THE SET OF BISEQUENSES OVER PRIMARY VARIANTS

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Abstract

In this paper , at the beginning we attempted to introduce some preliminary concepts for bisequences . After that we explained The collection of primary variants T_p Where each element t_p in T_p ($p \in Z$) is called primary variant , also (by definition of the set of all bisequences on finite set) , we obtained the set of all bisequences primary variant $X(t_j^*) = \{t_j^*\}_{j=-\infty}^{\infty}$ and collection of all bisequences of primary variants χ and some subset of χ such as χ^o , χ^e , χ^+ and χ^- , we consider A the collection of all abelian variants groups and P_k symmetric variants group also we introduce the homomorphism f_χ : Hom χ (B , G) $\longrightarrow$ Hom χ (A , G) , we introduce some of theorems and study some of their basic properties and at last we show that χ is a topological space .

Introduction

In this paper , we introduce the collection of primary variants T_p, we use the bisequences which defined in symbolic dynamic [4].

We obtained $X(t_i) = \{t_i\}_{i=-\infty}^{\infty}$ and say the set of all bisequences primary variant and denoted to the collection of the set of all bisequences primary variants by χ and we defined the sets χ^e, χ^+, χ^- which each of them is subset of χ .

We introduce some operations such as (• ,⊗ , ♦ , ⁎ , □) on the finite set Σ , infinite sets T_p , χ^- and χ , also we consider A the collection of all abelian primary variants group and symmetric variant group also we introduce the homomorphism f_χ from the Hom χ (B , G) group in to Hom χ (A , G) group where each of Hom χ (B , G) and Hom χ (A , G) are collection of all homomorphisms from A into G and from B into G and respectively ,where

A,B,G **in A and f a homomorphism from group A into group B , and we introduce some of theorems and study some of their basic properties .**

Finally we define a topology on the set χ

Definitions 1:

i) Let *S* be a finite set of n elements this finite set is often called the symbol set and each element in it called symbol or also called the alphabet [5] and in this case each element in it may be called letter.

ii)A doubly infinite sequence or bisequence *x* is a function from the set of Integers *Z* to alphabet or symbol set *S* that is each element x_i in it is a symbol or letter ([1] , [4]) .

Remarks 2:

i) To show zero image in doubly infinite sequence we put appoint to the left of the letter which represent the zero image that is a letter which represent zero image lies on the right of the

point [4] .

ii) the set of all doubly infinite sequences or bisequences on alphabet or symbol set *S* is denoted by *X*(*S*) .

iii) The topology which defined on *X*(*S*) is equivalent to product topology

$$S^{\infty} = \dots \quad S \times S \times S \times \dots$$

Where the topology which defined on alphabet *S* is the discrete topology [1]

Examples 3:

i) Shift map σ (which defined in [1]) is a homeomorphism from *X*(*S*) into itself and it is shifting a letter x_i one position to the left that is $[\sigma(x)]_i = x_{i+1}$.If *S* ={0,1}and if

$$x = \dots\ 0\ 0\ 0\ .1\ 0\ 0\ 0\ 0\dots$$

then $$\sigma(x) = \dots\ 0\ 0\ 1\ .0\ 0\ 0\ 0\ 0\dots$$

is a sequences above are sequences on symbol set or alphabet {0,1}.

ii) The following sequence

… ◊ # † # . † ◊ ◊ # ◊ …

is a sequence on symbole set {◊,#,†}.

iii) The following sequence

… a c d b . d a b a c …

is a sequence on alphabet {*a* , *b* , *c*, *d*}.

Definition 4: Let $T_p=\{t_i\}_i$ (such that $i\in z$ the set of integers) we say that T_p is primary variantsset and t_k primary variants , for each integer k

$$t_k=\begin{cases} t_{q^-} & k\le 0 \\ t_{q^+} & k\ge 0 \end{cases}$$ **where** $t_{q^-}=\{-q,1-q,\ldots,0\}$, $t_{q^+}=\{0,1,\ldots,q\}$ **for integer** $q\ge 0$ **and** $q=|k|$

Definition 5: Let • be an operation on the set T_p defined as fellow

$t_i \bullet t_j = t_{i+j}$ **for integers i and j .**

Theorem 6 : (T_p , •) is abelian (commutative) group

Proof :

i) See that $(t_i \bullet t_j) \bullet t_k = t_{i+j} \bullet t_k = t_{i+j+k} = t_i \bullet t_{i+j} = t_i \bullet (t_i \bullet t_k)$ and

$(t_i \bullet t_j) = t_{i+j} = t_{j+i} = (t_i \bullet t_i)$ that is • is associative and commutative .

ii) $t_0 \bullet t_i = t_i$ and $t_{-i} \bullet t_i = t_0$ so t_0 is identity of • and t_{-i} is the inverse of t_i .

