

# Relaxation and Thermodynamic Parameters of the Theory of Spin-Wave Damping in Low-Dimensional Magnetic Materials

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**Abstract**—Several basic parameters of the surface spectrum of a system are estimated with respect to the spin–spin interaction. An expression for the surface relaxation time and its numerical estimation are given. The dependence of the relaxation time on the geometric factor determined by the surface geometry only is presented. The conditions for the resonance absorption of spin-wave energy by the surface are considered. Certain expressions for the thermodynamic potentials taking the effect of surface resonance on the equilibrium properties of the system into account are given.

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## INTRODUCTION

One important direction in the physics of low-dimensional structures is the study of their magnetic properties [1]. Interest in the study of low-dimensional magnetism is due to the discovery of the extraordinary magnetic properties of high-temperature superconductors [2] and the possibility of varying the properties of nanomaterials and producing solid structures with controllable parameters based on them [3–9].

An important problem of low-dimensional magnetism (as, indeed, of the entire physics of low-dimensional structures) is the determination of relaxation and thermodynamic properties [7]. This is related to the fact that low-dimensional systems are noted for their significant nonuniformity in terms of the distribution of charge carriers. A nonuniform charge-density distribution is accompanied by that of spin density. As a consequence of this, surface effects in low-dimensional structures play a considerable role. So the damping of spin waves in a quasi-one-dimensional system with a step-like structure is studied in the recent paper [8]. It is shown that the spin–spin interaction becomes significant in low-dimensional structures, which leads to wave damping. In addition, the spin–phonon interaction [9] and the interaction of spins with surface plasmons [10] become significant in low-dimensional magnetic materials. It is shown, in our recent paper [11], that taking the local surface geometry into account in the framework of the classical theory of spin waves also leads to the damping of spin waves.

The aim of this paper is to estimate certain basic features of the spin spectrum obtained in [11].

## SURFACE SPIN SPECTRUM

In the framework of the classical theory of spin waves, the expression for the part of the exchange energy corresponding to the influence of the surface was obtained in [11]. This expression has the form

$$\Delta E_{s2} = -i \frac{\alpha^2 A}{I_0 d^2} \sum_k \left( \int \text{Im}(D_n^+(k, k)) dr \right) q_k^+ q_k + \frac{\alpha^2 A}{I_0 d^2} \sum_{k, k'} \left( \int [D_n(k, k) e_{kk'}^- + D_n^+(k, k) e_{kk'}^+] dr \right) q_k^+ q_{k'}, \quad (1)$$

where the functions  $D_n(k, k')$  and  $e_{k, k'}^\pm$  are defined as

$$D_n(k, k') = - \left( n_l (k^l - k'^l) \right) (n_j k^j) + i \left[ (\nabla^l n_l) (n_j k^j) + (\nabla^l n_j) (n_l k^l) \right],$$

$$D_n^+(k, k') = (D_n(k, k'))^+,$$

$$e_{k, k'}^- = \sum_{x_j, t} \exp \left\{ -i \left[ (\omega(k) - \omega(k')) t + (k^j - k'^j) x_j \right] \right\},$$

$$e_{kk'}^+ = (e_{kk'}^-)^+, \quad e_{kk}^+ = e_{kk}^- = 1.$$

Here,  $I_0$  is the spontaneous magnetization of a non-perturbed system,  $d$  is the average distance between neighboring crystal ions,  $A$  is the constant of exchange interaction between neighboring sites,  $\alpha = (2\mu I_0/V)^{1/2}$  is the normalization factor,  $\mu = g\hbar/2mc = g\mu_b$  is the  $z$  projection of the spin magnetic moment of the atomic magnetization carrier with the magnetomechanical ratio  $g$  and the spin quantum number  $s$  (strictly speaking, by  $\mu$ , the average magnetic moment

per lattice site should be understood), and  $l$  and  $j$  are the corresponding directions.

