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Soft(1, 2)-Strongly Open Maps in Bi-Topological Spaces

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Abstract. In this paper the concepts of soft (1, 2)-strongly open maps and soft (1, 2)-generality open maps are introduced and their relations with soft (1, 2)-open maps and soft (1, 2)-continuous maps are stated. We show that every soft (1, 2)-strongly open map is soft (1, 2)-generality open map and our work in this paper are examined. Furthermore, these our concepts are used to discuss the notion of soft (1, 2)-semi Hausdorff spaces in this work.

Keywords: Soft sets theory, bi-topological spaces, soft (1, 2)-continuous maps, soft (1, 2)-open maps.

Mathematics Subject Classification: 54A40, 03E72, 54E52, 54F65, 54G99.

1. Introduction

Molodtsov [24] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. In recent years, development in the fields of soft set theory and its application has been taking place in a rapid pace (See [7]-[22], [26]). Shabir and Naz [25] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Later, Zorlutuna et al. [27], Aygunoglu and Aygun [1], and Hussain et al. continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [2]. He investigated soft semiopen sets in soft topological spaces and studied some properties of them. After then Hussain [3] continue to give the properties of soft semi-open sets and soft semi-closed sets in soft topological spaces. He define soft semi-exterior, soft semi-boundary, soft semi-open neighborhood and soft semi-open neighborhood systems in soft topological spaces. Moreover he discuss the characterizations and properties of soft semi-interior, soft semi-exterior, soft semi-closure and soft semi-boundary in soft topological spaces. In 2014, Ittanagi [5] introduce and study the concept of soft bi-topological spaces which are defined over an initial universe with a fixed set of parameters then this concept is discussed with our new concepts in this work. The aim of this paper is



to introduce the notions of soft (1, 2)-strongly open maps and soft (1, 2)-generality open maps are introduced and their relations with soft (1, 2)-open maps and soft (1, 2)-continuous maps are stated.

2. Definitions and Notations

We will show some past results and basic definitions in this section. The following definitions have been used to obtain the results and properties developed in this paper.

2.1 Definition: ([24], [9]) Assume that U is an initial universe set and E is a set of parameters. Let F be a multi-valued function $F : A \rightarrow P(U)$, where $A \subseteq E$. We say (F, A) is a soft set over universe set U . In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(e)$, $e \in E$, from this family may be considered as the set of e -elements of the soft set (F, A) , or as the set of e -approximate elements of the soft set. Clearly, a soft set is not a set. For two soft sets (F, A) and (G, B) over the common universe U , we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \subseteq (G, B)$. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) . A soft set (F, A) over U is called a null soft set, denoted by $\Phi = (\phi, \phi)$, if $F(e) = \phi, \forall e \in A$. Similarly, it is called universal soft set, denoted by (U, E) , if $F(e) = U, \forall e \in A$. The collection of soft sets (F, A) over a universe set U and the parameter set A is a family of soft sets denoted by $SS(U_A)$.

2.2 Definition: ([7], [23]) The union of two soft sets (F, A) and (G, B) over X is the soft set (H, C) ,

where $C = A \cup B$ and for all $e \in C$, $H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$. We write $(F, A) \tilde{\cup} (G, B) = (H, C)$. The intersection (H, C) of (F, A) and (G, B) over X , denoted $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

2.3 Definition: ([27]) The soft set $(F, A) \in SS(U_A)$ is called a soft point in (U, A) , denoted by e_F , if for the element $e \in A$, $F(e) \neq \phi$ and $F(e') = \phi$ for all $e' \in A - \{e\}$. The soft point e_F is said to be in the soft set (G, A) , denoted by $e_F \tilde{\in} (G, A)$, if $e \in A$ and $F(e) \subseteq G(e)$.

2.4 Definition: ([25]) The difference (H, E) of two soft sets (F, E) and (G, E) over X , denoted by $(F, E) \setminus (G, E)$, is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

2.5 Definition: ([25]) Let (F, A) be a soft set over X . The complement of (F, A) with respect to the universal soft set (X, E) , denoted by $(F, A)^c$, is defined as (F^c, D) , where

$$D = E \setminus \{e \in A \mid F(e) = X\} = \{e \in A \mid F(e) = X\}^c, \text{ and } F^c(e) = X \setminus F(e) \text{ for all } e \in D,$$

2.6 Proposition: ([25]) Let (F, E) and (G, E) be the soft sets over X . Then

$$(1) ((F, E) \tilde{\cup} (G, E))^c = (F, E)^c \tilde{\cap} (G, E)^c$$

$$(2) ((F, E) \tilde{\cap} (G, E))^c = (F, E)^c \tilde{\cup} (G, E)^c.$$

2.7 Definition: ([8], [25])

Let τ be the collection of soft sets over X . Then τ is called a soft topology on X if τ satisfies the following axioms:

- (i) $\Phi, (X, E)$ belong to τ .
- (ii) The union of any number of soft sets in τ belongs to τ .
- (iii) The intersection of any two soft sets in τ belongs to τ .