Definition 7: Let χ be the collection of all bisequences of primary variants, that is each element in χ is all bisequences of primary variant that is

$\chi=\{X(t_i)\}_{i\in Z}$ we shall call χ the set of all bisequences of primary variants.

Example 8 : Let

$$x = \ldots\ 0\ 0\ 1\ .2\ 0\ 1\ 1\ \ldots$$

$$y = \ldots\ 0\ 0\ 3\ .1\ 0\ 2\ 1\ \ldots$$

see that $x\in X(t_2)$ and $x\in X(t_3)$ and $y\in X(t_3)$ but $y\notin X(t_2)$

Definitions 9 :

i) Let ∗ be an operation on χ defined by $X(t_i) * X(t_j) = X(t_k)$ where $k=\min\{|i|,|j|\}$.

ii) Let □ be an operation on χ defined by $X(t_i) \square X(t_j) = X(t_k)$ where $k=\max\{|i|,|j|\}$.

iii) Let ♦ be an operation on χ defined by $X(t_i) \blacklozenge X(t_j) = X(t_k)$ where $k=\min\{i, j\}$.

Definitions 10 :

i) $\chi^{e}=\{X(t_i);\ i$ **is even} ii)** $\chi^{+}=\{X(t_i);\ i>0$ **is integer} iii)** $\chi^{-}=\{X(t_i);\ i<0$ **is integer}**

Remarks 11 :

i) Both (χ , $*$) and (χ , $\square$) are commutative semi groups

ii) The system (χ^{-} , ♦) is commutative semi sub group with identity $X(t_{-1})$

iii) The system (χ^{+} , $\square$) is commutative semi sub group with identity $X(t_1)$

Definition 12 : Let † be an operation on χ defined as fellow

$$X(t_i) \dagger X(t_j) = X(t_{i+j})\ .$$

Theorem 13 : (χ , †) is abelian (commutative) group

Proof : $[X(t_i) \dagger X(t_j)] \dagger X(t_k) = X(t_{i+j}) \dagger X(t_k) = X(t_{i+j+k}) = X(t_i) \dagger X(t_{j+k})$

$= X(t_i) \dagger [X(t_j) \dagger X(t_k)]$ **then† is associative on χ.**

$X(t_0)$ **is identity of †**

$X(t_{-i})$ **is the inverse of** $X(t_i)$ **, for every integer** i

And it is clearly that † is commutative on χ because ($i+j$) = ($j+i$)

(note that (χ^{e} , †) is a subgroup of (χ , †))

Definition 14 : Let ‡ be an operation on χ defined as

$$X(t_i) \ddagger X(t_j) = X(t_{i+j-1})\ .$$

Theorem 15 : (χ , ‡) is abelian (commutative) group

Proof : $[X(t_i) \ddagger X(t_j)] \ddagger X(t_k) = X(t_{i+j-1}) \ddagger X(t_k) = X(t_{i+j-1+k-1}) = X(t_{i+j+k-2})$

$X(t_i) \ddagger [X(t_j) \ddagger X(t_k)] = X(t_i) \ddagger X(t_{j+k-1}) = X(t_{i+j+k-1-1}) = X(t_{i+j+k-2})$

Then ‡ is associative on χ.

$X(t_i) \ddagger X(t_1) = X(t_{i+1-1}) = X(t_i)$ **so is identity of ‡**

$X(t_{2-i})$ **is the inverse of** $X(t_i)$ **because for every integer** i **we have**

$X(t_{2-i}) \ddagger X(t_i) = X(t_{2-i+i-1}) = X(t_1)$

And since ($i+j-1$) = ($j+i-1$) so ‡ is commutative on χ

Definition 16 : Let $\Sigma = \{X(S_1), X(S_2), \ldots, X(S_r)\}$ ($r > 0$ is integer) the set of all bisequences of alphabets $S_1, S_2, \ldots, S_r$ such that $S_1 \subset S_2 \ldots \subset S_r$ where S_i ($i=1,\ldots,r$) is alphabet of i letter(s).

