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PROJECTIVE-RECURRENT VIASMAN-GRAY MANIFOLD

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AUTHORS' CONTRIBUTIONS
This work was carried out in collaboration between both authors. Author HMA gathered the initial data and help to interpret the results. Author HGAA designed the study, wrote the algorithm, methodology programming and interpreted the results. Both authors read and approved the final manuscript.

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ABSTRACT

The purpose of this work is to study the geometric properties of projective tensor of Viasman-Gray manifold. The necessary condition that the Viasman-Gray manifold is projective para-kahler manifold has been found. The projective-recurrent Viasman-Gray manifold has been studied, in particular, it has been proved the projective-recurrent Viasman-Gray manifold is either projective symmetrical manifold or it is a locally conformal Kähler manifold. Finally, an application about the projective-recurrent of Viasman-Gray manifold has been given.

Keywords: Almost Hermitian manifold; Viasman-Gray manifold; projective-recurrent tensor.

Mathematics Subject Classification: 53C55, 53B35.

1 Introduction

Almost Hermitian manifold is regarded as one of the most important subjects of differen-tial geometry. This subject classified into different components in an attempt to determine its specifications and features accurately. Then appeared important matter is the classification of the different classes of almost Hermitian manifold according to specific features. Many researchers studied the almost Hermitian manifold and they found that it had important geometrical properties. One of them is Russian researcher Kirichenko, when he studied the almost Hermitian manifold by using G-structure space that does not depend on a manifold itself but on a principle subfiber bundle of all complex frames which is called the adjoined G-structure space [1]. We used this method to study the projective tensor of the class Viasman-Gray manifold ($VG-$ manifold). This class denoted by $W_1 \oplus W_4$, where W_1 is the nearly Kähler manifold ($NK-$ manifold) and W_4 is the locally conformal Kähler manifold ($LCK-$ manifold) [2].

There are many researchers studied the geometrical properties of some kind of the curvature tensors on almost Hermitian manifold. Kirichnko, Rustanov and Shikhab [3] studied the geometry of conhormanic curvature tensor of almost Hermitian manifold. One of these curvature tensor is projective tensor. Kirichenko [4] proved that non-trivial projective-recurrent K-space of maximal rank is 6-dimensional manifold of constant curvature tensor, also he proved that projective-recurrent K-space of dimension $n > 2$

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is local symmetric or local equivalent to the product of Euclidean space and 2-dimensional Kähler manifold. Abood [5], studied the projective tensor of NK −manifold, and he proved that NK −manifold is a projective – parakahler manifold if and only if, either M is a Ricci flat Kähler manifold, or M is 6-dimensional proper NK −manifold.

2 Preliminaries

Let $X(M)$ be a module of smooth vector fields on M. $C^{\infty}(M)$ be a set of smooth functions on M . An almost Hermitian manifold (AH-manifold) is a set $\{M, J, g = \langle\cdot,\cdot\rangle\}$, where M is $2n$-dimensional $(n > 1)$ smooth manifold; J is an endomorphism of tangent space $T_p(M)$ with $\left(J_p\right)^2 = -id$, and $g =<.,.>$ is a Riemannian metric on M such that $\langle JX, JY\rangle = \langle X, Y\rangle; X, Y \in X(M)$ [6].

Suppose that $\{e_1, \dots, e_n, Je_1, \dots, Je_n\}$ is a basis of the tangent space $T_p(M)$ at the point $p \in M$. By using this basis we can construct a new basis $\{\varepsilon_1, \dots, \varepsilon_2, \bar{\varepsilon}_1, \dots, \bar{\varepsilon_n}\}$ of $T_p^c(M)$ which is called adapted basis. Its corresponding complex frame is $\{p, \varepsilon_1, \dots, \varepsilon_2, \bar{\varepsilon_1}, \dots, \bar{\varepsilon}_n\}$. The G-structure space is the principle fiber bundle of all complex frames of manifold M with structure group $U(n)$ which is called an adjoined G-structure space. Suppose that the indices i, j, k, l in the range $1,2,\dots,2n$ and the indices a, b, c, d, e, f in the range $1,2,\dots,n$ and $\hat{a} = a + n$.

The components matrices of complex structure J and Riemannian metric g in the adjoined G-structure space have the following forms:

$$\left(J_j^i\right) = \begin{pmatrix} \sqrt{-1}I_n & 0 \\ 0 & -\sqrt{-1}I_n \end{pmatrix}, \left(g_{ij}\right) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} \tag{2.1}$$

Where I_n is the unit matrix of order n [1].