Applying the diagonalization procedure to the canonical form for the “surface” part of the exchange energy and using certain mathematical manipulations, we calculated the spectrum of elementary excitations of the system in the zeroth approximation with respect to the interaction of spin waves, which makes sense, strictly speaking, only in a rather low temperature range when the excitation density is small. Thus, we obtained relations completely determining the “surface” spectrum of the system in the considered zeroth approximation with respect to the spin–spin interaction [11]:

$$\begin{aligned} \text{Im}(\Delta E_{s_2}) &= \left( \frac{\alpha^2 AV}{I_0^2 d^2} \right) \sum \lambda_n(k, \bar{r}), \\ \Delta E_{s_2} &= i \text{Im}(\Delta E_{s_2}). \end{aligned} \quad (2)$$

The function  $\lambda_n(k, \bar{r})$  is defined as [9]

$$V\lambda_n(k, \bar{r}) = 2 \text{Im} \left( D_n^-(k, \bar{r}) \right) - \int \text{Im} \left( D_n^+(k, k') \right) dr.$$

As can be seen from (1), this function yields the dispersion law, which in the detailed form is represented by the following equation (for the choice of the normalization constant  $\alpha = (2\mu I_0/V)^{1/2}$ ):

$$B^{-1} \omega_{s_2}(k) = \sum_{m_1, m_2} |g_{m_1}^{jl}(k, k)| \delta^{ll'} \tilde{F} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (r) \quad (3)$$

with the “surface” functions

$$\begin{aligned} \tilde{F} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (r) &= 2\tilde{P} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (r) - \tilde{S} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (r), \\ \tilde{S} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (\bar{r}) &= \frac{1}{V} \int_V r_l^{m_1} r_j^{m_2} dr \end{aligned}$$

and the constant  $B = 2dA/\hbar$ , ( $l, j = 1, 2, 3$ ). We pass directly to the estimation of certain basic parameters of the system under study.

## ESTIMATION OF CERTAIN RELAXATION PARAMETERS

We estimate certain basic parameters of the spin spectrum (1). The total shift of the frequency spectrum of the spin waves (equal to  $\Delta\omega(k) = \Delta\omega_R(k) + i\Delta\Gamma(k)$ ) reflects only an imaginary shift, because  $\Delta\omega_R(k) = 0$  [11]. The imaginary component  $\Delta\Gamma(k)$  makes it possible to find the expression for the surface relaxation time  $\tau_s$ :

$$\tau_s = \frac{1}{B\lambda_n(k, \bar{r})}, \quad (4)$$

where  $\mathbf{r} = \bar{\mathbf{r}}$  is the characteristic vector function. We rewrite the second factor in the denominator (4) in the form [9]:

$$\begin{aligned} \lambda_n(k, \bar{r}) &= k^j \cdot G(\bar{r}) \\ &= \sum_{m, m_2} k^j (1+m)(1+\hat{\pi}(l, j)) \delta^{ll'} \\ &\quad \times (\beta_{m_1})_l (\beta_{m_2})_j \tilde{F} \left\{ \begin{matrix} m_1, m_2 \\ l, j \end{matrix} \right\} (\mathbf{r}), \end{aligned} \quad (5)$$

where the coefficients  $(\beta_{m_1})_l = (\beta_{m_1})_l(r_0)$  are understood in the vector sense as a projection of the given vector  $\boldsymbol{\beta}_m(\mathbf{r}_0)$  on the  $l$ th direction ( $l, j = 1, 2, 3$ ) and  $\boldsymbol{\beta}_m(\mathbf{r}_0)$  are the coefficients of expansion in Taylor power series of the function of surface normals [11]. Taking the last expression into account, we obtain the following estimation for the relaxation time:

$$\tau_s \sim \frac{\hbar}{6\pi AG}, \quad (6)$$

or, for ordinary ferromagnets, taking into account that  $A \approx k_B \Theta$ ,  $\Theta \sim 10^3$  K ( $\Theta$  is the Curie ferromagnet temperature), we have the following expression in the numerical form:

$$\tau_s \cong 4.04 \times 10^{-16} G^{-1} \text{ s}. \quad (7)$$

As can be seen directly from (7), the numerical value of the relaxation time depends parametrically on the geometric factor  $G$ , which is completely determined only by the local surface geometry by means of the coefficients  $\boldsymbol{\beta}$  and the function  $\tilde{F}(\mathbf{r})$ , as is easily seen from (5). Thus, in the given theory, the factor  $G$  is precisely the parameter that determines the numerical values of the considered physical quantities and establishes their dependence on the surface system geometry. Result (7) is a consequence of the fact that, as can be seen from (5), the factor  $G$  specifies the quasimomentum  $\lambda_n(k)$  acquired by the “bulk” spin wave with the initial quasimomentum  $k$  in the presence of the surface restriction of the ferromagnetic system with the linear dimensions  $\bar{\mathbf{r}}$ .

