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A q-DIFFERENCE EQUATION APPROACH FOR GENERALIZED AL-SALAM-CARLITZ POLYNOMIALS

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Abstract. In this paper, we focus on using the q-difference equation approach to derive generating function, another generating function, an extension of the generating function, the Rogers formula and its extension, Mehler's formula, and a Srivastava-Agarwal type generating function for the polynomials $h_n(x, y, a, b|q)$. In addition, we establish connections between q-difference equations and a transformation formula. By assigning specific values to parameters in the polynomials identities for $h_n(x, y, a, b|q)$, another generating function, an extension of the generating function, the Rogers formula and its extension, Mehler's formula, and a Srivastava-Agarwal type generating function for the Al-Salam-Carlitz polynomials $U_n(x, y, a; q)$ are determined. Additionally, connections between q-difference equations and a transformational identity for the polynomials $U_n(x, y, a; q)$ are established.

1 Introduction

In this paper, we employ the same concepts and terminology as in [12], and we assume that $|q| < 1$. The q-shifted factorial is defined as [10, 12]

$$(a;q)_k = \begin{cases} 1, & \text{if } k = 0, \\ (1-a)(1-aq)\cdots(1-aq^{k-1}), & \text{if } k = 1, 2, 3, \cdots. \end{cases}$$

We also define

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

The following notation will be used for the multiple q-shifted factorials:

$$(a_1, a_2, \ldots, a_m; q)_n = (a_1;q)_n(a_2;q)_n \cdots (a_m;q)_n.$$
$$(a_1, a_2, \ldots, a_m; q)_\infty = (a_1;q)_\infty(a_2;q)_\infty \cdots (a_m;q)_\infty.$$

The generalized basic hypergeometric series ${}_r\phi_s$ is defined by [1, 5, 22]

$$\begin{aligned} {}_r\phi_s(a_1, \ldots, a_r; b_1, \ldots, b_s; q, x) &= {}_r\phi_s\left(\begin{matrix} a_1, \ldots, a_r \\ b_1, \ldots, b_s \end{matrix}; q, x\right) \\ &= \sum_{k=0}^{\infty} \frac{(a_1;q)_k \cdots (a_r;q)_k}{(q;q)_k\,(b_1;q)_k \cdots (b_s;q)_k} \left[(-1)^k q^{\binom{k}{2}}\right]^{1+s-r} x^k, \end{aligned}$$