Note that the elements of S may be numbers , letters or symbols like ∗ , # , … , etc.

Also note Σ may be a subset of χ if $S_i = t_{i-1}$ ($i=1,2,\ldots,r$)

Definition 17 : Let κ and η be positive integers such that ($1 \le \kappa \le r$ and $1 \le \eta \le r$) and let $\otimes$ be an operation on the set Σ defined as fellow

$$X(S_\kappa) \otimes X(S_\eta) = X(S_\rho) \text{ where } \rho = (\kappa+\eta-1)(\text{mod } r) .$$

Theorem 18 : (Σ , $\otimes$) is abelian (commutative) group

Proof : $\otimes$ is associative because

$\{X(S_\kappa) \otimes X(S_\eta)\} \otimes X(S_{\acute{\eta}}) = X(S_\nu) \otimes X(S_{\acute{\eta}})$ where $\nu=(\kappa+\eta-1)(\text{mod } r)=\kappa(\text{mod } r)+\eta(\text{mod } r)-1$.

$X(S_\nu) \otimes X(S_{\acute{\eta}})= X(S_\mu)$ where $\mu=(\nu+\acute{\eta}-1)(\text{mod } r) = \nu (\text{mod } r)+\acute{\eta}(\text{mod } r)-1$

$X(S_\kappa) \otimes \{X(S_\eta) \otimes X(S_{\acute{\eta}})\} = X(S_\kappa) \otimes X(S_q)$ where $q=(\eta+\acute{\eta}-1)(\text{mod } r)=\eta(\text{mod } r)+\acute{\eta}(\text{mod } r)-1$.

$X(S_\kappa) \otimes X(S_q) = X(S_\pi)$ where $\pi =(\kappa+q-1)(\text{mod } r)= \kappa(\text{mod } r)+q(\text{mod } r)-1$

but $\mu = \nu (\text{mod } r)+\acute{\eta}(\text{mod } r)-1= \kappa(\text{mod } r)+\eta(\text{mod } r)-1+\acute{\eta}(\text{mod } r)-1$

$=\kappa(\text{mod } r)+\eta(\text{mod } r) +\acute{\eta}(\text{mod } r)-1-1 =\kappa(\text{mod } r)+q(\text{mod } r)-1= \pi$

It is clearly that unique sequence on S_1 ($X(S_1)$) is identity of $\otimes$ and for every integer $1 \le m \le r$ there is an integer $1 \le q \le r$ such that $(m+q)$ (mod r) =2 . Then the inverse of $X(S_m)$ is $X(S_q)$ where $q=2-m+r$

$\rho=(\kappa+\eta-1)(\text{mod } r)=(\eta+\kappa-1)(\text{mod } r)$ that is $\otimes$ is commutative on Σ .

Definition 19 : Let ψ be a function defined from T_p to χ as

$$\psi(t_i) = X(t_i) .$$

Theorem 20 : $\psi: T_p \longrightarrow \chi$ is homomorphism

Proof : $\psi(t_i \bullet t_j)= \psi(t_{i+j}) = X(t_{i+j}) = X(t_i) \dagger X(t_j)$.

Definition 21 : Let φ be a function from T_p to Σ defined by

$\varphi(t_i) = X(S_\rho)$ where $\rho=(i)(\text{mod } r)+1$.

Theorem 22 : $\varphi: T_p \longrightarrow \Sigma$ is homomorphism .

Proof : $\varphi(t_i \bullet t_j) = \varphi(t_{i+j}) = X(S_\rho)$ where $\rho=(i+j)(\text{mod } r)+1=(i)(\text{mod } r)+(j)(\text{mod } r)+1$.

because $(i+j)(\text{mod } r) = (i)(\text{mod } r)+(j)(\text{mod } r)$ Since i & j are integers.