The triplet (X, E, τ) is called a soft topological space over X . The members of τ are called soft open sets in X and complements of them are called soft closed sets in X .

2.8 Definition :([4])

The soft closure of (F, A) is the intersection of all soft closed sets containing (F, A) . (i.e, The smallest soft closed set containing (F, A) and is denoted by $cl^S(F, A)$). The soft interior of (F, A) is the union of all soft open set is contained in (F, A) and is denoted by $int^S(F, A)$.

2.9 Definition ([2])

A soft set (F, A) in a soft topological space (X, E, τ) will be termed soft semi-open if and only if there exists a soft open set (G, B) such that $(G, B) \subseteq (F, A) \subseteq cl^S(G, B)$.

2.10 Remark ([2]) Every soft open set in a soft topological space is a soft semi-open. But the converses is not true in general.

2.11 Definition ([6]) Let (X, E) and (Y, K) be soft classes and let $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Then a mapping $f : (X, E) \rightarrow (Y, K)$ is called soft map and defined as: for a soft set (F, A) in (X, E) , $(f(F, A), B)$, $B = p(A) \subseteq K$ is a soft set in (Y, K) given by

$$f(F, A)(\beta) = \begin{cases} u \left(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} F(\alpha) \right), & \text{if } p^{-1}(\beta) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise,} \end{cases} \quad \text{for } \beta \in B \subseteq K$$

$(f(F, A), B)$ is called a soft image of a soft set (F, A) . If $B = K$, then we shall write $(f(F, A), K)$ as $f(F, A)$.

2.12 Definition ([6]) Let $f : (X, E) \rightarrow (Y, K)$ be a mapping from a soft class (X, E) to another soft class (Y, K) and (G, C) be a soft set in soft class (Y, K) where $C \subseteq K$. Let $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Then $(f^{-1}(G, C), D)$, $D = p^{-1}(C)$, is a soft set in the soft classes (X, E) defined as:

$$f^{-1}(G, C)(\alpha) = \begin{cases} u^{-1}(G(p(\alpha))), & p(\alpha) \in C, \\ \emptyset, & \text{otherwise} \end{cases} \quad \text{for } \alpha \in D \subseteq E$$

$(f^{-1}(G, C), D)$ is called a soft inverse image of (G, C) . Hereafter, we shall write $(f^{-1}(G, C), E)$ as $f^{-1}(G, C)$.

2.13 Theorem ([6]) Let $f : (X, E) \rightarrow (Y, K)$, $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Then for soft sets (F, A) , (G, B) and a family of soft sets (F_i, A_i) in the soft class (X, E) we have:

- (1) $f(\Phi) = \Phi$,

- (2) $f((X, E)) = (Y, K)$,
 (3) If $(F, A) \subseteq (G, B)$ then $f((F, A)) \subseteq f((G, B))$,
 (4) $f^{-1}(\Phi) = \Phi$,
 (5) $f^{-1}((Y, K)) = (X, E)$,
 (6) If $(F, A) \subseteq (G, B)$ then $f^{-1}((F, A)) \subseteq f^{-1}((G, B))$.

2.14 Definition ([6]) A soft mapping $f : (X, E, \tau_X) \rightarrow (Y, K, \tau_Y)$ is said to be soft continuous (briefly s-continuous) if the soft inverse image of each soft open set of (Y, K, τ_Y) is a soft open set in (X, E, τ_X) .

2.15 Definition ([6]) A soft mapping $f : (X, E, \tau_X) \rightarrow (Y, K, \tau_Y)$ is said to be soft open (briefly s-open) if soft image of each soft open set of (X, E, τ_X) is a soft open set in (Y, K, τ_Y) .

2.16 Definition ([6]) A soft continuous mapping $f : (X, E, \tau_X) \rightarrow (Y, K, \tau_Y)$ is said to be soft homeomorphism if f is onto, one to one and f^{-1} is soft continuous.

