculated wave function in the boson model, the β decay were investigated in wide regions [27–39]. As is known, the ft values depend on the nuclear structure of the parent and daughter nuclei. Here, the model wave functions obtained are reliable. The decay operators used were defined previously [29–31] as follows:

$$A_m^{(j)} = \zeta_j a_{jm}^\dagger + \sum_j \zeta_{jj} s^\dagger [\tilde{d} a_j^\dagger]_m^{(j)} \times (\Delta n_j = 1, \Delta N = 0), \quad (20)$$

$$B_m^{(j)} = \Theta_j s^\dagger \tilde{a}_{jm} + \sum_j \Theta_{jj} s [\tilde{d} \tilde{a}_j]_m^{(j)} \times (\Delta n_j = -1, \Delta N = 1). \quad (21)$$

The former creates a fermion, while the latter annihilates a fermion simultaneously creating a boson. Either operator increases the quantity $n_j + 2N$ by one unit. The conjugate operators are

$$\tilde{A}_m^{(j)} = (-1)^{j-m} \{A_{-m}^{(j)}\}^\dagger = \zeta_j^* \tilde{a}_{jm} + \sum_j \zeta_{jj}^* s [d^\dagger \tilde{a}_j]_m^{(j)} \times (\Delta n_j = -1, \Delta N = 0), \quad (22)$$

$$\tilde{B}_m^{(j)} = (-1)^{j-m} \{B_{-m}^{(j)}\}^\dagger = -\Theta_j^* s a_{jm}^\dagger - \sum_j \Theta_{jj}^* s [\tilde{d} a_j^\dagger]_m^{(j)} \times (\Delta n_j = 1, \Delta N = -1). \quad (23)$$

The Fermi and the Gamow-Teller transition operators between the initial J_i state and the final J_f state are

$$Q^F = \sum_j -\sqrt{2j+1} [P_\pi^{(j)} P_\nu^{(j)}]^{(0)}, \quad (24)$$

$$Q^{GT} = \sum_{jj} \eta_{jj} [P_\pi^{(j)} P_\nu^{(j)}]^{(1)}, \quad (25)$$

TABLE IV. Theoretical and experimental [16] branching ratios I_γ in $^{203,205}\text{Po}$ isotopes, the energy levels are in MeV unit.

^{203}Po						^{205}Po					
E_i	J_i^π	E_f	J_f^π	IBFM-2	EXP	E_i	J_i^π	E_f	J_f^π	IBFM-2	EXP
0.062	$3/2^-$	0.000	$5/2^-$	100	100	0.143	$1/2^-$	0.000	$5/2^-$	100	100(7)
0.133	$1/2^-$	0.000	$5/2^-$	100		0.154	$3/2^-$	0.143	$1/2^-$	30.5	0.42(3)
		0.062	$3/2^-$	0.1				0.000	$5/2^-$	100	100(8)
0.531	$5/2^-$	0.133	$1/2^-$	1.5		0.384	$3/2^-$	0.154	$3/2^-$	2.6	25.4(23)
		0.062	$3/2^-$	100	15.2(13)			0.000	$5/2^-$	100	100(5)
		0.000	$5/2^-$	81.5	100(3)	0.669	$9/2^-$	0.000	$5/2^-$	100	100
0.639	$7/2^-$	0.531	$5/2^-$	0.9		0.719	$9/2^-$	0.000	$5/2^-$	100	100
		0.133	$3/2^-$	6.1	9.80(20)	0.783	$7/2^-$	0.669	$9/2^-$	0.00	≈ 0.55
		0.000	$5/2^-$	100	100			0.154	$3/2^-$	51.7	100(7)
0.686	$5/2^-$	0.531	$5/2^-$	15.9				0.000	$5/2^-$	100	
		0.133	$3/2^-$	100	100(4)	0.799	$5/2^-$	0.384	$3/2^-$	41.4	89(10)
		0.000	$5/2^-$	60.2	60(2)			0.154	$3/2^-$	10.6	100(7)
0.719	$7/2^-$	0.686	$5/2^-$	16.0				0.000	$5/2^-$	100	
		0.639	$7/2^-$	0.7		0.872	$7/2^-$	0.669	$9/2^-$	11.9	10.5(13)
		0.133	$3/2^-$	100	100(1)			0.384	$3/2^-$	0.6	17.7(9)
		0.000	$5/2^-$	36.1	51.9(15)			0.000	$5/2^-$	100	100(15)
0.803	$9/2^-$	0.639	$7/2^-$	0.3		0.902	$7/2^-$	0.154	$3/2^-$	13.9	67(9)
		0.000	$5/2^-$	100	100			0.000	$5/2^-$	100	100(5)

where

$$\eta_{jj} = -\frac{1}{\sqrt{3}} \left\langle \ell, \frac{1}{2}, j \left\| \sigma \left\| \ell, \frac{1}{2}, j \right\rangle \right. \right. \\ = -\delta_{ll} \sqrt{2(2j+1)(2j+1)} W \left(\ell j \frac{1}{2} 1; \frac{1}{2} j \right). \quad (26)$$

