

# A Study of Neutrosophic Regular Generalized $b$ -Closed Sets in Neutrosophic Topological Spaces

Nadia Abbas<sup>a</sup>, Abu Firas Al Musawi<sup>b</sup>, Shuker Alsalem<sup>\*c</sup>

<sup>a</sup> Ministry of Education, Directorate General of Education, Al-Kark/3, Baghdad, Iraq; <sup>b</sup>Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Iraq; <sup>c</sup> Department of Mathematics, College of Science, University of Basrah, Basrah 61004, Iraq.

## ABSTRACT

In neutrosophic topological space ( $NTS$ ), The neutrosophic setting for regular generalized  $b$  – closed set is shown as a novel kind of neutrosophic closed set ( $NCS$ ) Moreover, Several new ideas of ( $NCS$ ) are given and discussed, like neutrosophic generalized preregular closed set, neutrosophic regular generalized  $\alpha$  – closed set, neutrosophic generalized  $b$  – closed set, neutrosophic generalized  $\alpha$  – closed set, neutrosophic generalized semi pre – closed set, neutrosophic generalized  $\alpha b$  – closed set, neutrosophic semi generalized  $b$  – closed set. in neutrosophic topological spaces. Moreover, the relationships between the neutrosophic regular generalized  $b$  – closed and these new classes in ( $NTS$ ) are explained and studied their characters.

**Keywords:** Neutrosophic  $b$  – open, Neutrosophic  $b$  – closed, Neutrosophic semi closed, Neutrosophic sets theory.

## 1. INTRODUCTION

Smarandache<sup>1</sup> proposed and characterized neutrosophic set (NS) as a new non-classical set, and this meaning is an extension of Fuzzy sets (FS), where (FS) is supplied by Zadeh<sup>2</sup>. Atanassav<sup>3</sup> also demonstrates the implication of Intuitionistic Fuzzy set (IFS). Following that, Salama and Alblowi<sup>4</sup> proposed a new idea known as a neutrosophic topological space ( $NTS$ ), that has just lately been inspected in this type of topological space. Rao and Srinivasa<sup>5</sup> then studied the meaning of a neutrosophic per-closed. In 2018, Ebenanjar M et al<sup>6</sup>. published a paper in ( $NTS$ ) describing neutrosophic  $b$  – closed. In 2020, ( $NTS$ )<sup>7</sup> introduced and investigated the connotation of a neutrosophic  $bg$  – closed. Some non-classical sets like, soft<sup>8-13</sup>, fuzzy<sup>14-17</sup>, nano<sup>18</sup>, permutation<sup>19-26</sup>, and other sets in different fields<sup>27-34</sup> are discussed in last years. To investigate our non-classical expansion, we'll use the concept of neutrosophic.

The neutrosophic interior /closur ( $cl^N(D)/int^N(D)$ ) to each (NS)  $D$  in ( $NTS$ ) ( $\Psi, \tau$ ) are defined by  $cl^N(D) = \cap \{D \subseteq B; B^c \in \tau\}$  and  $int^N(D) = \cup \{B \subseteq D; B \in \tau\}$ , respectively. The major goal of this work is to demonstrate and investigate new kinds of (NS), such as neutrosophic generalized preregular closed set, neutrosophic regular generalized  $\alpha$  – closed set neutrosophic generalized  $b$  – closed set, neutrosophic generalized  $\alpha$  – closed set, neutrosophic generalized semi pre – closed set, neutrosophic generalized  $\alpha b$  – closed set, neutrosophic regular generalized  $b$  – closed, neutrosophic semi generalized  $b$  – closed set. in neutrosophic topological spaces.

\*shuker.alsalem@gmail.com

## 2. PRELIMINARY

This section's core definitions are based on the research works in <sup>5,35-39</sup>.

