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Generalized Homogeneous q-Shift Operator Applied for Generalized Cigler's Polynomials

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ABSTRACT: In this paper, we establish the generalized homogeneous q-shift operator $E(q^{\alpha}z\theta_{xy})$ and the generalised Cigler's polynomials $\mathbb{D}_n^{(\alpha-n)}(\sigma_1,\cdots,\sigma_r,\rho_1,\cdots,\rho_s,x,y,z)$. Then, we apply this operator to derive some q-identities such as: the generating function and its extension, Rogers formula and its extension, Mehler's formula and its extension, Srivastava-Agarwal type bilinear generating functions to the polynomials $\mathbb{D}_n^{(\alpha-n)}(\sigma_1,\cdots,\sigma_r,\rho_1,\cdots,\rho_s,x,y,z)$. In addition, we supply some special values for the identities of $\mathbb{D}_n^{(\alpha-n)}(\sigma_1,\cdots,\sigma_r,\rho_1,\cdots,\rho_s,x,y,z)$ in order to establish the same identities for the polynomials $D_n^{(\alpha-n)}(x,y,b)$ and $\Psi_n^{(\mathbf{a},\mathbf{b})}(x,y,z|q)$.

Key Words: the homogeneous q-shift operator, Cigler's polynomials, generating function, Rogers formula, Mehler's formula, Srivastava-Agarwal type bilinear generating function.

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1. Introduction

In this paper, we will follow common notations and definitions for the q-series that used in [11]. We assume that $0<q<1$.

For $\delta\in\mathbb{C}$, the q-shifted factorial is defined by [9,10,11]

$$(\delta;q)_0=1,\quad (\delta;q)_n=\prod_{k=0}^{n-1}(1-\delta q^k),\quad (\delta;q)_\infty=\prod_{k=0}^{\infty}(1-\delta q^k),$$

and the multiple q-shifted factorials by:

$$(\delta_1,\delta_2,\ldots,\delta_r;q)_m=(\delta_1;q)_m(\delta_2;q)_m\cdots(\delta_r;q)_m,$$

where $m\in\mathbb{Z}$ or ∞.

The basic hypergeometric series ${}_r\phi_s$ is presented as follows [11,15]:

$${}_r\phi_s\left(\begin{matrix}a_1,\ldots,a_r\\ b_1,\ldots,b_s\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(a_1,\ldots,a_r;q)_n}{(q,b_1,\ldots,b_s;q)_n}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-r}x^n,$$

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