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Cauchy Problem for Fractional Ricatti Differential Equations Type with Alpha Order Caputo Fractional Derivatives

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Abstract

In this paper, we investigate solution of the fractional Ricatti differential equations (FRDEs) with alpha order Caputo fractional derivatives. In fact, FRDEs are analogous of the Ricatti ordinary differential equations. The multi power series method is used to obtain a useful formula that is implemented to find an explicit solution of Cauchy problem for FRDEs without solving any integral. This formula is explicit and easy to compute by using Maple software to get explicit solution. Also, it is shown that the proposed formula can be used to solve the Cauchy problem for Ricatti ordinary differential equations.

Keywords: Ricatti differential equation; Caputo fractional derivatives**;** multi power series method; Cauchy problems.

1. Introduction

The history of fractional calculus is back to 1695 when L'Hospital asked Leibniz about $\frac{d^{1/2}u(x)}{dx^{1/2}}$. Leibniz replied, "It will lead to a paradox." But he added prophetically, "From this apparent paradox, one day useful consequences will be drawn" [5]. In the past, only pure mathematician deal with fractional calculus because they are believed there is no applications to the fractional derivatives [14]. Based on the fact that a reasonable modeling of many physical phenomena having to depend on the time instant to gather with the prior time history, fractional calculus can be used successfully. Therefore, in recent year, fractional derivatives have been used in many phenomena in electromagnetic theory, fluid mechanics, viscoelasticity, circuit theory, control theory, biology, atmospheric physics, etc., [10] and [21]. However, there are two difficulties raised in the study of fractional derivatives, fractional derivatives cannot be expressed to a tangent direction as the classical first derivative. The second difficulty comes from complex integro-differential definitions which make the chain rule not valid for all type of fractional derivatives.

Fractional differential equations (FDEs) have been used to describe many real world problems such as damping laws, fluid mechanics, rheology, physics, mathematical biology, diffusion processes, electrochemistry, and so on. The solvability of a wide fractional differential equation types has been attracted many researchers [4], [6], [7], [11], [12], [16], [17], [19], [20], and [21]. Some theorems related