### Definition 2.1:

Let  $\Psi \neq \emptyset, E = \{\langle c, \gamma_E(c), \rho_E(c), r_E(c) \rangle : c \in \Psi\}$  and  $M = \{\langle c, \gamma_M(c), \rho_M(c), r_M(c) \rangle : c \in \Psi\}$  be neutrosophic sets (NSs), Then;

- (1)  $M^c = \{\langle c, r_M(c), 1 - \rho_M(c), \gamma_M(c) \rangle : c \in \Psi\}$ ,
- (2)  $M \subseteq E$  iff  $\gamma_M(c) \leq \gamma_E(c), \rho_M(c) \geq \rho_E(c)$  and  $r_M(c) \geq r_E(c)$ ,
- (3)  $M \cup E = \{\langle c, \max\{\gamma_M(c), \gamma_E(c)\}, \min\{\rho_M(c), \rho_E(c)\}, \min\{r_M(c), r_E(c)\} \rangle : c \in \Psi\}$ ,
- (4)  $M \cap E = \{\langle c, \min\{\gamma_M(c), \gamma_E(c)\}, \max\{\rho_M(c), \rho_E(c)\}, \max\{r_M(c), r_E(c)\} \rangle : c \in \Psi\}$ .

**Definition 2.2:** Assume that  $\tau = \{t_j | j \in \Delta\}$  is a collection of neutrosophic sets (NSs) of  $\Psi$ . then  $(\Psi, \tau)$  is a neutrosophic topological space (NTS) if  $\tau$  satisfies:

- (1)  $t_m \cap t_k \in \tau, \forall t_m, t_k \in \tau$ ,
- (2)  $0_N = \{\langle \varepsilon, (0,1,1) \rangle : \varepsilon \in \Psi\} \in \tau$ , &  $1_N = \{\langle \varepsilon, (1,0,0) \rangle : \varepsilon \in \Psi\} \in \tau$ .
- (3)  $\bigcup_{j \in \Delta} t_j \in \tau$ , for any  $\Delta \subseteq \Delta$ . Also, if  $t_j \in \tau$ , we have  $t_j$  is called a neutrosophic open set (NOS), and  $t_j^c$  is called neutrosophic closed set (NCS).

## 3. NEW CLASSES OF NEUTROSOPHIC CLOSED SETS

First, a variation of neutrosophic generalized closed sets will be defined.

**Definition 3.1.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called a neutrosophic generalized preregular closed

set [briefly, (NgprCS)] if  $pcl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in NRO(\Psi, \tau)$ .

**Definition 3.2.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called a neutrosophic regular generalized  $\alpha$  – closed set [briefly, (Nrg $\alpha$ CS)] if  $acl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in NRO(\Psi, \tau)$ .

**Definition 3.3.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called neutrosophic generalized  $b$  – closed [briefly, (NgbCS)] if  $bcl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in \tau$ .

**Definition 3.4.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called neutrosophic generalized  $\alpha$  – closed set [briefly, (Ng $\alpha$ CS)] if  $acl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in \tau^\alpha$ .

**Definition 3.5.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called neutrosophic generalized semi pre – closed set

[briefly, (NgspCS)] if  $spcl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in \tau$ .

**Definition 3.6.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called neutrosophic generalized  $\alpha b$  – closed set [briefly, (NgabCS)] if  $bcl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in \tau^\alpha$ .

**Definition 3.7** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  is called neutrosophic regular generalized  $b$  – closed [briefly, (NrgbCS)] in  $\Psi$  if and only if  $bcl^N D \subseteq C$ , whenever  $D \subseteq C$  and  $C \in NRO(\Psi, \tau)$ .

**Definition 3.8.** Suppose that  $(\Psi, \tau)$  is a (NTS). A (NS)  $D$  of  $\Psi$  is called neutrosophic semi generalized  $b$  – closed set [briefly, (NsgbCS)] if  $bcl^N D \subseteq C$  whenever  $D \subseteq C$  and  $C \in NSO(\Psi, \tau)$ .