The operator P_ρ^j form depends on the parent and daughter nuclei as in Eqs. (20)–(23). The ft value is calculated

$$ft = \frac{6163}{M_F^2 + (G_A/G_V)^2 M_{GT}^2} \quad (27)$$

in units of seconds where $(G_A/G_V)^2 = 1.59$ and the matrix elements

$$M_F^2 = \frac{1}{2J_i + 1} |\langle I_f \| Q^F \| I_i \rangle|^2, \quad (28)$$

$$M_{GT}^2 = \frac{1}{2J_i + 1} |\langle I_f \| Q^{GT} \| I_i \rangle|^2. \quad (29)$$

According to the model calculation, each of the parent-nuclei ground state $J^\pi = 9/2^-$ decays to daughter states $J^\pi = 7/2^-, 9/2^-$ and $11/2^-$ as clear in Table V. At the beginning of the discussion, there are no adjustable parameters in calculating β decay. On the other side, the quadrupole and exchange interactions have approximately the same order in the calculations of the parent and daughter nuclei. Therefore, the wave functions, especially for the low-lying states, are not very mixed. As mentioned above, the ground state of ^{203}At is $h_{9/2}$ single-particle orbital, while the mixture of $h_{9/2}$ (1–5 %) in all $J = 7/2^-, 9/2^-$, and $11/2^-$ states up to 1.5 MeV, except the $9/2^-$ state, which has 79% $h_{9/2}$ for daughter states. The excellent reproduction of $7/2^-$ ($i = 1-7$)'s excitation energy as shown in Table V gives a good wave function. The calculated $\log ft$

values are in good agreement with experimental data [16]. For example, the theoretical $\log ft$ values for $9/2^- \rightarrow 7/2^-$ and $9/2^- \rightarrow 7/2^-$ are 6.584 and 7.712, that is, they are close to the experimental ones, which are 6.82 and 7.38 [16], respectively. The calculated Gamow-Teller (GT) matrix elements show that maximum values $|M^{GT}|^2 = 1.011 \times 10^{-3}$ and minimum $|M^{GT}|^2 = 7.530 \times 10^{-5}$ for $7/2^-$ and $7/2^-$ states, respectively. These values reflect the dominance of the 88% $f_{5/2}$ orbital in the wave function of the $7/2^-$ state, while the largest mixing ratio (64% $p_{3/2} + 25\% f_{5/2}$) is present in the wave function of the $7/2^-$ state.

For the $9/2^- \rightarrow 9/2^-$ decay, the $|M^F| > |M^{GT}|$ except for the first transition. As shown in Table V, the calculated $\log ft(9/2^- \rightarrow 9/2^-)$ is equal to 5.033 compared to the experimental one of 7.41, where $|M^F|^2 = 3.444 \times 10^{-2}$ and $|M^{GT}|^2 = 3.590 \times 10^{-2}$. The decrease in $\log ft$ values for the decay to $9/2^-$ follows a development of its fermion contributions to the wave functions, which are approximately dominating $h_{9/2}$ in both the initial and final states, i.e., $h_{9/2} \rightarrow h_{9/2}$, which is ℓ allowed. The predicted 1.205 MeV and 1.470 MeV ($11/2^-$) states have dominant components of (83% $f_{5/2} + 13\% p_{3/2}$) and (20% $f_{5/2} + 64\% p_{3/2}$); the calculated $\log ft$ values of the two branches are 7.691 and 6.473, respectively.

It is interesting to note that the model performs well and reproduces the available experimental data for the $^{205}\text{At} \rightarrow ^{205}\text{Po}$ transitions, or at least their overall trend. Examining the wave functions of the ^{205}At states reveals that $h_{9/2}$ is dominant in the ground state. For the daughter nucleus, the first three $J^\pi = 7/2^-$ states are predicted to be below 0.75 MeV, with $f_{5/2}$ percent contribution of 88, 71, and 35, respectively. It is found that the $|M^{GT}|^2 = 3.855 \times 10^{-4}$ and 1.218×10^{-6} transition are of the $J^\pi = 7/2^-$ and $J^\pi = 7/2^-$ states, respectively. The known $9/2^-$ most heavily feeds two $7/2^-$ states at 0.783 and 0.872 MeV in ^{205}Po . The predicted $\log ft$ values are

TABLE V. Comparison of the IBFM-2 predicted and available experimental data [16] of $\log ft$ values in the decays $^A\text{At} \rightarrow ^A\text{Po}$.