Definition 2.1 [7]. An AH −manifold is called VG-manifold, LCK − manifold and NK − manifold, if in the adjoined G-structure space, the following conditions are respectively hold:

$$B^{abc} = B^{-bac} \quad , \; B^{ab}{}_c = \alpha^{[a}\delta_c^{b]} \; ;$$
$$B^{abc} = 0 \; , B^{ab}{}_c = \alpha^{[a}\delta_c^{b]} \; ;$$
$$B^{abc} = -B^{bac}, B^{ab}{}_c = 0 \; ,$$

where $B^{abc} = \frac{\sqrt{-1}}{2} J^a_{[\hat{b},\hat{c}]}$, $B^{ab}{}_c = -\frac{\sqrt{-1}}{2} J^a_{\hat{b},c}$ and $\alpha = \frac{1}{n-1}\delta F \circ J$, F is a Kähler form which def-ined by $F(X,Y) =< JX, Y >$, δ is a codrivative and $X, Y \in X(M)$ and the bracket [ ] denote to the antisymmetric operation.

Theorem 2.2 [8]. In the adjoined G-structure space, the family of the structure equations of VG-manifold has the following forms:

1) $d\omega^a = \omega^a_b \Lambda \omega^b + B^{ab}{}_c \omega^c \Lambda \omega_b + B^{abc}\omega_b \Lambda \omega_c$;
2) $d\omega_a = -\omega^b_a \Lambda \omega_b + B_{ab}{}^c \omega_c \Lambda \omega^b + B_{abc}\omega^b \Lambda \omega^c$;
3) $d\omega^a_b = \omega^a_c \Lambda \omega^c_b + \left(2B^{adh}B_{hbc} + A^{ad}_{bc}\right)\omega^c \Lambda \omega_d + \left(B^{ah}{}_{[c}B_{d]bh} + A^a_{bcd}\right)\omega^c \Lambda \omega^d$
$+\left(B_{bh}{}^{[c}B^{d]ah} + A^{acd}_b\right)\omega_c \Lambda \omega_d$;

where $\{\omega^i\}$are the components of mixture form, $\{\omega^i_j\}$ are the components of Riemannian conn-ecion of metric g, $\{A^a_{bcd}, A^{acd}_b\}$ are some functions on adjoined G-structure space and $\{A^{ad}_{bc}\}$ are system of functions which are symmetric by the lower and upper indices and are called the components of holomorphic sectional curvature tensor.

Definition 2.3 [9]. A Riemannian curvature tensor R of smooth manifold M is an 4-covariant tensor $R: T_p(M) \times T_p(M) \times T_p(M) \times T_p(M) \to \mathbb{R}$ which is defined by

$$R(X, Y, Z, W) = g(R(Z, W)Y, X),$$

where $R(X, Y)Z = \left([\nabla_X, \nabla_Y] - \nabla_{[X,Y]}\right)Z;\ X, Y, Z, W \in T_p(M)$ and satisfies the following pro-perties:

1) $R(X, Y, Z, W) = -R(Y, X, Z, W);$
2) $R(X, Y, Z, W) = -R(X, Y, W, Z);$
3) $R(X, Y, Z, W) + R(X, Z, W, Y) + R(X, W, Y, Z) = 0;$
4) $R(X, Y, Z, W) = R(Z, W, X, Y).$

Theorem 2.4 [8]. In the adjoined G −structure space, the components of Riemannian cur- vature tensor R of VG −Manifold are given as follows:

1) $R_{abcd} = 2(B_{ab[cd]} + \alpha_{[a} B_{b]cd})$;
2) $R_{\hat{a}bcd} = 2\mathrm{A}^{a}_{bcd}$;
3) $R_{\hat{a}\hat{b}cd} = 2(-B^{abh} B_{hcd} + \alpha^{[a}_{[c} \delta^{b]}_{d]})$;
4) $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh} B_{hbc} - B^{ah}{}_{c} B_{hb}{}^{d}$,

where $\{\alpha^{a}{}_{b}, \alpha_{a}{}^{b}, \alpha_{ab}, \alpha^{ab}\}$ are some functions on adjoined G −structure space such that

$$d\alpha_{\mathrm{a}} + \alpha_b \omega^b_a = \alpha^b_a \omega_b + \alpha_{ab} \omega^b$$

and

$$d\alpha^a - \alpha^b \omega^a_b = \alpha^a_b \omega^b + \alpha^{ab} \omega_b$$

The other components of Riemannian curvature tensor R can be obtained by the property of symmetry for R.