**Example 3.9:** Assume that  $\Psi = \{\alpha, \beta, \sigma\}$  and  $K, H$  are (NSs), where  $K = \{\langle \alpha, (1,0,1) \rangle, \langle \beta, (0,1,1) \rangle, \langle \sigma, (1,1,0) \rangle\}$  &  $H = \{\langle \alpha, (1,1,1) \rangle, \langle \beta, (1,0,0) \rangle, \langle \sigma, (0,0,1) \rangle$ . Now, assume that  $\tau = \{1_N, 0_N, K, H\}$ , thus  $(\Psi, \tau)$  is a (NTS) and  $H$  is a (NS) in

$\Psi$  such that  $\alpha cl^N H = Pcl^N H = bcl^N H = Scl^N H = H$ . Hence  $H$  is a  $(NrgbCS)$ ,  $(Nrg\alpha CS)$ ,  $(NgprCS)$ ,  $(NgbCS)$ ,  $(NgspCS)$ ,  $(Ng\alpha CS)$ ,  $(NgabCS)$ , and  $(NsrgbCS)$  in  $\Psi$ .

**Proposition 3.10.** Let  $T$  be  $(NCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $T$  is a  $(NCS)$  in  $(NTS)$   $\Psi$  satisfies  $T \subseteq F$ , where  $F \in NRO(\Psi, \tau)$ . Since  $T$  is  $(NCS)$ , thus  $cl^N T = T$ . However  $bcl^N D \subseteq cl^N D$ , where  $D$  is any in  $\Psi$ . Hence  $bcl^N T \subseteq F$ . Then  $T$  is  $(NrgbCS)$  in  $\Psi$ .

**Proposition 3.11.** Let  $T$  be  $(NbCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $D$  is a  $(NbCS)$  in  $(NTS)$   $\Psi$  with  $D \subseteq C$ . where  $C \in NRO(\Psi, \tau)$ . Since  $D$  is  $(NbCS)$ , hence  $bcl^N D = D$ . Then  $bcl^N D \subseteq C$ . Therefore  $D$  is  $(NrgbCS)$ .

**Proposition 3.12** Let  $T$  be  $(N\alpha CS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof** Suppose that  $K$  is a  $(N\alpha CS)$  in  $\Psi$  and  $B$  is a  $(NROS)$  containing  $K$ . Since  $K$  is  $(N\alpha CS)$ , thus  $\alpha cl^N K = K$ . However  $bcl^N D \subseteq \alpha cl^N D$ , where  $D$  is any  $(NS)$  in  $\Psi$ . So,  $bcl^N K \subseteq B$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.13.** Let  $T$  be  $(NSCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $K$  is a  $(NSCS)$  in  $\Psi$  and  $B$  is a  $(NROS)$  containing  $K$ . Since  $K$  is  $(NSCS)$ , thus  $Scl^N K = K$ . However  $cl^N D \subseteq Scl^N D$ , where  $D$  is any  $(NS)$  in  $\Psi$ . Thus  $bcl^N K \subseteq B$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.14.** Let  $T$  be  $(NPCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $K$  is a  $(NPCS)$  in  $\Psi$  and  $B$  is a  $(NROS)$  containing  $K$ . Since  $K$  is  $(NPCS)$ , thus  $Pcl^N K = K$ . However  $cl^N D \subseteq Pcl^N D$ , where  $D$  is any  $(NS)$  in  $\Psi$ . So,  $bcl^N K \subseteq B$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.15.** Let  $T$  be  $(NgCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $H$  is a  $(NgCS)$  in  $\Psi$  and  $C$  is a  $(NROS)$  containing  $H$ . Also, any  $(NROS)$  is  $(NOS)$  and  $H$  is  $(NgCS)$ , so  $cl^N H \subseteq C$ . However  $cl^N D \subseteq cl^N D$ , for any  $(NS)$   $D$  in  $\Psi$ . So,  $bcl^N H \subseteq C$ . Then  $H$  is  $(NrgbCS)$ .