2.17 Definition ([5])

Let τ_1 and τ_2 be the two different soft topologies on (X, E) . Then (X, E, τ_1, τ_2) is called a soft bitopological space.

2.18 Definition ([5]) Let (X, E, τ_1, τ_2) be a soft bi-topological space. Then a soft subset (F, A) of (X, E) is called soft (1,2)-open set if $(F, A) = (F_1, A_1) \tilde{\cup} (F_2, A_2)$, where $(F_1, A_1) \in t_1$ and $(F_2, A_2) \in t_2$. The complement of soft (1, 2)-open set is called soft (1, 2)-closed set.

2.19 Example

Let $U = \{a, b, c, d, e\}$ and $E = \{m, n\}$, and let $\tau_1 = \{(U, E), \Phi, (F, E)\}$ and $\tau_2 = \{(U, E), \Phi, (G, E)\}$ be soft topologies over U , where $F(m) = \{a, c, e\}$, $F(n) = U$ and $G(m) = \{b, c\}$, $G(n) = \{a, c, d\}$. The soft sets $(U, E), \Phi, (F, E), (G, E)$ are soft (1,2)-open sets in (U, E, τ_1, τ_2) . Also, (H, E) is a soft (1,2)-open sets in (U, E, τ_1, τ_2) where $H(m) = \{a, b, c, e\}$ and $H(n) = U$.

2.20 Definition ([5]) Let $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a map from a soft bi-topological space (X, E, τ_1, τ_2) into $(Y, K, \lambda_1, \lambda_2)$, we say that f is soft (1, 2)-continuous map if $f^{-1}(G, B)$ is soft (1, 2)-open set in (X, E, τ_1, τ_2) for each soft (1, 2)-open set (G, B) in $(Y, K, \lambda_1, \lambda_2)$.

2.21 Definition ([5]) Let $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a map from a soft bi-topological space (X, E, τ_1, τ_2) into $(Y, K, \lambda_1, \lambda_2)$, we say that f is soft (1, 2)-open map if $f(F, A)$ is soft (1, 2)-open set in $(Y, K, \lambda_1, \lambda_2)$ for each soft (1, 2)-open set (F, A) in (X, E, τ_1, τ_2) .

3. Soft Strongly Open Maps:

3.1 Definition

Let $f: (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a map from a soft bi-topological space (X, E, τ_1, τ_2) into $(Y, K, \lambda_1, \lambda_2)$, we say that f is soft (1, 2)-strongly open map if $f((F, A)) \in \lambda_1 \cup \lambda_2$ for each soft semi open set (F, A) in $(X, E, \tau_1 \cap \tau_2)$.

3.2 Example

Let $f: (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a map from any soft bi-topological space (X, E) into (Y, K) , where λ_1 or λ_2 is a soft discrete topology, then we consider that $\lambda_1 \cup \lambda_2 = SS(U_E)$. Hence, for any soft semi open set (F, A) in $(X, E, \tau_1 \cap \tau_2)$ we have $f((F, A)) \in \lambda_1 \cup \lambda_2$ and this implies that f is a soft (1, 2)-strongly open map.

3.3 Remark

It is clear that every soft (1, 2)-strongly open map is soft (1, 2)-open map but the converse is not true. (see example 3.4).

3.4 Example

Let $X = \{s_1, s_2, s_3, s_4\}$ be the set of cars under consideration. Let $E = \{\text{red color } (b_1); \text{ black color } (b_2); \text{ white color } (b_3); \text{ yellow color } (b_4)\}$ be the set of parameters framed to buy the best car. Suppose that the soft set (F, A) describing the Mr. X opinion to choose the best color is defined by $A = \{b_1, b_2\}$, $F(b_1) = \{s_2\}$, $F(b_2) = \{s_1, s_2\}$ and the soft set (G, B) describing the Mr. S opinion to choose the best color is defined by $B = \{b_1, b_2, b_4\}$, $G(b_1) = \{s_2, s_4\}$, $G(b_2) = X$, $G(b_4) = \{s_3\}$. We consider that:

$\tau_1 = \{\Phi, (X, E), (F, A)\}$ and $\tilde{\tau}_1 = \{\Phi, (X, E), (F, A), (G, B)\}$. Moreover, let the set of students under consideration be $Y = \{a_1, a_2, a_3, a_4\}$. Let $K = \{\text{pleasing personality } (e_1); \text{ conduct } (e_2); \text{ good result } (e_3); \text{ sincerity } (e_4)\}$ be the set of parameters framed to choose the best student. Suppose that the soft sets (H, C) , (G, B) and (T, K) describing the opinions of three teachers Mr. Z, Mr. W and Mr. Q respectively, to choose the best student of an academic year was defined by $C = \{e_1, e_4\}$, $H(e_1) = \{a_1\}$, $H(e_4) = \{a_1, a_2, a_3\}$, $B = \{e_1, e_3, e_4\}$, $G(e_1) = \{a_1, a_3\}$, $G(e_3) = \{a_1, a_2, a_3\}$, $G(e_4) = \{a_1, a_2, a_3\}$ and $T(e_1) = \{a_1, a_2, a_3\}$, $T(e_2) = \{a_1, a_2, a_3\}$, $T(e_3) = \{a_1, a_2, a_3\}$, $T(e_4) = \{a_1, a_2, a_3\}$. Then we consider that: $\tau_2 = \{\Phi, (Y, K), (T, K), (H, C), (G, B)\}$, let $\tau_2 = \tilde{\tau}_2$, $u: X \rightarrow Y$ and $p: E \rightarrow K$ be two mappings defined by $u(s_1) = a_1, u(s_2) = a_2, u(s_3) = a_3, u(s_4) = a_4$ and $p(b_1) = e_1, p(b_2) = e_2, p(b_3) = e_3, p(b_4) = e_4$. Then $f: (X, E, \tau_1, \tilde{\tau}_1) \rightarrow (Y, K, \tau_2, \tilde{\tau}_2)$ is a soft (1, 2)-open map but not soft (1, 2)-strongly open map, since there is a soft semi open set (N, R) in $(X, E, \tau_1 \cap \tilde{\tau}_1)$ where $R = \{b_1, b_2, b_3\}$, $N(b_1) = \{s_2\}$, $N(b_2) = \{s_1, s_2, s_4\}$, $N(b_3) = \{s_4\}$, but $f((N, R)) = (\Gamma, O) \notin \tau_2$ and hence $f((N, R)) \notin \tau_2 \cup \tilde{\tau}_2$, where $O = \{e_1, e_2, e_3\}$ and $\Gamma(e_1) = \{a_2\}$, $\Gamma(e_2) = \{a_1, a_2, a_4\}$, $\Gamma(e_3) = \{a_4\}$.

3.5 Theorem: If $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ is a soft (1, 2)-open map from soft discrete bi-topological space (X, E, τ_1, τ_2) into soft bi-topological space $(Y, K, \lambda_1, \lambda_2)$, then f is soft (1, 2)-strongly open map.

Proof: suppose (F, A) is soft semi open set in $(X, E, \tau_1 \cap \tau_2)$ we have (F, A) is soft (1, 2)-open set in (X, E, τ_1, τ_2) (since (X, E, τ_1, τ_2) is soft discrete bi-topological space). Hence $f((F, A))$ is soft (1, 2)-open set in $(Y, K, \lambda_1, \lambda_2)$ (since f is soft (1, 2)-open map), thus f is soft (1, 2)-strongly open map.

3.6 Definition: Let $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a soft (1, 2)-map from a soft bi-topological space (X, E, τ_1, τ_2) into $(Y, K, \lambda_1, \lambda_2)$, we say that f is a soft (1, 2)-generality open map if $f((F, A))$ is soft semi open set in $(Y, K, \lambda_1 \cap \lambda_2)$ for each (F, A) soft semi open set in $(X, E, \tau_1 \cap \tau_2)$.

3.7 Example

Let $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a map from any soft bi-topological space (X, E) into (Y, K) , where τ_1 or τ_2 is a soft indiscrete topology, then we consider that $\tau_1 \cap \tau_2 = \{\Phi, (X, E)\}$. Hence, for any soft semi open set (F, A) in $(X, E, \tau_1 \cap \tau_2)$ we have $(F, A) = \Phi$ or (X, E) and this implies that $f(F, A) = \Phi$ or (X, E) (by theorem (2.13,1-2)). Then f is a soft (1, 2)-generality open map.

3.8 Remark: It is clearly every soft (1, 2)-strongly open map is soft (1, 2)-generality open map but the converse is not true. (see example 3.9).