Definition 2.5 [9]. A tensor of type $(2,0)$ which is defined as $r_{ij} = R^k_{ijk} = g^{kl} R_{kijl}$ is called a Ricci tensor.

Theorem 2.6 [8]. In the adjoined G-structure space, the components of the Ricci tensor of VG-manifold are given as the following forms:

1) $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a \alpha_b);$
2) $r_{\hat{a}b} = 3\mathrm{B}^{cah} \mathrm{B}_{cbh} - \mathrm{A}^{ca}_{bc} + \frac{n-1}{2}(\alpha^a \alpha_b - \alpha^h \alpha_h) - \frac{1}{2}\alpha^{h}{}_{h} \delta^a_b + (n-2)\alpha^{a}{}_{b}).$

And the others are conjugate to the above components.

3 Main Results

Definition 3.1 [4]. A projective tensor of an AH-manifold is a tensor P of type $(4,0)$which is defined by the form:

$$P_{ijkl} = R_{ijkl} + \frac{1}{2n-1}\left(r_{ik} g_{jl} - r_{jk} g_{il}\right),$$

where, R_{ijkl} , r_{ij} and g_{ij} are respectively the components of Riemannian curvature tensor, Ricci tensor and Riemannian metric.

This tensor has properties similar to those of Riemannian curvature tensor, this means

$$P_{ijkl} = -P_{jikl} = -P_{ijlk} = P_{klij}$$

Lemma 3.2. In the adjoined G- structure space, the components of the projective tensor of VG-manifold are given as the following forms:

1) $P_{abcd} = 2\left(B_{ab[cd]} + \alpha_{[a}B_{b]cd}\right)$.
2) $P_{\hat{a}bcd} = 2\mathrm{A}^{a}_{bcd} - \frac{1}{2n-1} r_{bd}\delta^{a}_{d}$.
3) $P_{\hat{a}\hat{b}cd} = 2\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) + \frac{2}{2n-1} r^{[a}_{c}\delta^{b]}_{d}$.
4) $P_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2n-1} r^{a}_{c}\delta^{d}_{b}$.

Proof.

1) For $i = a$, $j = b$, $k = c$, and $l = d$, then

$$P_{abcd} = R_{abcd} + \frac{1}{2n-1}(r_{ac}g_{bd} - r_{bc}g_{ad})$$

According to the relaions(2.1), we have

$$P_{abcd} = R_{abcd}$$

2) For $i = \hat{a}$, $j = b$, $k = c$ and $l = d$, we obtain

$$P_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2n-1}(r_{\hat{a}c}g_{bd} - r_{bc}g_{\hat{a}d})$$
$$P_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2n-1} r_{bc}\delta^{a}_{d}$$

3) For $i = \hat{a}$, $j = \hat{b}$, $k = c$ and $l = d$, it follows that

$$\begin{aligned} P_{\hat{a}\hat{b}cd} &= R_{\hat{a}\hat{b}cd} + \frac{1}{2n-1}(r_{\hat{a}c}g_{\hat{b}d} - r_{\hat{b}c}g_{\hat{a}d}) \\ &= R_{\hat{a}\hat{b}cd} + \frac{1}{2n-1}\left(r^{a}_{c}\delta^{b}_{d} - r^{b}_{c}\delta^{a}_{d}\right) \\ &= R_{\hat{a}\hat{b}cd} + \frac{2}{2n-1} r^{[a}_{c}\delta^{b]}_{d} \end{aligned}$$

4) For $i = \hat{a}$, $j = b$, $k = c$, and $l = \hat{d}$, we have

$$\begin{aligned} P_{\hat{a}bc\hat{d}} &= R_{\hat{a}bc\hat{d}} + \frac{1}{2n-1}(r_{\hat{a}c}g_{b\hat{d}} - r_{bc}g_{\hat{a}\hat{d}}) \\ &= R_{\hat{a}bc\hat{d}} + \frac{1}{2n-1} r^{a}_{c}\delta^{d}_{b} \end{aligned}$$

Definition 3.3. An AH-manifold is called a projective flat if the projective tensor is equal to zero.

Definition 3.4 [5]. An VG-manifold is called a projective parakahler manifold if the com-ponent $P_{\hat{a}\hat{b}cd}$ of projective tensor P is equal to zero.

Definition 3.5 [10]. An AH-manifold has J-invariant Ricci tensor, if $J \circ r = r \circ J$.

Lemma 3.6 [11]. An AH-manifold has J-invariant Ricci tensor if and only if, in the adjoined G-structure space $r_b^{\hat{a}} = r_{ab} = 0$.