To clarify the peculiarities of the character of the kinetics of “surface” spin waves, it is necessary to compare the numerical estimation (7) with the characteristic frequencies of the initial spin spectrum. Estimating the upper limit of the frequency spin spectrum as  $\bar{\omega} \approx 5.2 \times 10^{12} \text{ s}^{-1}$ , we obtain  $\bar{\omega}\tau_s \approx 2.1 \times 10^{-3} G^{-1}$ . Consequently, for  $G \sim 1$  and  $\bar{\omega}\tau_s \ll 1$ , the influence of the surface on the spin spectrum also turns out to be very insignificant. However, in the range of values where  $G \sim 10^{-3}$  and  $\bar{\omega}\tau_s \sim 1$ , the calculation shows the resonance absorption of the spin-wave energy by the surface (the conditions under which this can occur are given below); in this case, the maximum of the absorbed energy is at the upper edge of the spin fre-

quencies. As the geometric factor  $G$  decreases further ( $G < 10^{-3}$ ), a shift of the resonance-absorption range beyond the edge of the frequency spectrum toward the range of lower spin frequencies must be observed. An energy gap with additional upper and lower frequencies  $\bar{\omega}_1$  and  $\underline{\omega}_1$  ( $\underline{\omega}_1 < \bar{\omega}_1 < \bar{\omega}$ ) can form under such conditions,  $\Delta\omega_1 = \bar{\omega}_1 - \underline{\omega}_1 \sim (\Delta\Gamma)^{-1}$ , it is caused by the direct interaction of the surface with the spin waves of the system. Here, under direct surface interaction, the energy-momentum exchange between spin waves of the system and the internal energy of the surface subsystem should, apparently, be understood; in turn, it occurs through the field interaction between magnons and lattice vibrations on the surface. The problem of the microscopic mechanism of such an interaction falls outside the scope of this paper and is of independent interest. However, we indicate that the initial point of the model microscopic theory is the expression

$$\Delta E_{\text{ex}} = \int \left\{ \frac{2A}{I_0^2 d} [(\nabla\gamma_x)^2 + (\nabla\gamma_y)^2] + \frac{h}{2I_0} (\gamma_x^2 + \gamma_y^2) \right\} d\mathbf{r} + \frac{A}{d} \oint \nabla_n j_z^2 d\sigma,$$

which is given after the transformation of the second terms of the sum in accordance with the Ostrogradskii–Gauss formula

$$\oint \nabla_n j^2 d\sigma \rightarrow \frac{1}{d} \int \text{div}(\nabla_n j^2 \mathbf{n}) d\mathbf{r}, \quad n_i = (\mathbf{n} \cdot \mathbf{e}) = \cos \alpha_i, \\ i = x, y, z$$

in the form of the space integral

$$\Delta E_{\text{ex}} = \int \left\{ \frac{2A}{I_0^2 d} [(\nabla\gamma_x)^2 + (\nabla\gamma_y)^2] + \frac{h}{2I_0} (\gamma_x^2 + \gamma_y^2) + \frac{A}{d^2} \left[ \nabla_x (\nabla_n j^2 \cos \alpha_x) + \nabla_y (\nabla_n j^2 \cos \alpha_y) + \nabla_z (\nabla_n j^2 \cos \alpha_z) \right] \right\} d\mathbf{r},$$

where we can pass to field operators later.

Under the condition  $G < 10^{-3}$ , the correlation between spin waves in the spatial distribution can be established because of the presence of the gap in the spectrum. In this case, the direct surface interaction leads to the indirect interaction of spin waves through virtual magnon exchanges of “surface” phonons, i.e., the consistent redistribution of surface and spin-wave energies. In the case of an energy gain for such exchanges, effective attraction leading to the formation of spin complexes can be observed between spin waves. The fact that the propagation rate of the spin wave in the  $j$ th direction (which is  $u_j = 2.7 \times 10^4 G_i$  m/s in accordance with estimates) decreases significantly to  $\sim 0.3 \times 10^2$  m/s or less in the resonance absorption range is in favor of the spatial localization of two cor-

relating spin waves (which, generally, requires consideration within the framework of microscopic theory). Undoubtedly, this leads to a significant increase in the density of spin excitations compared with that without the influence of the factor  $G$  at the same temperature; however, unlike the latter, this increase is not accompanied by a corresponding enhancement in the direct spin–spin interaction.