Let $\varphi(t_i)=X(S_\mu)$ where $\mu=(i)(\text{mod } r)+1$ and let $\varphi(t_j)=X(S_\eta)$ where $\eta=(i)(\text{mod } r)+1$.

Then $\varphi(t_i) \otimes \varphi(t_j) = X(S_\mu) \otimes X(S_\eta) = X(S_\pi)$ where $\pi = (\mu+\eta)(\text{mod } r) -1$

Then $\pi = \mu(\text{mod } r)+\eta(\text{mod } r)-1=(i)(\text{mod } r)+1+(j)(\text{mod } r)+1-1$

$=(i)(\text{mod } r)+(j)(\text{mod } r)+1=\rho$

Theorem 23 : Let $h:(T_p, \bullet) \longrightarrow (\chi^e, \dagger)$ be a map defined by $h(t_i) = X(t_{2i})$, for each integer i then h is isomorphism .

Proof : for every $t_i, t_j \in T_p$ *we* have

$h(t_i \bullet t_j)=h(t_{i+j})=X(t_{2(i+j)})=X(t_{2i+2j})=X(t_{2i}) \dagger X(t_{2j})=h(t_i) \dagger h(t_j) \Rightarrow h$ is homomorphism.

If $h(t_i)=h(t_j) \Rightarrow X(t_{2i})=X(t_{2j}) \Rightarrow t_{2i}=t_{2j} \Rightarrow t_i=t_j \Rightarrow h$ is monomorphism.

Now suppose that i is even integer then there exist integer j such that $j = i/2$ then $h(t_j)=X(t_i)$ therefore h is epimorphism.

Hence h is isomorphism .

Definitions 24 :

1) Let $\mathbf{A}$ ={ A is abelian group : either A is a subgroup of χ or A is a subgroup of T_p } , we say D is variants group for each $D \in \mathbf{A}$

2) Let A and B be any two elements in $\mathbf{A}$,we define Hom $\chi(A, B)$ be the set of all homomorphisms $f : A \longrightarrow B$

Remark 25 : The zero homomorphism $0 : A \longrightarrow B$ defined by $0(a)=e_B$, for every element $a \in A$, where e_B is identity element in Group B

Definition 26 : Let $*_B$ be an operation of Group B and let $\oplus$ be an operation on the set

Hom $\chi(A, B)$ defined by $(f \oplus g)(a) = f(a) *_B g(a)$ for every $a \in A$

Remark 27 : Note that $(f \oplus g)(a)$ is a function in Hom $\chi(A, B)$ and let us assume that

$(f \oplus g)(a) = f(a) *_B g(a) = h(a)$

Theorem 28 : The system (Hom $\chi(A, B)$, $\oplus$) is commutative group

Proof :$\oplus$ is associative operation since $*_B$ is associative (*B* is Group)

$(f \oplus 0)(a) = f(a) *_B 0(a) = f(a) *_B e_B = f(a)$ $\forall a \in A$ and $\forall f \in$ Hom $\chi(A,B)$

Then zero homomorphism $0 \in$ Hom $\chi(A, B)$ is identity element of $\oplus$

Let $f \in$ Hom $\chi(A, B) \Rightarrow f(a) \in B$ since *B* is group hence it is has to contain an inverse of any non identity element in *B*. Let $\bar{f}$ is an inverse of f , Then for each element $f \in$ Hom $\chi(A, B)$ there is inverse element $\bar{f} \in$ Hom $\chi(A, B)$ such that $(f \oplus \bar{f})(a) = e_B = 0(a)$.

By definition of A *B* is commutative group $\Rightarrow$

$(f \oplus g)(a) = f(a) *_B g(a) = g(a) *_B f(a) = (g \oplus f)(a)$ that is $\oplus$ is commutative .

Example 29 : Let h_1 and h_2 be two homomorphisms from $(T_p, \bullet)$ to $(\chi^e, \dagger)$ defined by

$h_1(t_i) = X(t_{2i})$ and $h_2(t_i) = X(t_{4i})$ $\forall t_i \in T_p$.