Theorem 3.7. Let M be an VG-manifold, If M is a projective parakahler manifold, then M is $LCK-$ manifold if and only if, M is J-invariant Ricci tensor.

Proof:

Put $i = \hat{a}$, $j = \hat{b}$, $k = c$ and $l = d$, it follows that

$$P_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} + \frac{1}{2n-1}(r_{\hat{a}c}g_{\hat{b}d} - r_{\hat{b}c}g_{\hat{a}d})$$

Suppose that M is projective parakahler manifold, so according to the definition 3.4 we have

$$-2B^{abh}B_{hcd} + 2\alpha_{[c}^{[a}\delta_{d]}^{b]} + \frac{1}{2n-1}(r_{\hat{a}c}g_{\hat{b}d} - r_{\hat{b}c}g_{\hat{a}d}) = 0 \tag{3.4}$$

Since M is J- invariant Ricci tensor, consequently we deduce

$$-2B^{abh}B_{hcd} + 2\alpha_{[c}^{[a}\delta_{d]}^{b]} = 0$$

Contracting by the indices (b, d), obtained

$$-2B^{abh}B_{hcb} + 2n\alpha_c^a = 0 \tag{3.5}$$

Symmetrization and antisymmetrization by the indices (a, b) we get:

$$2n\alpha_c^a = 0 \tag{3.6}$$

Making use of the equations (3.5) and (3.6), it follows that

$$-2B^{abh}B_{hcb} = 0$$

Contracting by the indices (a, c) we get

$$-2B^{abh}B_{hab} = 0$$

$$B^{abh}B_{abh} = 0 \Longrightarrow \bar{B}_{abh}B_{abh} = 0 \Longrightarrow \sum_{a,b,h} \|B_{abh}\|^2 = 0 \Leftrightarrow B_{abh} = 0$$

Therefore, according to the definition 2.1 we have M is LCK $-$manifold.

Conversely, by using (3.4) and since M is LCK $-$manifold we get

$$2\alpha_{[c}^{[a}\delta_{d]}^{b]} + \frac{1}{2n-1}(r_c^a\delta_d^b - r_c^b\delta_d^a) = 0$$

Symmetrization by the indices (c, d) , it follows that

$$\frac{1}{2(2n-1)}(r_c^a\delta_d^b + r_d^a\delta_c^b - r_c^b\delta_d^a - r_d^b\delta_c^a) = 0$$

Contracting by the indices (b, d) we get

$$\frac{1}{2(2n-1)}(r_c^a\delta_b^b + r_b^a\delta_c^b - r_c^b\delta_b^a - r_b^b\delta_c^a) = 0$$
$$\frac{1}{2(2n-1)}(nr_c^a + r_c^a - r_c^a - r_b^b\delta_c^a) = 0$$
$$\frac{1}{2(2n-1)}(nr_c^a - r_b^b\delta_c^a) = 0$$

Symmetrization and antisymmetrization by the indices (b, a) we obtain

$$\frac{n}{2(2n-1)}r_c^a = 0$$

Consequently, we get

$$r_c^a = 0$$

Therefore, M is J-invariant Ricci tensor.

Definition 3.8 [4]. Let M be a Riemannian manifold, t be a non-zero tensor field of the type (r, s)on M. The tensor t is said to be a recurrent if there is 1-form ρ on M such that $\nabla t = \rho \otimes t$, where ∇ is the Riemannian connection on M. The 1-form ρ is called a recurrence convector, and the Riemannian manifold which allows a field of the recurrent tensor t is called t-recurrent.

Remark 3.9 [4]. If $\rho = 0$, then the manifold is called t-symmetrical, and if $\rho \neq 0$ then it is called nontrivially t-recurrent.

Definition 3.10. Let M be an VG-manifold, M is called Pr-recurrent VG-manifold if M is P-recurrent and r-recurrent with the same recurrence convector.

Theorem 3.11. Suppose that M is Pr-recurrent VG-manifold then M either projective symmetrical manifold or projective recurrent LCK $-$manifold.

Proof:

Suppose that M is Pr-recurrent VG-manifold.

According to the definition 3.10, we have M is P-recurrent and r-recurrent VG-manifold.

