Two New Predictor-Corrector Iterative Methods with Third- and Ninth-Order Convergence for Solving Nonlinear Equations

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Abstract

In this paper, we suggest and analyze two new predictor-corrector iterative methods with third and ninth-order convergence for solving nonlinear equations. The first method is a development of [M. A. Noor, K. I. Noor and K. Aftab, Some New Iterative Methods for Solving Nonlinear Equations, World Applied Science Journal, 20(6),(2012):870-874.] based on the trapezoidal integration rule and the centroid mean. The second method is an improvement of the first new proposed method by using the technique of updating the solution. The order of convergence and corresponding error equations of new proposed methods are proved. Several numerical examples are given to illustrate the efficiency and performance of these new methods and compared them with the Newton's method and other relevant iterative methods.

Keywords: Nonlinear equations, Predictor–corrector methods, Trapezoidal integral rule, Centroid mean, Technique of updating the solution; Order of convergence.

1. Introduction

The fundamental problem, which arise in various fields of pure and applied sciences, is the exact solution of the nonlinear equation of the form:

$$f(x) = 0 \tag{1}$$

Where $f: I \subseteq R \rightarrow R$ for an open interval I. In recent years, many authors developed several iterative methods for solving nonlinear equation (1) by using some numerical techniques as Taylor's series, quadrature formulas, homotopy, decomposition or predictor-corrector technique [1-8, 10-27, 29-59]. Taylor's series expansion of $f(x)$ around a given initial point $x = x_0$, yields the important methods. The famous Newton's method (N for simplicity) is a one of these methods that used to solve equation (1) by the iterative scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \ldots \tag{2}$$

This method has quadratic convergence in some neighborhood of a simple root α of f. Moreover, it has efficiency index is 1.41421; (see [10]). In addition, using Taylor expansion of the second order for the function $f(x)$, gives the well-known iterative method, defined by

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'^2(x_n) - f(x_n)f''(x_n)}, \quad n = 0, 1, \ldots \tag{3}$$

This is known as Halley's (H) method [15, 16, 19, 22, 43], which has cubic convergence and its efficiency index equals 1.44225. Newton's method and Halley's method are members of a family of one-step (explicit) iterative methods.