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The Generalized q-Operator ${}_r\Phi_s$ and its Applications in q-Integrals

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ABSTRACT: In this paper, we use the generalzed q-operator ${}_r\Phi_s$ to generalize some well-known q-integrals such as Askey-beta Integral, Anderws-Askey Integral, Gasper integral and Askey-Roy integral.

Keywords: The q-operator ${}_r\Phi_s$, Askey-beta integral, Gasper integral, Andrews-Askey integral.

Contents

1. Defintions and Basic Results

We'll follow the notations that used in [17] with assuming that $|q| < 1$.
Let a be a complex variable. The q-shifted factorial is defined by [1,2,14]

$$(a;q)_0 = 1, \qquad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), \qquad (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k),$$

and adopting the following compact notation for the multiple q-shifted factorial [9,30,31]:

$$(a_1,\ldots,a_r;q)_n = (a_1;q)_n \ldots (a_r;q)_n,$$

where n is an integer or ∞.

The basic hypergeometric series ${}_r\phi_s$ is defined by [17,18,34]:

$$\begin{aligned} {}_r\phi_s(a_1,\ldots,a_r;b_1,\ldots,b_s;q,x) &= {}_r\phi_s\left(\begin{matrix} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{matrix};q,x\right) \\ &= \sum_{k=0}^{\infty}\frac{(a_1;q)_k\cdots(a_r;q)_k}{(q;q)_k\,(b_1;q)_k\cdots(b_s;q)_k}\left[(-1)^k q^{\binom{k}{2}}\right]^{1+s-r} x^k, \end{aligned}$$

where $r,s\in\mathbb{N}$; $a_1,\ldots,a_r,b_1,\ldots,b_s\in\mathbb{C}$; and none of the denominator factors evaluate to zero. The above series is absolutely convergent for all $x\in\mathbb{C}$ if $r<s+1$, for $|x|<1$ if $r=s+1$ and for $x=0$ if $r>s+1$.

The most important case of the above series is when $r=s+1$ [6,7,8]

$$\begin{aligned} {}_{s+1}\phi_s(a_1,\ldots,a_{s+1};b_1,\ldots,b_s;q,x) &= {}_{s+1}\phi_s\left(\begin{matrix} a_1,\ldots,a_{s+1} \\ b_1,\ldots,b_s \end{matrix};q,x\right) \\ &= \sum_{k=0}^{\infty}\frac{(a_1;q)_k\ldots(a_{s+1};q)_k}{(q;q)_k\,(b_1;q)_k\cdots(b_s;q)_k}\,x^k, \quad |x|<1. \end{aligned}$$

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