## THERMODYNAMIC CHARACTERISTICS

We illustrate the foregoing using some expressions for thermodynamic potentials showing the possible influence of surface resonance on the equilibrium properties of the system. For simplicity, we consider only the  $\delta$ -shaped surface resonance in the case of resonance broadening  $\Delta = \hbar\Delta\omega_1 \rightarrow 0$  in the frequency spectrum. In the case of no external magnetic field and negligibly small magnetic anisotropy where the average thermal energy is significantly larger than the system energy in the “Lorenz” field ( $k_B T \gg 2\pi g\mu_B I_0$ ), the thermodynamic potential of the ferromagnetic gas is

$$\Omega(T, 0; \delta) = \frac{V(k_B T)^{5/2}}{6\pi^2 (\hbar\xi)^{3/2}} \int_0^\infty \frac{x^{3/2} dx}{\exp(x - \delta) - 1}, \quad (8)$$

where  $\xi = 4Ag\mu_B/\hbar I_0 d$  and  $\delta = \Delta/k_B T$ .

Under the assumption of  $\delta$ -shaped broadening, we calculate the integral in (8) as follows. First, we rewrite the integral, transforming the integrand in the form

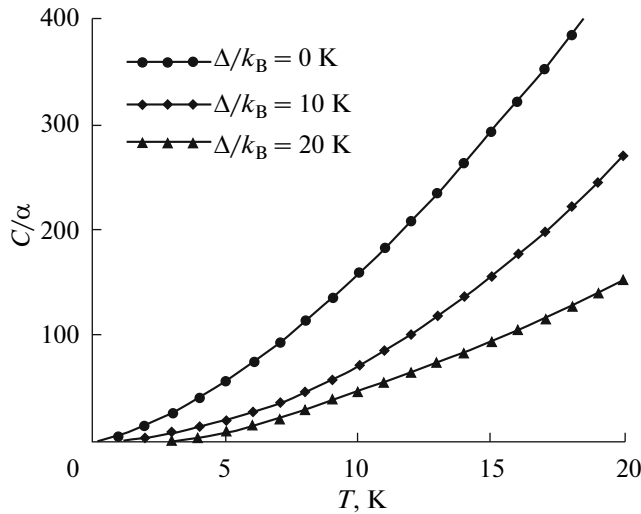
$$\int_0^\infty \frac{x^{3/2} dx}{\exp(x + \delta) - 1} = \int_0^\infty \frac{\exp(-x)}{\exp(\delta) - \exp(-x)} x^{\frac{5}{2}-1} dx.$$

The fraction in the integrand can be decomposed as follows:

$$\frac{\exp(-x)}{\exp(\delta) - \exp(-x)} = \exp(-(x + \delta)) + \exp(-2(x + \delta)) + \exp(-3(x + \delta)) + \dots$$

That is,

$$\int_0^\infty \frac{\exp(-x)}{\exp(\delta) - \exp(-x)} x^{\frac{5}{2}-1} dx \\ = \sum_{n=1}^\infty \int_0^\infty \exp(-n(x + \delta)) x^{\frac{5}{2}-1} dx \\ = \sum_{n=1}^\infty e^{-n\delta} \int_0^\infty \exp(-nx) x^{\frac{5}{2}-1} dx.$$



Temperature dependence of the heat capacity for different values of the energy gap  $\Delta$ .

If the  $\gamma$ -function is used, the last expression can be re-written as

$$\sum_{n=1}^{\infty} e^{-n\delta} \int_0^{\infty} \exp(-nx) x^{\frac{5}{2}-1} dx = \Gamma\left(\frac{5}{2}\right) \sum_{n=1}^{\infty} \frac{e^{-n\delta}}{n^{5/2}}.$$

Thus, for the sought integral, we obtain the following expression:

$$\int_0^{\infty} \frac{x^{3/2} dx}{\exp(x + \delta) - 1} = \Gamma\left(\frac{5}{2}\right) \sum_{n=1}^{\infty} \frac{e^{-n\delta}}{n^{5/2}}. \quad (9)$$

Substituting (9) into (8) and calculating the corresponding derivatives with respect to the temperature and the magnetic field in accordance with the well-known formulas

$$C_V(T, H) = -T \frac{\partial^2 \Omega}{\partial T^2}, \quad I(T, H) = -\frac{1}{V} \frac{\partial \Omega}{\partial H} + I_0$$

we find the required expressions for the heat capacity and the magnetization of the ferromagnetic sample in the case (that is most general here) where  $\Delta = \Delta(T, H)$ .