$(h_1 \oplus h_2)(t_i) = h_1(t_i) \oplus h_2(t_i) = X(t_{2i}) \dagger X(t_{4i}) = X(t_{6i}) = h(t_i) = (h_2 \oplus h_1)(t_i)$, then we have

i. **$h:(T_p, \bullet) \longrightarrow (\chi^e, \dagger)$ defined by $h(t_i) = X(t_{6i})$, $\forall t_i \in T_p$ and hence $h \in$ Hom $\chi(T_p, \chi^e)$.**
ii. **The zero homomorphism $0:(T_p, \bullet) \longrightarrow (\chi^e, \dagger)$ defined by $0(t_i) = X(t_o)$, $\forall t_i \in T_p$.**
iii. **$\bar{h}_1:(T_p, \bullet) \longrightarrow (\chi^e, \dagger)$ and $\bar{h}_2:(T_p, \bullet) \longrightarrow (\chi^e, \dagger)$ are two homomorphisms and they are**
iv. **inverse of h_1 and h_2 respectively where $\bar{h}_1(t_i) = X(t_{-2i})$ and $\bar{h}_2(t_i) = X(t_{-4i})$,$\forall t_i \in T_p$.**

Lemma 30 : If $f \in$ Hom $\chi(A, B)$ and *G* is an other abelian group then f induces a homomorphism f_χ: Hom $\chi(B, G) \longrightarrow$ Hom $\chi(A, G)$ which is given by

$f_\chi(g) = g \circ f$ $\forall g \in$ Hom $\chi(B, G)$

Proof :Let $g: B \longrightarrow G$ and $h: B \longrightarrow G$ be two homomorphisms in Hom $\chi(B, G)$ we have $f_\chi(g \oplus h) = (g \oplus h) \circ f = (g \oplus h)(f) = g(f) *_B h(f) = (g \circ f) *_B (h \circ f) = f_\chi(g) *_B f_\chi(h) = f_\chi(g) \oplus f_\chi(h)$.

Remarks 31 :

i. **If $f: A \longrightarrow B$, $g: B \longrightarrow G$ and $h: B \longrightarrow C$ then we have**

f_χ: Hom $\chi(B, G) \longrightarrow$ Hom $\chi(A, G)$ and h_χ: Hom $\chi(C, G) \longrightarrow$ Hom $\chi(B, G)$.

ii. **$f_\chi \circ h_\chi = (h \circ f)_\chi$ and $(f_1)_\chi \circ (f_2)_\chi \circ \ldots \circ (f_n)_\chi = (f_n \circ f_{n-1} \circ \ldots \circ f_3 \circ f_2 \circ f_1)_\chi$.**

Lemma 32 : If $f \in$ Hom $\chi(A, B)$, $g \in$ Hom $\chi(B, C)$ and $G \in$ A with $h \circ f = 1_A$, where $h \in$ Hom $\chi(B, A)$ $\Rightarrow f_\chi \circ h_\chi = 1_{\text{Hom}\,\chi(A, G)}$.

Proof : Let $\rho \in \chi(A, G)$. We have

$f_\chi \circ h_\chi(\rho) = f_\chi(\rho \circ h) = (\rho \circ h) \circ f = \rho \circ (h \circ f) = \rho \circ 1_A = 1_{\mathrm{Hom}\,\chi(A,G)}$.

Theorem 33 : If $f \in \mathrm{Hom}\,\chi(A, B)$, $g \in \mathrm{Hom}\,\chi(B, C)$ and $G \in$ A with $h \circ f = 1_A$ where $h \in \mathrm{Hom}\,\chi(B, A)$. Then

1) f_χ is an epimorphism
2) if $g : B \longrightarrow C$ epimorphism $\Rightarrow g_\chi$ monomorphism .

Proof :

1) From Lemma(32) we have $f_\chi \circ h_\chi = 1_{\mathrm{Hom}\,\chi(A,G)}$.

But $1_{\mathrm{Hom}\,\chi(A,G)}$ is an isomorphism therefore f_χ is an epimorphism .

2) We must prove that Ker $g_\chi = \{0\}$ (0 is zero homomorphism) .

Let $\delta \in$ Ker g_χ we have $g_\chi(\delta) = 0 \Rightarrow \delta \circ g = 0$.