**Proposition 3.16.** Let  $T$  be  $(NpgCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $H$  is a  $(NpgCS)$  in  $\Psi$  and  $C$  is a  $(NROS)$  containing  $H$ . Also, any  $(NROS)$  is  $(NPOS)$  and  $H$  is  $(NpgCS)$ , so  $Pcl^N H \subseteq C$ . However  $bcl^N D \subseteq Pcl^N D$ , for any  $(NS)$   $D$  in  $\Psi$ , thus  $bcl^N H \subseteq C$ . Then  $H$  is  $(NrgbCS)$ .

**Proposition 3.17.** Let  $T$  be  $(NgbCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $H$  is a  $(NgbCS)$  in  $\Psi$  and  $C$  is a  $(NROS)$  containing  $H$ . Also, any  $(NROS)$  is  $(NOS)$ , and  $H$  is  $(NgbCS)$ , so  $bcl^N H \subseteq C$ . Then  $H$  is  $(NrgbCS)$ .

**Proposition 3.18.** Let  $T$  be  $(NgspCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $H$  is a  $(NgspCS)$  in  $\Psi$  and  $C$  is a  $(NROS)$  containing  $H$ . Also, any  $(NROS)$  is  $(NOS)$  and  $H$  is  $(NgspCS)$ , so  $SPcl^N H \subseteq C$ . However  $SPcl^N H = H \cup int^N(cl^N(int^N H))$ , thus  $H \cup int^N(cl^N(int^N H)) \subseteq C$ . Also,  $int^N(cl^N(int^N H)) \subseteq cl^N(int^N H) \cap int^N(cl^N H) \subseteq int^N(cl^N H)$ . However,  $SPcl^N D \subseteq bcl^N D \subseteq scl^N D$ , where  $D$  is any  $(NS)$  in  $\Psi$ . Since  $C \in NRO(\Psi, \tau)$  and  $H \subseteq C$ , so  $int^N(cl^N C) = C$  and  $Scl^N H \subseteq Scl^N C$ . Further, we get  $H \cup int^N(cl^N H) = Scl^N H \subseteq Scl^N C = C \cup int^N(cl^N K(C)) = C$ , thus  $bcl^N H \subseteq C$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.19.** Let  $T$  be  $(Ng\alpha CS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $K$  is a  $(Ng\alpha CS)$  in  $\Psi$  and  $B$  is a  $(NROS)$  containing  $K$ . Also, any  $(NROS)$  is  $(N\alpha OS)$ , and  $K$  is  $(Ng\alpha CS)$ , so  $\alpha cl^N K \subseteq B$ . However  $bcl^N D \subseteq \alpha cl^N D$ , for any  $(NS)$   $D$  in  $\Psi$ , thus  $bcl^N K \subseteq B$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.20.** Let  $T$  be  $(NgabCS)$ , then  $T$  is  $(NrgbCS)$ .

**Proof.** Suppose that  $K$  is a  $(NgabCS)$  in  $\Psi$  and  $B$  is a  $(NROS)$  containing  $K$ . Also, any  $(NROS)$  is  $(N\alpha OS)$  and  $K$  is  $(NgabCS)$ , so  $bcl^N K \subseteq B$ . Then  $K$  is  $(NrgbCS)$ .

**Proposition 3.21.** Let  $T$  be ( $NsbGCS$ ), then  $T$  is ( $NrgbCS$ ).

**Proof.** Suppose that  $K$  is a ( $NsbGCS$ ) in  $\Psi$  and  $B$  is a ( $NROS$ ) containing  $K$ . Also, any ( $NROS$ ) is ( $NSOS$ ) and  $K$  is ( $NsgbCS$ ), so  $bcl^N K \subseteq B$ . Then  $K$  is ( $NrgbCS$ ).

