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Analytic Solution of Linear Fractional Differential Equations with Constant Coefficient

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Abstract

This paper presents direct methods for obtaining the explicit general solution to a linear sequential fractional differential equation (LSFDE), involving Jumarie's modification of Riemann–Liouville derivative, with constant coefficients. The general solution to a homogenous LSFDE with constant coefficients is obtained by using the roots of the characteristic polynomial of the corresponding homogeneous equation. For the non-homogeneous case, two methods, undetermined coefficients and variation of parameter, are investigated to find the particular solution. The method of undetermined coefficients is independent of the integral transforms while the method of variation of parameter is not. Moreover, several examples are illustrative for demonstrating the advantage of our approach.

Keywords: Fractional differential equations, Riemann–Liouville derivative, Caputo derivative, undetermined coefficients, variation of parameter.

1. Introduction

Fractional Calculus is a field of applied mathematics that deals with derivatives and integrals of any arbitrary real or complex order. The History of fractional derivatives were planted over 300 years ago. Since that time the fractional calculus has drawn the attention of many great mathematicians (pure and applied) of their times, such as N. H. Abel, M. Caputo, L. Euler, J. Fourier, A.K. Grunwald, J. Hadamard, G. H. Hardy, O. Heaviside, H. J. Holmgren, P. S. Laplace, G. W. Leibniz, A. V. Letnikov, J. Liouville, B. Riemann, M. Riesz, and H. Weyl (Sabatier et al, 2007). But during this last decades fractional calculus have been applied in widespread fields of science and engineering (Machado et al, 2011).

Fractional differential equations arise in many complex systems in nature and society with many dynamics, such as charge transport in amorphous semiconductors, the spread of contaminants in underground water, relaxation in viscoelastic materials like polymers, the diffusion of pollution in the atmosphere, and many more (Podlubny, 1999; Kilbas et al, 2006). However, the problem of studying fractional differential equations has been dealt with by numerous authors throughout history, particularly in recent years(Mophou,2010; Rajeev and Kushwaha,2013, Khudair 2013, Khudair and Mahdi 2016. Eidelman and Kochubei, 2004; Xue et al, 2008; Guo et al,2012; Molliq et al, 2009). A wide description of the existence and uniqueness of solutions of initial value problem for fractional order differential equations together with its applications can be found in the literature (Samko, et al, 1993; Delbosco, 1996; Podlubny, 1999, Dielhelm,2002).