3.9 Example: take $f : (X, E, \tau_1, \check{\tau}_1) \rightarrow (Y, K, \tau_2, \check{\tau}_2)$ in Example (3.4), we have f is a soft (1, 2)-open map but not soft (1, 2)-strongly open map. For any soft semi open set (F, A) in $(X, E, \tau_1 \cap \check{\tau}_1)$, we have there exists soft open set (G, B) in $(X, E, \tau_1 \cap \check{\tau}_1)$ satisfies $(G, B) \subseteq (F, A) \subseteq cl^S(G, B)$. But, $(G, B) \in \tau_1 \cap \check{\tau}_1$, thus (G, B) can be written as union of two soft open sets $(G, B) \in \tau_1$ and $(G, B) \in \check{\tau}_1$. Then (G, B) is soft (1,2)-open set in $(X, E, \tau_1, \check{\tau}_1)$. Therefore $f((G, B))$ is soft (1,2)-open set in $(Y, K, \tau_2, \check{\tau}_2)$ [since f is a soft (1, 2)-open map]. Also, $f((G, B)) \subseteq f((F, A)) \subseteq f(cl^S(G, B)) \subseteq cl^S(f((G, B)))$. But $f((G, B)) \in \tau_2 \cap \check{\tau}_2$, then $f((F, A))$ is semi open set in $(Y, K, \tau_2 \cap \check{\tau}_2)$. Hence f is soft (1, 2)-generality open map but not soft (1, 2)-strongly open map.

3.10 Theorem: Every soft (1, 2)-continuous and soft (1, 2)-open map is soft (1, 2)-generality open map.

Proof: Let $f : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a soft (1, 2)-continuous and soft (1, 2)-open map from a soft bi-topological space (X, E, τ_1, τ_2) into $(Y, K, \lambda_1, \lambda_2)$, suppose (F, A) is soft semi open set in $(X, E, \tau_1 \cap \tau_2)$, then there exists soft open set (G, B) in $(X, E, \tau_1 \cap \tau_2)$ such that: $(G, B) \subseteq (F, A) \subseteq cl^S(G, B) \Rightarrow f((G, B)) \subseteq f((F, A)) \subseteq f(cl^S(G, B))$. Furthermore, the soft open set (G, B) is a soft (1, 2)-open set in bi-topological space (X, E, τ_1, τ_2) since $(G, B) \in \tau_1 \cap \tau_2$, this implies that $f(cl^S(G, B)) \subseteq cl^S(f((G, B)))$ [since each f is soft (1, 2)-continuous map]. Also,

$f((G, B))$ is soft (1, 2)-open set in $(Y, K, \lambda_1, \lambda_2)$ [since f soft (1, 2)-open map] , therefore $f((F, A))$ is soft semi open set in $(Y, K, \lambda_1 \cap \lambda_2)$. Hence f is soft (1, 2)-generality open map.

3.11 Remark: By the above results we have the following diagram:

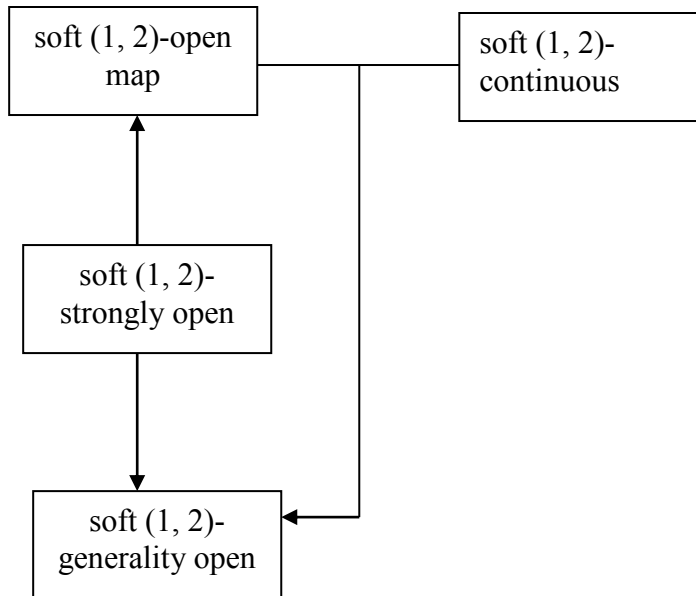


Figure 1: diagram showing relationships among some of the soft (1, 2)-maps

3.12 Definition: Let (X, E, τ_1, τ_2) be a soft bi-topological space and $e_F, e'_G \tilde{\in} (X, E)$ such that $e_F \neq e'_G$. Then (X, E, τ_1, τ_2) is called a soft (1, 2)-semi Hausdorff spaces or soft (1, 2)-semi T_2 -space if there exist two soft sets (F, A) and (G, B) where (F, A) is a semi open soft set in (X, E, τ_1) and (G, B) is a semi open soft set in (X, E, τ_2) such that $e_F \tilde{\in} (F, A)$, $e'_G \tilde{\in} (G, B)$ and $(F, A) \tilde{\cap} (G, B) = \Phi$.

