

THE HOMOGENEOUS q -SHIFT OPERATOR FOR POLYNOMIALS WITH DOUBLE q -BINOMIAL COEFFICIENTS

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ABSTRACT. Within this document, a homogeneous q -shift operator is constructed and we form a polynomial with double q -binomial coefficients $L_n^{(\tilde{j}, \tilde{k})}(\beta, u, m, o, r, s, p)$. The generating function, the Rogers type, Mehler's formula, and mixed generating functions and all of its extension are inferred for the polynomials $L_n^{(\tilde{j}, \tilde{k})}(\beta, u, m, o, r, s, p)$. When we take a few special values for a polynomials $L_n^{(\tilde{j}, \tilde{k})}(\beta, u, m, o, r, s, p)$ are provided for the purpose of find the polynomials' identities for $L_{(\tilde{j}, \tilde{k})}(\beta, u, o, r)$ and $L_n^{(\tilde{j}, \tilde{k})}(\beta, u, m, o, r)$. Also we develop the generating functions of a class mixed. Using generating functions for $L_n^{(\tilde{j}, \tilde{k})}(\beta, u, m, o, r, s, p)$, a transformation formula is derived.

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Key words and phrases. homogeneous q -shift operator; generating function; Rogers formula; Mehler's formula; double q -binomial coefficients; mixed generating function.

1. INTRODUCTION

This section, the q -series symbols and definitions used in [15] shall be adhered to. Our hypothesis is that $|q| < 1$.

The q -shifted factorial for $c \in \mathbb{C}$ is defined as [13–15].

$$(c; q)_0 = 1, \quad (c; q)_m = \prod_{k=0}^{m-1} (1 - cq^k), \quad (c; q)_\infty = \prod_{k=0}^{\infty} (1 - cq^k),$$

The following relation is true:

$$(c_1, c_2, \dots, c_r; q)_g = (c_1; q)_g (c_2; q)_g \cdots (c_r; q)_g,$$

where $g \in \mathbb{Z}$ or ∞ .