**Remark 3.22** From Example (3.23), we show that the opposite of the above Propositions is not necessary hold.

**Example 3.23.** Assume that  $\Psi = \{\alpha, \beta, \sigma\}$  and  $K, H$  are (NSs), where  $K = \{\langle \alpha, (1,0.8,0.2) \rangle, \langle \beta, (0.5,0,0.4) \rangle, \langle \sigma, (0.7,0.2,0.5) \rangle\}$ , and  $H = \{\langle \alpha, (0.4,0.9,0.6) \rangle, \langle \beta, (0.3,0.1,0.7) \rangle, \langle \sigma, (0.5,0.4,1) \rangle\}$ . Now, assume that  $t = \{1_N, 0_N, K, H\}$ , so  $(\Psi, \tau)$  is a (NTS). Then  $S_1, S_2, S_3$  and  $S_4$  are all (NCSs) over  $\Psi$  can be found as the form:

$$S_1 = \{\langle \alpha, (1,0,0) \rangle, \langle \beta, (1,0,0) \rangle, \langle \sigma, (1,0,0) \rangle\},$$

$$S_2 = \{\langle \alpha, (0,1,1) \rangle, \langle \beta, (0,1,1) \rangle, \langle \sigma, (0,1,1) \rangle\},$$

$$S_3 = \{\langle \alpha, (0.6,0.1,0.4) \rangle, \langle \beta, (0.7,0.9,0.3) \rangle, \langle \sigma, (1,0.6,0.5) \rangle\},$$

$$S_4 = \{\langle \alpha, (0.2,0.2,1) \rangle, \langle \beta, (0.4,1,0.5) \rangle, \langle \sigma, (0.5,0.8,0.7) \rangle\}.$$

Clearly for the (NS)  $H$ ,  $H \subseteq H$ ,  $H \in \tau \subseteq \tau^\alpha \subseteq NSO(\Psi, \tau)$  and  $H \in \tau \subseteq NSPO(\Psi, \tau)$ . However  $cl^N(H) = bcl^N(H) = SPcl^N(H) = Pcl^N(H) = acl^N(H) = S_1 \not\subseteq H$ . Moreover,  $NRO(\Psi, \tau) = \{\emptyset, M_1\}$ . Thus  $H$  is ( $NrgbCS$ ). However it is not ( $NgbCS$ ), ( $NgspCS$ ), ( $Ng\alpha CS$ ), ( $NgabCS$ ), ( $NsgbCS$ ), ( $NgCS$ ), and ( $NpgCS$ ).

**Proposition 3.24.** Let  $T$  be ( $NrgCS$ ), then  $T$  is ( $NrgbCS$ ).

**Proof.** Suppose that  $K$  is a ( $NrgCS$ ) in  $\Psi$  and  $B$  is a ( $NROS$ ) containing  $K$ . Since  $K$  is ( $NrgCS$ ) and  $B \in NRO(\Psi, \tau)$ , so  $cl^N K \subseteq B$ . However  $cl^N D \subseteq cl^N D$ , for each (NS)  $D$  in  $\Psi$ , so  $bcl^N K \subseteq B$ . Then  $K$  is ( $NrgbCS$ ).

**Proposition 3.25.** Let  $T$  be ( $NgprCS$ ), then  $T$  is ( $NrgbCS$ ).

**Proof.** Suppose that  $T$  is a ( $NgprCS$ ) in  $\Psi$  and  $C$  is a ( $NROS$ ) containing  $T$ . Since  $C \in NRO(\Psi, \tau)$  and  $T$  is ( $NgprCS$ ), so  $Pcl^N T \subseteq C$ . However  $bcl^N D \subseteq Pcl^N D$ , where  $D$  is any (NS) in  $\Psi$ . Thus  $bcl^N T \subseteq C$ . Then  $T$  is ( $NrgbCS$ ).

**Proposition 3.26.** Let  $T$  be ( $Nrg\alpha CS$ ), then  $T$  is ( $NrgbCS$ ).

**Proof.** Suppose that  $T$  is a ( $Nrg\alpha CS$ ) in  $\Psi$  and  $C$  is a ( $NROS$ ) containing  $T$ . Since  $T$  is ( $Nrg\alpha CS$ ) and  $C \in NRO(\Psi, \tau)$ , then  $acl^N T \subseteq C$ . However  $cl^N D \subseteq acl^N D$ , for any (NS)  $D$  in  $\Psi$ , thus  $bcl^N T \subseteq C$ . Hence  $T$  is ( $NrgbCS$ ).

