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# Studying of COVID-19 fractional model: Stability analysis

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A B S T R A C T

This article focuses on the recent epidemic caused by COVID-19 and takes into account several measures that have been taken by governments, including complete closure, media coverage, and attention to public hygiene. It is well known that mathematical models in epidemiology have helped determine the best strategies for disease control. This motivates us to construct a fractional mathematical model that includes quarantine categories as well as government sanctions. In this article, we prove the existence and uniqueness of positive bounded solutions for the suggested model. Also, we investigate the stability of the disease-free and endemic equilibriums by using the basic reproduction number (BRN). Moreover, we investigate the stability of the considering model in the sense of Ulam–Hyers criteria. To underpin and demonstrate this study, we provide a numerical simulation, whose results are consistent with the analysis presented in this article.

## 1. Introduction

To combat the spread of COVID-19, all governments around the world have made significant efforts and taken preventive measures.[1–3] In Wuhan, the capital of Hubei province, China, COVID-19 was first detected which is a new strain of SARS-CoV-2.[4,5] In the months following its discovery, the number of patients grew at an exponential rate. According to the World Health Organization's situation report, there were 5 304 772 total cases and 342 029 deaths worldwide as of May 25, 2020. The use of mathematical models in epidemics is very important for understanding the nature of these epidemics as well as for designing effective strategies for controlling them.[6–8] As a contribution from some mathematicians to reduce the COVID-19 pandemic, many researchers have adopted the development of models for this emerging epidemic. Where some researchers took from developing some models of the spread of epidemics such as SEIR and SIR to design a model that simulates the spread of Corona disease.[9–19] The COVID-19 severity was calculated by Wu et al. using the dynamics of transition in Ref. 20. There have been investigations into random transition models in Refs. 21, 22. The general multi-group SEIRA model for modeling COVID-19 diffusion in a heterogeneous population was represented and numerically tested in Ref. 23. Differential equations in their various forms (ordinary, randomly detected, partial, fractional, or with delay) are an essential mathematical tool for modeling many epidemics.[8] Many research attempts have been made to prevent epidemic outbreaks via optimum control.[24–26] Mathematical studies of epidemic illnesses have become more relevant.[27–29] Several studies have been introduced to control HIV,[30] dengue fever,[31] TB,[32] SIR,[33,34] and SIRS.[35]

Fractional differential equations (FDEs) provided an accurate description of the dynamics of epidemiological models,[36] taking into account information about a population's memory and learning mechanisms, which influence disease spread. In this paper, we used a fractional mathematical model that includes the quarantine category and methods taken by the government to prevent the spread of disease and demonstrated the existence of non-negative and bounded solutions.

Many authors prefer to use FDEs to describe epidemic models since they carry more memory information and provide a learning mechanism for the spread of disease in the population compared to the ordinary differential equation, which is incapable of serving this purpose. Also, the region of stability for the FDEs is larger than that for the ordinary differential equations. Furthermore, the fractional derivative is a non-local operator, whereas the classical derivative is a local operator. In other words, the description of epidemic models by using fractional differential equations takes into account all historical and current states, which makes them more realistic and more general in nature.[37,38] This prompted us to develop the Caputo fractional mathematical model for COVID-19 and introduce details about the existence of a unique positive solution and its behavior. In spite of the fact that there are many definitions of a fractional derivative, many scholars prefer to use the Caputo derivative to describe mathematical models by means of FDEs. In fact, due to the initial conditions of FDEs with Caputo derivatives containing integer order derivatives with physical meanings like distance, speed, and acceleration, FDEs with Caputo derivatives are widely used in real-world applications.[39]

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