^{203}At J_p^-	^{203}Po J_d^-	Energy EXP	IBFM-2	$\log ft$ EXP	IBFM-2	^{205}At J_p^-	^{205}Po J_d^-	Energy EXP	IBFM-2	$\log ft$ EXP	IBFM-2
9/2 ⁻	7/2 ⁻	0.639	0.629	6.82	6.584	9/2 ⁻	7/2 ⁻		0.494		7.002
		0.719	0.894	6.88	6.867				0.716		9.407
		1.029	1.053	7.36	7.048				0.749		9.503
		1.112	1.110	7.30	6.930			0.783	0.786	7.23	7.778
		1.153	1.190	7.38	7.712			0.872	0.879	7.45	7.930
		1.369	1.321	7.68	7.293			0.902	0.903	8.30	7.861
		1.428	1.360	7.23	7.450			1.167	1.164	7.55	8.353
	9/2 ⁻	0.803	0.845	7.46	6.369		9/2 ⁻	0.669	0.694	7.31	5.603
		1.056	1.061	7.41	5.033			0.719	0.741	7.19	6.365
		1.174	1.163	7.40	6.855				0.808		6.943
			1.233		7.204				0.878		6.701
	11/2 ⁻		1.389		6.627		11/2 ⁻		1.018		6.398
			1.205		7.691				0.776		8.034
			1.470		6.473			1.030	0.956	7.22	7.273

7.778 and 7.930, whereas the observed values are 7.23 and 7.45 [16], respectively. The calculated $|M^F|^2 = 3.588 \times 10^{-5}$ and $|M^F|^2 = 2.065 \times 10^{-4}$ for $11/2_{1,2}^-$ states. The $11/2_2^-$ state appeared at 1.030 MeV with $\log ft$ smaller than one of $11/2_1^-$. The $\log ft(9/2_1^- \rightarrow 11/2_2^-)$ is equal to 7.22 and 7.273 in the experiments [16] and model results, respectively.

VI. CONCLUSION

We have investigated the nuclear structure of $^{203,205}\text{At}$ and $^{203,205}\text{Po}$ isotopes in the interacting boson fermion model. All the ground-state angular momentum and parity are reproduced. It is found that the low-lying energy levels in these nuclei are consistent with experimental ones. In most energy levels, the root-mean-square deviations are very small with $\sigma = [\frac{1}{N} \sum (E_{\text{Cal}} - E_{\text{Exp}})^2]^{1/2}$; for example, it is $\sigma(J = 7/2_{i=1-7}^-) = 0.075$ for ^{203}Po isotope. Therefore, the used values of the parameters and single-particle orbital energy are good. The $(1/2^-)$ state at 0.133 MeV in ^{203}Po and $(3/2^-)$ state 0.384 MeV in the ^{205}Po are built on the $p_{1/2}$ and $f_{5/2}$ single-particle orbits, respectively. On the other hand, it is not

tempting to assign the $J^\pi = (5/2)^-$ state at 0.531 MeV in the ^{203}Po isotope as a $5/2^-$ state, where the first theoretical $J^\pi = 5/2^-$ appears at 0.656 MeV and close to the experimental value at 0.686 MeV [16]. Concurrently, the $3/2_2^-$ and $1/2_2^-$ states appeared at 0.530 and 0.548 MeV, respectively. The observed set of $\log ft$ values, as well as their dependence on the mass numbers, are well calculated in this study, except for β decays into $9/2^-$ states, where there is some discrepancy between the experimental and theoretical $\log ft$ values. This discrepancy indicates that the wave function of the $9/2^-$ states of the Po isotopes have significant contributions outside the boson-fermion space.

ACKNOWLEDGMENT

The authors would like to thank Professor N. Yoshida for his interest in the subject and his many helpful suggestions.

DATA AVAILABILITY

The data that support the findings of this article are openly available [15,16].

