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An Efficient Third-Order Scheme Based on Runge–Kutta and Taylor Series Expansion for Solving Initial Value Problems

Noori Y. Abdul-Hassan [1], Zainab J. Kadum [2] and Ali Hasan Ali [1,3,*]

[1] Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Basrah 61001, Iraq
[2] Basrah Education Directorate, Ministry of Education, Basrah 61001, Iraq
[3] Institute of Mathematics, University of Debrecen, Pf. 400, H-4002 Debrecen, Hungary
* Correspondence: ali.hasan@science.unideb.hu

Abstract: In this paper, we propose a new numerical scheme based on a variation of the standard formulation of the Runge–Kutta method using Taylor series expansion for solving initial value problems (IVPs) in ordinary differential equations. Analytically, the accuracy, consistency, and absolute stability of the new method are discussed. It is established that the new method is consistent and stable and has third-order convergence. Numerically, we present two models involving applications from physics and engineering to illustrate the efficiency and accuracy of our new method and compare it with further pertinent techniques carried out in the same order.

Keywords: numerical methods; initial value problem; autonomous equation; local truncation error; consistency; stability

MSC: 65L05; 65L07; 65L09; 65L12; 65L20

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1. Introduction

Numerical analysis is the area of mathematics that deals with computational techniques for studying and finding solutions to math problems. It is an offshoot of computer science and mathematics that develops, analyzes, and generates algorithms for numerically solving mathematical problems. Numerical methods are typically centered on the implementation of numerical algorithms. The goal of these numerical methods is to provide systematic procedures for numerically solving mathematical problems. The ordinary differential equation (ODE) is a mathematical equation that is used in natural physical processes, chemistry, engineering, and other sciences. Ordinary differential equations are notoriously difficult to approximate analytically, so the numerical treatment is crucial because it offers a powerful alternate solution method for resolving the differential equation.

We frequently use initial value problems (IVPs), such that

$$y' = f(x, y(x)),\ y(x_0) = y_0 \tag{1}$$

where x is the independent variable, which might also indicate time in physical problems, and $y(x)$ is the solution. Furthermore, because $y(x)$ can be a vector-valued function with N-dimensions, the domain and range of f and the solution y are given by

$$f : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$$

$$y : \mathbb{R} \to \mathbb{R}^N$$