**Remark 3.27.** The opposite of Propositions [(3.24), (3.25), and (3.26)] might not be true.

**Example 3.28.** Assume that  $\Psi = \{\alpha, \beta, \sigma, \vartheta\}$  and  $K, T, D$  are (NSs), where  $K = \{\langle \alpha, (0.4,0.2,1) \rangle, \langle \beta, (0,1,0.6) \rangle,$

$$\langle \sigma, (0.8,1,1) \rangle, \langle \vartheta, (0,0.4,0.7) \rangle\}, T = \{\langle \alpha, (0,1,0.9) \rangle,$$

$$\langle \beta, (0.7,0.2,1) \rangle, \langle \sigma, (0,0.5,0.9) \rangle, \langle \vartheta, (0.6,1,1) \rangle\}, \text{ and}$$

$$D = \{\langle \alpha, (0,4,0.2, 0.9) \rangle, \langle \beta, (0.7,0.2,0.6) \rangle, \langle \sigma, (0.8,0.5,0.9) \rangle,$$

$\langle \vartheta, (0.6,0.4,0.7) \rangle\}$ . Now, assume that  $t = \{1_N, 0_N, K, H, D\}$ , so  $(\Psi, \tau)$  is a (NTS). Then  $S_1, S_2, S_3, S_4$  and  $S_5$  are all (NCSs) in  $\Psi$  can be found as the form:

$$S_1 = \{\langle \alpha, (1,0,0) \rangle, \langle \beta, (1,0,0) \rangle, \langle \sigma, (1,0,0) \rangle, \langle \vartheta, (1,0,0) \rangle\} = 1_N,$$

$$S_2 = \{\langle \alpha, (0,1,1) \rangle, \langle \beta, (0,1,1) \rangle, \langle \sigma, (0,1,1) \rangle, \langle \vartheta, (0,1,1) \rangle\} = 0_N,$$

$$S_3 = \{\langle \alpha, (1,0.8,0.4) \rangle, \langle \beta, (0.6,0,0) \rangle, \langle \sigma, (1,0,0.8) \rangle, \langle \vartheta, (0.7,0.6,0) \rangle\},$$

$$S_4 = \{\langle \alpha, (0.9,0,0) \rangle, \langle \beta, (1,0.8,0.7) \rangle, \langle \sigma, (0.9,0.5,0) \rangle, \langle \vartheta, (1,0,0.6) \rangle\},$$

$$S_5 = \{\langle \alpha, (0.9,0.8,0.4) \rangle, \langle \beta, (0.6,0.8,0.7) \rangle, \langle \sigma, (0.9,0.5,0.8) \rangle, \langle \vartheta, (0.7,0.6,0.6) \rangle\}.$$

Clearly for the (NS)  $T, T \subseteq T, T \in NRO(\Psi, \tau)$ . However  $cl^N(T) = Pcl^N(T) = \alpha cl^N(H) = S_3 \not\subseteq T$ . Also, there are only each of  $S_2$  and  $T$  is ( $NrOS$ ) and containing  $H$ . Furthermore,  $bcl^N(T) = T \subseteq S_2, bcl^N(T) = T \subseteq T$ . Then  $T$  is ( $NrgbCS$ ). However it is not ( $NgprCS$ ) or ( $NrgCS$ ) or ( $Nrg\alpha CS$ ).

**Proposition 3.29.** Assume that  $D$  is a ( $NrgbCS$ ) of a (NTS)  $(\Psi, \tau)$ . Then there is no ( $NRCS$ )  $C \neq \emptyset$  such that  $C \subseteq bcl^N D - D$ .