Since $g : B \longrightarrow C$ is an epimorphism , then for every $c \in C$ there exists $b \in B$ such that $g(b) = c$, since $\delta \circ g = 0$ (0 is zero homomorphism) then $\delta \circ g(b) = 0(b) = e_g$ $\Rightarrow \delta(g(b)) = e_G \Rightarrow \delta(c) = e_G$, $\forall c \in C \Rightarrow \delta$ is zero homomorphism .This is show g_χ is monomorphism .

Definition 34 : A bijective mapping $f : t_k \longrightarrow t_k$ have the property that the set $\{a : f(a) \neq a$ for some $a \in t_k\}$ is finite , is called a permutation of t_k .

Remark 35 : The order of t_k denoted by $|t_k|$ is called the degree of the permutations of t_k .

Theorem 36 : The set of all the permutations of t_k ,

$P_k = \{f : t_k \longrightarrow t_k : f$ is bijective and $f(a) \neq a$ for some $a \in t_k\}$ is a permutation group under composition

Proof :

1) Composition functions is an associative operation .

2) The identity map $I = e$ is identity element for composition .

3) For each $f : t_k \longrightarrow t_k$, we have for every $j \in t_k$ there exists $i \in t_k$ such that $f(j) = i$ we could be defined inverse of f in P_k by $f^{-1}(i) = j$ for each $i \in t_k$ so there exists inverse of f

$f^{-1} : t_k \longrightarrow t_k$ for composition .

Remarks 37 :

1) $(P_k, \circ)$ is called symmetric variants group .

2) $|t_k| = \begin{cases} k+1 & k \geq 0 \\ 1-k & k < 0 \end{cases}$

3) $|P_k| = \begin{cases} k!+1 & k \geq 0 \\ 1+(-k)! & k < 0 \end{cases}$

4) $|P_k| = |P_l| \Rightarrow k = l$ **or** $k = -l$

Theorem 38 : The two symmetric groups (P_k , ○) and (P_l , ○) are isomorphic if and only if they have the same degree .

Proof :

Suppose(P_k , ○) isomorphic to (P_l , ○) and $|P_k| = m$, $|P_l| = n$ then we have isomorphism $h : P_l \longrightarrow P_k$ and $P_k = \{ f_i : i=1,2,\ldots, m\}$, $P_l = \{ g_i : i=1,2,\ldots, n\}$, h is epimorphism and monomorphism then h send each element in group P_l to exactly one element in group P_k therefore P_k and P_l have the same number of elements thus $|P_k| = |P_l|$

Now suppose $|P_k| = m = |P_l|$, that is $P_k = \{ f_i : i=1,2,\ldots, m\}$ and $P_l = \{ g_i : i=1,2,\ldots, m\}$ contains m elements , so we can define isomorphism from the group (P_k , ○) to the group (P_l , ○) by $h (f_i) = g_i$ $\forall i=1 , 2 , \ldots , m$, we have h is one to one homomorphism from the group (P_l , ○) on to the group (P_k , ○) $\Rightarrow$ (P_k , ○) and (P_l , ○) are isomorphic .

Definition 39 : Let t_{q^-} and t_{q^+} for positive integer q be the sets which defined in Definition 4 we shall denote to the collection of all the sets t_{q^-} by symbol A_{q^-} that is

$A_{q^-} = \{t_{q^-}\}_{q=0,1,2.3,\ldots}$ **and** $A_{q^+} = \{t_{q^+}\}_{q=0,1,2.3,\ldots}$

Remarks 40 :

1)The intersections of any two sets in A_{q^-} is one of them that is if for positive integers q and r

$t_{q^-}, t_{r^-} \in A_{q^-} \Rightarrow t_{q^-} \cap t_{r^-} = t_{k^-}$ **such that k=min{q , r} so as intersection of any two elements in A_{q^+} is one of them that is also for** $t_{q^+}, t_{r^+} \in A_{q^+} \Rightarrow t_{q^+} \cap t_{r^+} = t_{k^+}$ **, k=min{q , r} .**

2) The union of any two sets in A_{q^-} is one of them that is if for positive integers q and r

$t_{q^-}, t_{r^-} \in A_{q^-} \Rightarrow t_{q^-} \cup t_{r^-} = t_{k^-}$ **such that k=max{q , r} so as union of any two elements in A_{q^+} is one of them that is also for** $t_{q^+}, t_{r^+} \in A_{q^+} \Rightarrow t_{q^+} \cup t_{r^+} = t_{k^+}$ **.**

