Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–23.
ISSN-0037-8712
doi:10.5269/bspm.77414

Generalized q-Difference Equation for Generalized Cigler's Polynomials

Samaher A. Abdul-Ghani and Husam L. Saad*

ABSTRACT: This work establishes the generalised Cigler's polynomials and the generalised homogeneous q-shift operator. The q-difference equation is then utilized to demonstrate numerous polynomial q-identities, including the generating function and its extension, Rogers' formula and its extension, and Mehler's formula and its extension for the generalized Cigler's polynomials. Also presented a transformational identity involving generating functions for generalised Cigler's polynomials.

Key Words: Cigler's polynomials, homogeneous q-shift operator, q-difference equation, generating function, Mehler's formula, Rogers' formula, transformational identity.

Contents

1. Introduction

The notations and definitions used in [15] are followed here. It is assumed that $0<|q|<1$.
The q-shifted factorial is defined for $a\in\mathbb{C}$ as [13,14,15]:

$$(a;q)_0=1,\quad (a;q)_n=\prod_{k=0}^{n-1}(1-aq^k),\quad (a;q)_\infty=\prod_{k=0}^{\infty}(1-aq^k).$$

The multiple q-shifted factorials is given by [15]:

$$(a_1,a_2,\ldots,a_r;q)_m=(a_1;q)_m(a_2;q)_m\cdots(a_r;q)_m,$$

where $m\in\mathbb{Z}$ or ∞.

The basic hypergeometric series ${}_r\phi_s$ is presented as follows [15]:

$${}_r\phi_s\left(\begin{matrix}\alpha_1,\ldots,\alpha_r\\ \beta_1,\ldots,\beta_s\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(\alpha_1,\ldots,\alpha_r;q)_n}{(q,\beta_1,\ldots,\beta_s;q)_n}\left[(-1)^nq^{\binom{n}{2}}\right]^{1+s-r}x^n,$$

Note that

$${}_{r+1}\phi_r\left(\begin{matrix}\alpha_1,\ldots,\alpha_{r+1}\\ \beta_1,\ldots,\beta_r\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(\alpha_1,\ldots,\alpha_{r+1};q)_n}{(q,\beta_1,\ldots,\beta_r;q)_n}x^n,\quad |x|<1.$$

* Corresponding author
2010 *Mathematics Subject Classification:* 05A30, 33D45.
Submitted June 20, 2025. Published September 01, 2025