**Proof.** Assume that  $C \in NRC(\Psi, \tau)$  such that  $C \subseteq bcl^N D - D$ . Also, any ( $NRCS$ ) is ( $NCS$ ). So  $\Psi - C$  is ( $NOS$ ),  $D \subseteq \Psi - C$  and  $D$  is ( $NrgbCS$ ), it follows that  $bcl^N D \subseteq \Psi - C$  and thus  $C \subseteq \Psi - bcl^N D$ . Therefore,  $C \subseteq (\Psi - bcl^N D) \cap (bcl^N D - D) = \emptyset$ .

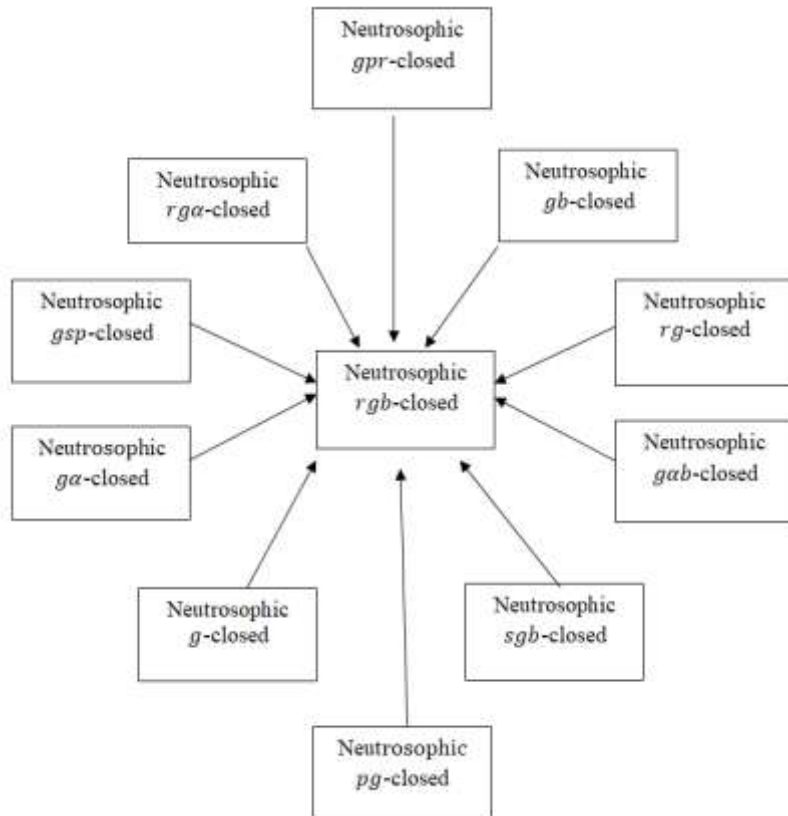
**Corollary 3.30.** Suppose that  $D$  is a ( $NrgbCS$ ). Then  $D$  is ( $NbCS$ ) if and only if  $bcl^N D - D$  is ( $NRCS$ ).

**Proof.** Assume that  $D$  is a ( $NrgbCS$ ). If  $D$  is ( $NbCS$ ), so we have  $bcl^N D - D = \emptyset$  which is ( $NRCS$ ). Conversely, assume that  $bcl^N D - D$  is ( $NRCS$ ). Hence, from Proposition (3.29),  $bcl^N D - D$  does not contain any non-empty ( $NRCS$ ) and since  $bcl^N D - D$  is ( $NRCS$ ) and subset of itself, so  $bcl^N D - D = \emptyset$ . Therefore,  $D = bcl^N D$  and so  $D$  is ( $NbCS$ ).

### 4. RESULTS AND DISCUSSION

This paper analyses several forms of neutrosophic closed sets (NTS) and their relationships, which can be explained by Fig (1), which is based on all premises given in this work:

**Figure (1):** The interrelationships between some types of (NS)



## 4. CONCLUSION

This work investigates several kinds of neutrosophic closed sets (NTS). Furthermore, their relationships are discussed. We can use neutrosophic maps in the future to investigate the extensions of our classes in the setting of a neutrosophic image.

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