- [1] A. M. Oros, K. Heyde, C. De Coster, B. Decroix, R. Wyss, B. R. Barrett, and P. Navratil, *Nucl. Phys. A* **645**, 107 (1999).
- [2] R. Julin, K. Helariutta, and M. Muikku, *J. Phys. G: Nucl. Part. Phys.* **27**, R109 (2001).
- [3] E. Caurier, M. Rejmund, and H. Grawe, *Phys. Rev. C* **67**, 054310 (2003).
- [4] K. Higashiyama and N. Yoshinaga, *EPJ Web Conf.* **66**, 02050 (2014).
- [5] N. Kesteloot *et al.*, *Phys. Rev. C* **92**, 054301 (2015).
- [6] E. Teruya, K. Higashiyama, and N. Yoshinaga, *Phys. Rev. C* **93**, 064327 (2016).
- [7] V. Karayonchev *et al.*, *Phys. Rev. C* **103**, 044309 (2021).
- [8] D. Kalaydjieva *et al.*, *Phys. Rev. C* **104**, 024311 (2021).
- [9] S. Shukla, P. C. Srivastava, K. Nomura, and L. Zamick, *Nucl. Phys. A* **1044**, 122842 (2024).
- [10] F. Iachello and P. Van Isacker, *The interacting boson-fermion model* (Cambridge University Press, Cambridge, (1991).
- [11] F. H. Al-Khudair and G. L. Long, *Commun. Theor. Phys.*, **37**, 699 (2002).
- [12] B. Decroix, C. De Coster, K. Heyde, A. M. Oros, and J. De Beule, *Phys. Rev. C* **58**, 232 (1998).
- [13] J. Bardeen, N. L. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
- [14] O. Scholten, The interacting boson approximation model and applications, Ph.D. thesis, University of Groningen (1980).

- [15] T. Otsuka and N. Yoshida, Program NPBOS, Japan Atomic Energy Research Institute Report JAERI-M85-094 (1985).
- [16] ENSDF, Evaluated Nuclear Structure Data File, <http://www.nndc.bnl.gov/ensdf> (2025).
- [17] B. S. Reehal and R. A. Sorensen, *Phys. Rev. C* **2**, 819 (1970).
- [18] S. Chatterjee, B. Mondal, A. Ghosh, D. Arora, S. Das, S. Samanta, R. Raut, S. S. Ghugre, P. C. Srivastava, A. K. Sinha, U. Garg, H. K. Singh, Neelam, K. Rojeeta Devi, A. Sharma, S. S. Bhattacharjee, R. Garg, I. Bala, R. P. Singh *et al.*, *Phys. Rev. C* **106**, 044329 (2022).
- [19] B. Pritychenko, M. Birch, B. Singh, and M. Horoi, *At. Data Nucl. Data Tables* **107**, 1 (2016).
- [20] M. Stoyanova *et al.*, *Phys. Rev. C* **100**, 064304 (2019).
- [21] J. M. Arias, C. E. Alonso, and R. Bijker, *Nucl. Phys. A* **445**, 333 (1985).
- [22] C. E. Alonso, J. M. Arias, and M. Lozano, *J. Phys. G: Nucl. Phys.* **13**, 1269 (1987).
- [23] J. Rikowska, N. J. Stone, P. M. Walker, and W. B. Walters, *Nucl. Phys. A* **505**, 145 (1989).
- [24] A. Bohr and B. R. Mottelson, *Nuclear Structure* (World Scientific, Singapore, 1998).
- [25] A. Gizon *et al.*, *Nucl. Phys. A* **605**, 301 (1996).
- [26] F. H. Al-Khudair, *Phys. Rev. C* **91**, 054304 (2015).
- [27] F. Dellagiacoma and F. Iachello, *Phys. Lett. B* **218**, 399 (1989).
- [28] A. K. Singh and G. Gangopadhyay, *Phys. Rev. C* **55**, 2734 (1997).
- [29] N. Yoshida, L. Zuffi, and S. Brant, *Phys. Rev. C* **66**, 014306 (2002).
- [30] L. Zuffi, S. Brant, and N. Yoshida, *Phys. Rev. C* **68**, 034308 (2003).
- [31] S. Brant, N. Yoshida, and L. Zuffi, *Phys. Rev. C* **70**, 054301 (2004).
- [32] F. H. Al-Khudair, G. L. Long, and Y. Sun, *Phys. Rev. C* **77**, 034303 (2008).
- [33] F. H. Al-Khudair, *Phys. Rev. C* **80**, 014306 (2009).
- [34] N. Yoshida and F. Iachello, *Prog. Theor. Exp. Phys.* **2013**, 043D01 (2013).
- [35] N. Yoshida, *J. Phys.: Conf. Ser.* **533**, 012038 (2014).
- [36] E. Mardones, J. Barea, C. E. Alonso, and J. M. Arias, *Phys. Rev. C* **93**, 034332 (2016).
- [37] J. Ferretti, J. Kotila, R. I. Magaña Vsevolodovna, and E. Santopinto, *Phys. Rev. C* **102**, 054329 (2020).
- [38] K. Nomura, *Phys. Rev. C* **109**, 034319 (2024).
- [39] M. Homma and K. Nomura, *Phys. Rev. C* **110**, 014303 (2024).