3.13 Example:

Let (X, E, τ_1, τ_2) be a soft bi-topological space, where $X = Z = \{0, \bar{+}1, \bar{+}2, \dots\}$ and $E = N = \{0, 1, 2, \dots\}$, $\tau_1 = \{\Phi, (X, E)\} \cup \{(F, A_e) | e \in N\}$, $\tau_2 = \{\Phi, (X, E)\} \cup \{(G, A_e) | e \in N\}$,

$$A_e = \{0, 1, \dots, e\}, \quad F(x) = \begin{cases} Z_e(\text{even integers set}), & \text{if } x \text{ is even} \\ Z_o(\text{odd integers set}) & \text{if } x \text{ is odd} \end{cases}, \quad \forall x \in A_e, \quad \text{and}$$

$$G(x) = \begin{cases} Z_e(\text{even integers set}), & \text{if } x \text{ is odd} \\ Z_o(\text{odd integers set}) & \text{if } x \text{ is even} \end{cases}, \quad \forall x \in A_e. \quad \text{Then } F(x) \neq G(x), \quad \text{for any}$$

$x \in A_e = \{0, 1, \dots, e\}$. This implies that $(F, A_e) \tilde{\cap} (G, A_e) = \Phi$ for any $A_e \subseteq E$. Also, for any $e_F \neq e'_G \tilde{\in} (X, E)$, there are two disjoint soft semi open sets $e_F \tilde{\in} (F, A_e), e'_G \tilde{\in} (G, A_e)$ if $e = e'$. But, if $e \neq e' \in E \rightarrow e > e'$ or $e < e'$. Thus $e', e \in A_e$ or $e', e \in A_{e'}$. Hence, there are two disjoint soft semi

open sets $[e_F \tilde{\in} (F, A_e), e'_G \tilde{\in} (G, A_e), \text{ if } e > e']$ or $[e_F \tilde{\in} (F, A_e), e'_G \tilde{\in} (G, A_e), \text{ if } e < e']$. Then (X, E, τ_1, τ_2) is soft (1, 2)-semi Hausdorff spaces.

3.14 Theorem:

Let $f: (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \lambda_1, \lambda_2)$ be a soft (1, 2)-homeomorphism from soft (1, 2)-semi T_2 – space (X, E, τ_1, τ_2) into soft bi-topological space $(Y, K, \lambda_1, \lambda_2)$, then $(Y, K, \lambda_1, \lambda_2)$ is soft (1, 2)-semi T_2 – space.

Proof:

Let $y_1 \neq y_2 \tilde{\in} (Y, K)$, since f onto, then there exist two soft points such that $x_1, x_2 \tilde{\in} (X, E) \ni f(x_1) = y_1 \neq y_2 = f(x_2) \Rightarrow x_1 \neq x_2$ [by definition the soft (1, 2)-map]. Now, since (X, E, τ_1, τ_2) a soft (1, 2)-semi T_2 – space, then there exist $(F, A), (G, B)$ two disjoint soft (1,2)-semi open sets in (X, E, τ_1, τ_2) such that $x_1 \tilde{\in} (F, A), x_2 \tilde{\in} (G, B) \Rightarrow y_1 = f(x_1) \tilde{\in} f((F, A)), y_2 = f(x_2) \tilde{\in} f((G, B))$. Since f soft (1, 2)-continuous and soft (1, 2)-open map, then f is soft (1, 2)-generality open map by (3,10), therefore $f((F, A))$ and $f((G, B))$ are soft (1, 2)-semi open sets in $(Y, K, \lambda_1, \lambda_2)$, suppose $f((F, A)) \tilde{\cap} f((G, B)) \neq \Phi$, then there exist some soft point b in (Y, K) such that $b \tilde{\in} f((F, A)) \& b \tilde{\in} f((G, B))$, but this contradiction since f onto and $(F, A) \tilde{\cap} (G, B) = \Phi$, then $(Y, K, \lambda_1, \lambda_2)$ is soft (1, 2)-semi T_2 – space.

3.15 Theorem:

Let (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ be two soft bi-topological spaces if (F, A) soft (1, 2)-semi open set in (X, E, τ_1, τ_2) and (G, B) soft (1, 2)-semi open set in $(Y, K, \lambda_1, \lambda_2)$. Then $(F, A) \times (G, B)$ is also soft (1, 2)-semi open set in $(X, E) \times (Y, K)$.