3) The intersections of a set in A_{q^-} with another set in A_{q^+} is t_0 and the union of a set in A_{q^-} with another set in A_{q^+} is the set $B_q = \{-q, 1-q, \ldots, 0, 1, \ldots r\}$ **for positive integers q and r**

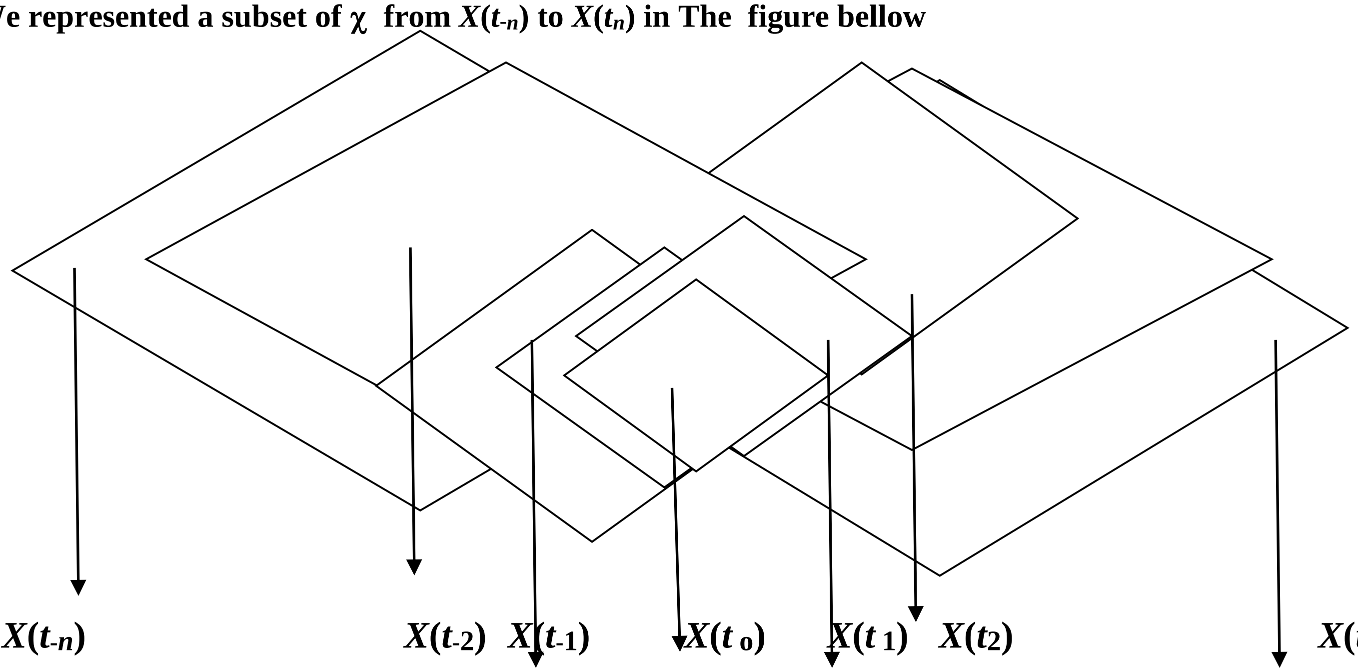
Definition 41: $\chi_{B_q} = \{X(B_q)\}_q = \{X(\{-q, 1-q, ..., 0, 1, ...q+k\})\}_q$ **for** $q \in z^+$ **and** $k \in z$.

Remark 42 : If $\chi_{A_{q^-}} = \{X(t_{q^-})\}_{q=0,1,2.3,...}$ **and** $\chi_{A_{q+}} = \{X(t_{q^+})\}_{q=0,1,2.3,...}$

Hence $\chi_{A_{q^-}} = \chi^-$ **and** $\chi_{A_{q+}} = \chi^+$

Definition 43: The set $\tau_\chi = \{\phi, \chi, \chi^-, \chi^+, \chi_{B_q}\}$ **is topology on** χ

We represented a subset of χ **from** $X(t_{-n})$ **to** $X(t_n)$ **in The figure bellow**



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