THE HOMOGENEOUS q-SHIFT OPERATOR FOR CIGLER'S POLYNOMIALS

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Abstract. We define a homogeneous q-shift operator$\mathcal{T}\left(q^{-\alpha};q,\rho D_{\chi\psi}\right)$. Subsequently, we utilise this operator to derive various polynomial q-identities, including the generating function, Rogers-type formula, as well as Mehler's formula along with it's extension and two mixed generating functions for Cigler's polynomials $\mathcal{C}_\nu^{(\alpha-\nu)}(\chi,\psi,\rho|q)$. Also, we obtain a transformational identity involving generating functions of $\mathcal{C}_\nu^{(\alpha-\nu)}(\chi,\psi,\rho|q)$. We provide some special values for $\mathcal{C}_\nu^{(\alpha-\nu)}(\chi,\psi,\rho|q)$ identities in order to establish identities for the bivariate Rogers-Szegö polynomials $h_n(x,y|q)$ and Al-Salam-Carlitz polynomials $U_n(x,y,a|q)$.

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Key words and phrases. q-shift operator; Cigler's polynomials; bivariate Rogers-Szegö polynomials; generating function; Al-Salam-Carlitz polynomials; Rogers formula; Mehler's formula.

1.. Introduction

This study will adhere to standard notations and definitions regarding the q-series utilised in [14]. Note that $0<|q|<1$. Let $b\in\mathbb{C}$, the q-shifted factorial is defined as: [10,11,14]

$$(b;q)_0=1,\quad (b;q)_n=(1-b)(1-bq)...(1-bq^{n-1}),\quad (b;q)_\infty=\prod_{k=0}^{\infty}(1-bq^k)$$

the multiple q-shifted factorials, along with:

$$(b_1,b_2,\ldots,b_r;q)_n=(b_1;q)_n(b_2;q)_n\cdots(b_r;q)_n,$$

where $n\in\mathbb{Z}$ or ∞.

The basic hypergeometric series ${}_r\phi_s$ is presented as follows [14]:

$${}_r\phi_k\left(\begin{array}{c}b_1,\ldots,b_r\\ c_1,\ldots,c_k\end{array};q,x\right)=\sum_{n=0}^{\infty}\frac{(b_1,\ldots,b_r;q)_n}{(q,c_1,\ldots,c_k;q)_n}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+k-r}x^n,$$

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