Proof:

Since (F, A) and (G, B) are soft (1, 2)-semi open sets in (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ respectively, then there are two soft (1, 2)-open sets (D, M) and (H, C) in (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ respectively, such that $(D, M) \tilde{\subseteq} (F, A) \tilde{\subseteq} cl(D, M)$ and $(H, C) \tilde{\subseteq} (G, B) \tilde{\subseteq} cl(H, C)$. However, $cl(D, M) \times cl(H, C) = cl((D, M) \times (H, C))$. Hence $(F, A) \times (G, B)$ is also soft (1, 2)-semi open set in $(X, E) \times (Y, K)$.

3.16 Theorem:

Let $(X, E) \times (Y, K)$ be a soft bi-topological space, if $H = (F, A) \times (G, B)$ soft (1, 2)-semi open set in $(X, E) \times (Y, K)$, then (F, A) is soft (1, 2)-semi open set in (X, E, τ_1, τ_2) and (G, B) is soft (1, 2)-semi open set in $(Y, K, \lambda_1, \lambda_2)$.

Proof:

Since $H = (F, A) \times (G, B)$ is soft (1, 2)-semi open set in $(X, E) \times (Y, K)$, then there exists soft (1,2)-open set $D = U^\# \times V^\#$ in $(X, E) \times (Y, K)$ such that $D \subseteq H \subseteq cl(D) \Rightarrow U^\# \times V^\# \subseteq (F, A) \times (G, B) \subseteq cl(U^\# \times V^\#) = cl(U^\#) \times cl(V^\#)$. Then we consider that $U^\# \subseteq (F, A) \subseteq cl(U^\#)$ and $V^\# \subseteq (G, B) \subseteq cl(V^\#)$, also $U^\#$ soft (1, 2)-open set in (X, E, τ_1, τ_2) and $V^\#$ soft (1, 2)-open set in $(Y, K, \lambda_1, \lambda_2)$, thus (F, A) is soft (1, 2)-semi open set in (X, E, τ_1, τ_2) and (G, B) is soft semi open set in $(Y, K, \lambda_1, \lambda_2)$.

3.17 Theorem:

Let (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ be two soft bi-topological spaces. Then $(X, E) \times (Y, K)$ is a soft (1, 2)-semi T_2 – space, if, and only if, (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ are soft (1, 2)-semi T_2 – spaces.

Proof:

Suppose $(X, E) \times (Y, K)$ is a soft (1, 2)-semi T_2 – space. Let $P_1: (X, E) \times (Y, K) \rightarrow (X, E)$, where $P_1(x, y) = x, \forall (x, y) \in (X, E) \times (Y, K)$ and $P_2: (X, E) \times (Y, K) \rightarrow (Y, K)$, where $P_2(x, y) = y, \forall (x, y) \in (X, E) \times (Y, K)$ be projection maps, it is clear that P_1, P_2 are soft (1, 2)-continuous, onto and soft (1, 2)-open maps, suppose $x_1 \neq x_2 \in (X, E), y_1 \neq y_2 \in (Y, K)$ since P_1, P_2 onto (1,2)-maps, then there exist some soft point $(a, b), (c, d), (e, f), (m, n) \in (X, E) \times (Y, K) \ni P_1(a, b) = x_1, P_1(c, d) = x_2, P_2(e, f) = y_1, P_2(m, n) = y_2$, since $P_1(a, b) = x_1 \neq x_2 = P_1(c, d), P_2(e, f) = y_1 \neq y_2 = P_2(m, n)$ we have $(a, b) \neq (c, d), (e, f) \neq (m, n)$ [by definition the (1, 2)-map], then there exist two disjoint soft (1, 2)-semi open sets A_1, A_2 in $(X, E) \times (Y, K)$ where $(a, b) \in A_1, (c, d) \in A_2$, also there exist two disjoint soft (1, 2)-semi open sets B_1, B_2 in $(X, E) \times (Y, K)$ where $(e, f) \in B_1, (m, n) \in B_2$, we have $x_1 = P_1(a, b) \in P_1(A_1), x_2 = P_1(c, d) \in P_1(A_2), y_1 = P_2(e, f) \in P_2(B_1), y_2 = P_2(m, n) \in P_2(B_2)$. Since P_1, P_2 are soft (1, 2)-generality open maps by (3,10) we have $P_1(A_1), P_1(A_2)$ are soft (1, 2)-semi open sets in (X, E, τ_1, τ_2) and $P_2(B_1), P_2(B_2)$ are soft (1, 2)-semi open sets in $(Y, K, \lambda_1, \lambda_2)$, suppose $P_1(A_1) \cap P_1(A_2) \neq \Phi$, then there exist some soft point x in (X, E) such that $x \in P_1(A_1)$ and $x \in P_1(A_2)$, suppose $t = P_1^{-1}(x) \Rightarrow t \in P_1^{-1}P_1(A_1) = A_1, t \in P_1^{-1}P_1(A_2) = A_2$ [since P_1 is onto (1, 2)-map], thus $t \in A_1 \cap A_2 \neq \Phi$ but this contradiction since A_1, A_2 are disjoint soft sets, therefore $P_1(A_1) \cap P_1(A_2) = \Phi$, also it is clear that $P_2(B_1) \cap P_2(B_2) = \Phi$. Then (X, E, τ_1, τ_2) and $(Y, K, \lambda_1, \lambda_2)$ are soft (1, 2)-semi T_2 – spaces. Now, suppose (X, E) and (Y, K) are soft (1, 2)-semi T_2 – spaces. Let $(x_1, y_1) \neq (x_2, y_2) \in (X, E) \times (Y, K) \Rightarrow x_1, x_2 \in (X, E), y_1, y_2 \in (Y, K)$, it is clear that we have one of each is satisfy