According to the definition 3.8, we have

$$\nabla P = \rho \otimes P$$

which has the following coordinate form:

$$P_{ijkl,h} = \rho_h P_{ijkl}$$

Consider this equation in the adjoined G-structure space, it follows

$$P_{\hat{a}\hat{b}cd,h} = \rho_h P_{\hat{a}\hat{b}cd}$$

By using the Lemma 3.2. we obtained:

$$-2B^{abh}B_{hcd,k}+2\alpha_{[c}^{[a}\delta_{d],k}^{b]}+\frac{1}{2n-1}(r_{c,k}^{a}\delta_{d}^{b}-r_{c,k}^{b}\delta_{d}^{a})=\rho_h[-2B^{abh}B_{hcd}+2\alpha_{[c}^{[a}\delta_{d]}^{b]}+\frac{1}{2n-1}(r_{c}^{a}\delta_{d}^{b}-r_{c}^{b}\delta_{d}^{a})]$$

According to the definition 3.10, we get

$$-2B^{abh}B_{hcd,k}+2\alpha_{[c}^{[a}\delta_{d],k}^{b]}=\rho_h(-2B^{abh}B_{hcd}+2\alpha_{[c}^{[a}\delta_{d]}^{b]})$$

Since M is VG-manifold, then we obtain

$$-2B^{abh}B_{hcd,k}+2\alpha_{[c}^{[a}\delta_{d],k}^{b]}=\rho_h(2B^{ahb}B_{hcd}+2\alpha_{[c}^{[a}\delta_{d]}^{b]})$$

By the Symmetrization and antisymmetrization by the indices (a,b) , it follows that

$$2\rho_h B^{ahb}B_{hcd}=0$$

Contracting by the indices (a,c) and (b,d) we have

$$2\rho_h B^{ahb}B_{hab}=0$$

Consequently either,

$\rho_h=0$; it follows $\nabla P=0$ which means that M is projective symmetrical manifold.

Or, $\quad B^{ahb}B_{hab}=0$

Since M is VG-manifold, then we get

$$B^{abh}B_{abh}=0 \Longrightarrow \bar{B}_{abh}B_{abh}=0 \Longrightarrow \sum_{a,b,h}\|B_{abh}\|^2=0 \Leftrightarrow B_{abh}=0$$

Making use of the definition 2.1, it follows that M is LCK −manifold.

Hence M is projective recurrent LCK −manifold.

Lemma 3.12 [4]. Let M be an AH-manifold. If the sectional curvature tensor A_{bc}^{ad} is recurrent $\left(A_{bc,h}^{ad}=\right.$ $\rho hAbcad$, then M is locally holomorphic isometrical to the product manifold of the 2-dimensional Kähler manifold M by the complex Euclidean space $\mathbb{C}^{n-1}$.

Finally, the next result gives an interesting application about projective-recurrent VG-manifold.

Theorem 3.13. Suppose that M is Pr-recurrent VG-manifold then M is locally holomorphic isometrically to the product manifold of the 2-dimensional Kähler manifold by the complex Euclidean space $\mathbb{C}^{n-1}$.

Proof:

Let M be Pr-recurrent VG-manifold.

According to the definition 3.10, we have M is P-recurrent and r-recurrent VG-manifold.

According to the definition 3.8, we have

$$\nabla P=\rho\otimes P$$

which has the following coordinate form:

$$P_{ijkl,h}=\rho_h P_{ijkl}$$

Consider this equation in the adjoined G-structure space, it follows

$$P_{\hat{a}bc\hat{d},h} = \rho_h P_{\hat{a}bc\hat{d}}$$

By using the Lemma 3.2. we obtained

$$A^{ad}_{bc,k} + B^{adh}B_{hbc,k} - B^{ah}{}_{c}B_{hb}{}^{d}{}_{,k} + \frac{1}{2n-1}r^{a}_{c,k}\delta^{d}_{b} = \rho_h(A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2n-1}r^{a}_{c}\delta^{d}_{b})$$

According to the definition 3.8, we have

$$A^{ad}_{bc,k} + B^{adh}B_{hbc,k} - B^{ah}{}_{c}B_{hb}{}^{d}{}_{,k} = \rho_h\left(A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d}\right) \quad (3.7)$$

Symmetrization and antisymmetrization by the indices (h, b) we get

$$A^{ad}_{bc,k} = \rho_h A^{ad}_{bc} \quad (3.8)$$

According to the Lemma 3.12. we obtained that the manifold M is locally holomorphic iso-metrically to the product manifold of the 2-dimentional Kähler manifold by the complex Euclidean space $\mathbb{C}^{n-1}$.

4 Conclusion

This paper is devoted to study the geometric properties of projective tensor of Viasman-Gray Manifold. In particular, we have found the properties of projective-recurrent Viasman-Gray manifold. Related to this properties we have got interesting application of projective-recurrent Viasman-Gray manifold.

Competing Interests

Authors have declared that no competing interests exist.

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