For the heat capacity, we find the expression

$$C_V(T, \Delta) = \frac{15}{4} \alpha T^{3/2} \sum_{n=1}^{\infty} \frac{e^{-n \frac{\Delta}{k_B T}}}{n^{5/2}} - 3\alpha T^{1/2} \frac{\Delta}{k_B} \sum_{n=1}^{\infty} \frac{n e^{-n \frac{\Delta}{k_B T}}}{n^{5/2}} + \alpha T^{-1/2} \frac{\Delta^2}{k_B^2} \sum_{n=1}^{\infty} \frac{n^2 e^{-n \frac{\Delta}{k_B T}}}{n^{5/2}}, \quad (10)$$

$$\text{where } \alpha = \frac{V k_B^{5/2}}{8(\pi \hbar \xi)^{3/2}}. \quad (11)$$

As can be seen from expression (10), the presence of the gap in the energy spectrum leads to deviation of the temperature dependence of the heat capacity from the Bloch three-halves power law. In the case where the gap  $\Delta = 0$ , taking into account that  $\hbar \xi = 2Ad^2 = k_B \Theta d^2$  and taking the value for the coefficient from (11), we obtain

$$C_V(T, 0) = \frac{15}{32} \frac{\zeta\left(\frac{5}{2}\right)}{\pi^{3/2}} \frac{V k_B}{d^3} \left(\frac{T}{\Theta}\right)^{3/2} = \alpha_1 T^{3/2}, \quad (12)$$

where  $\zeta\left(\frac{5}{2}\right)$  is the Riemann zeta function. Thus, for  $\Delta = 0$ , we return to the Bloch three-halves power law.

For clarity of the obtained result, we represent (10) graphically. Instead of the heat capacity  $C_V(T, \Delta)$ , we plot the reduced value  $\frac{C_V}{\alpha}$  of the heat capacity along the vertical axis. In this case, the temperature dependence of the reduced heat capacity is given for three different values of the gap  $\Delta$ .

As can be seen from the figure, the increase in the gap leads to the temperature dependence of the heat capacity having weakened. The curve corresponding to zero gap coincides exactly with that of the Bloch three-halves power law.

Similarly, the expression for spontaneous magnetization can also be obtained:

$$I_s(T) = I_0 (1 - b_1 T^{3/2} + b_2 (\delta) \Delta^{3/2}), \quad (13)$$

with the coefficients  $b_1$  and  $b_2$

$$b_1 = \frac{\zeta(3/2)}{8\pi^{3/2} \Theta^{3/2}}, \quad b_2 = \frac{\zeta(3/2)}{3\pi^2 (k_B \Theta)^{3/2}}, \quad (14)$$

where  $\zeta(3/2)$  is the Riemann zeta function.

## CONCLUSIONS

As should be expected, the presence of the gap in the spectrum leads to a decrease in the temperature dependence of the heat capacity of the ferromagnon gas and to an increase in its magnetization, which is a consequence of the total decrease in the energy of spin waves under surface resonance conditions. Other specific cases of the behavior of thermodynamic functions for low thermal energies of the system (compared to its "Lorenz" energy) or in the presence of weak and strong magnetic fields and also more subtle cases of dimensional effects of the system in quantizing magnetic fields in which the linear dimensions of the system in the quantization direction turn out to be slightly larger than the interatomic distance can be represented in a similar way.

## REFERENCES

1. A. A. Katanin and V. Yu. Irkhin, Phys. Usp. **50**, 613 (2007).
2. R. J. Birgeneau, D. R. Gabbe, H. P. Jenssen, et al., Phys. Rev. B **38**, 6614 (1988).
3. Yu. E. Lozovik and S. Yu. Volkov, Phys. Solid State **45**, 364 (2003).
4. R. Z. Vitlina and A. V. Chaplik, JETP Lett. **78**, 651 (2003).
5. R. P. Meilanov, B. A. Abramova, G. M. Musaeu, et al., Phys. Solid State **46**, 1107 (2004).
6. Z. Z. Alisultanov and R. P. Meilanov, J. Surf. Invest. **7**, 46 (2013).
7. M. N. Magomedov, J. Surf. Invest. **6**, 86 (2012).
8. Yu. V. Piskunov, V. V. Ogloblichev, and S. V. Verkhovskii, JETP Lett. **86**, 740 (2007).
9. S. A. Al'tshuler, Usp. Fiz. Nauk **104**, 685 (1971).
10. A. O. Govorov, Phys. Rev. B **82**, 155322 (2010).
11. A. K. Nukhov, G. M. Musaeu, and K. K. Kazbekov, Mosc. Univ. Phys. Bull. **66**, 416 (2011).

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