ISSN 2347-1921
Volume 12 Number10
Journal of Advances in Mathematics

On the Remes Algorithm for Rational Approximations

Husam L. Saad, Noori Yasir Abdul-Hassan
Department of Mathematics, College of Science, University of Basrah, Basrah, Iraq
husam.saad@uobasrah.edu.iq
Department of Mathematics, College of Education for Pure Science, University of Basrah, Basrah, Iraq
noori.hassan@uobasrah.edu.iq

Abstract

This paper is concerned with the minimax approximation of a discrete data set by rational functions. The second algorithm of Remes is applied. A crucial stage of this algorithm is solving the nonlinear system of leveling equations. In this paper, we will give a new approach for this purpose. In this approach, no initial guesses are required. Illustrative numerical example is presented.

Keywords: Minimax approximation, Rational functions, Remes algorithm, Nonlinear system of leveling Equations, The dual monomial Vandermond system, The leveled reference error.

1. Introduction

Let $f(x)$ be a given real-valued function define on a discrete point set $X_N = \{x_i\}_{i=1}^N$. We shall consider the problem of approximating the values $\{f(x_i)\}_{i=1}^N$ by functions of the form:

$$R(x) = \frac{P_n(x)}{Q_m(x)} = \frac{\sum_{r=1}^{n+1} a_r x^{r-1}}{\sum_{r=1}^{m+1} b_r x^{r-1}} \qquad (1)$$

Where n and m are nonnegative integers.

The rational minimax approximation for the values $\{f(x_i)\}_{i=1}^N$ is to determine the coefficients $\{a_r\}_{r=1}^{n+1}$ and $\{b_r\}_{r=1}^{m+1}$ which minimize the expression:

$$\max \left| f(x_i) - \frac{P_n(x_i)}{Q_m(x_i)} \right| \qquad (2)$$

Such that $Q_m(x_i) > 0$, $i = 1,2,\dots,N$.

The best approximation problem can be found in almost every book on approximation theory (see [4, 6, 10, 11, 13, 14, 15]).This problem was studied from the second half of the 19th century to the early 20th century, and by 1915 the main results had been established (see Steffens [16]). Also a good introduction and another vision on this subject is in Pachón and Trefethen [12].

Existence of a best approximation is not guaranteed [17, p.193]. If ∂P_n and ∂Q_m denotes the actual degree of P_n and Q_n respectively then characterization theorem can be stated as follows.

Theorem: $R^*(x) = \frac{P_n^*}{Q_m^*}$ is the best approximation to $f(x)$ on X_N if and only if $f(x) - R^*(x)$ has an alternating set consisting of at least (2 + max(n + ∂P_n , m + ∂Q_m)) points of X_N.

In [1] Barrodale and Mason gave a computational experience with two algorithms for rational approximation on a discrete point sets. In [8] three algorithms are described and discussed for discrete rational approximation. In [9] a comparison of eight algorithms is given for obtaining rational minimax approximations. The results of the study indicated that the Remes algorithm is one of two satisfactory methods to be used in practice. This algorithm is considered here. We shall assume that the interval include the origin, then we may choose b_1=1. If $R^*(x)$ is non-degenerate then by characterization theorem the error alternate at least ℓ=n+m+1 times. At the iteration a set is defined. The algorithm consists of the two following stages:

Stage (1). An approximation $R^{(k)}$ is obtained by solving the system of the leveling equations

$$E\left(x_i^{(k)}\right) = f\left(x_i^{(k)}\right) - R^{(k)}\left(x_i^{(k)}\right) = (-1)^i \lambda^{(k)}, i = 1,2,\dots,\ell \qquad (3)$$

Where $\lambda^{(k)}$ is the leveled reference error.

Stage (2). The extreme points of the error function $E\left(x_i^{(k)}\right)$ yield a new reference set $X^{(k+1)}$.

2. New Approach for Solving the Nonlinear System of Leveling Equations

We will concentrate on stage **(1)** of the algorithm with a new approach to solve the leveling equations