- (1) – $x_1 \neq x_2 \wedge y_1 \neq y_2$
- (2) – $x_1 = x_2 \wedge y_1 \neq y_2$
- (3) – $x_1 \neq x_2 \wedge y_1 = y_2$

Now, if (1) is hold, then there exist two disjoint soft (1, 2)-semi open sets G_1, G_2 in (X, E, τ_1, τ_2) where $x_1 \tilde{\in} G_1, x_2 \tilde{\in} G_2$, also there exist two disjoint soft (1, 2)-semi open sets h_1, h_2 in $(Y, K, \lambda_1, \lambda_2)$ where $y_1 \tilde{\in} h_1, y_2 \tilde{\in} h_2 \Rightarrow (x_1, y_1) \tilde{\in} G_1 \times h_1, (x_2, y_2) \tilde{\in} G_2 \times h_2$, also $G_1 \times h_1$ and $G_2 \times h_2$ are soft (1, 2)-semi open sets in $(X, E) \times (Y, K)$ by (3,15), suppose $(G_1 \times h_1) \tilde{\cap} (G_2 \times h_2) \neq \Phi$, then there exist some soft point (x, y) in $(X, E) \times (Y, K)$ such that $(x, y) \tilde{\in} G_1 \times h_1 \wedge (x, y) \tilde{\in} G_2 \times h_2 \Rightarrow x \tilde{\in} G_1 \tilde{\cap} G_2, y \tilde{\in} h_1 \tilde{\cap} h_2$, but this contradiction since G_1, G_2 are disjoint soft sets also h_1, h_2 are disjoint soft sets, then $(G_1 \times h_1) \tilde{\cap} (G_2 \times h_2) = \Phi \Rightarrow (X, E) \times (Y, K)$ is soft (1, 2)-semi T_2 – space .

If (2) satisfy, let $d \tilde{\in} (X, E) - \{x_1\} \Rightarrow d \neq x_1 \tilde{\in} (X, E)$, then there exist two disjoint soft (1, 2)-semi open sets D, G in (X, E) where $d \tilde{\in} D, x_1 \tilde{\in} G$ also there exist two disjoint soft semi open sets h_1, h_2 in (Y, K) where $y_1 \tilde{\in} h_1, y_2 \tilde{\in} h_2 \Rightarrow (x_1, y_1) \tilde{\in} G \times h_1, (x_1, y_2) = (x_2, y_2) \tilde{\in} G \times h_2$, also $G \times h_1$ and $G \times h_2$ are soft (1, 2)-semi open sets in $(X, E) \times (Y, K)$ by (3,15), suppose $(G \times h_1) \tilde{\cap} (G \times h_2) \neq \Phi$. Hence there exist some soft point (x, y) in $(X, E) \times (Y, K)$ such that $(x, y) \tilde{\in} G \times h_1 \wedge (x, y) \tilde{\in} G \times h_2 \Rightarrow y \tilde{\in} h_1 \tilde{\cap} h_2$ but this contradiction since h_1 and h_2 are disjoint soft sets, then $(X, E) \times (Y, K)$ is soft (1, 2)-semi T_2 – space. If (3) satisfy, we can prove that by similar (2).

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