The English Language and its Encoding in Topological Spaces

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ABSTRACT

The aim of this paper is to transform any sentence in English language to permutation B in symmetric group S_{26} and then consider B-sets which are so important to help us to hid this sentence. Furthermore, new technical is given to generate non-classical topological space (Ω, τ^B) is generated using B-sets. Moreover, classical topological space is given by consider graph G(B), and then we can define subbasis by given a family $D \subseteq G(B)$ that define a subbasis on G(B) The union of all subsets in the subbasis must be equal to the set G(B). After that, the basis that defines a topology T on G(B) by taking all possible finite intersections of subsets of the subbasis. Next, we take all possible unions of sets from the basis.

Keywords: Topological spaces, symmetric groups, permutation, subbasis, graphs, edges.

1. INTRODUCTION

Hill devised the Hill cipher in 1929¹. It is a block cipher with various advantages, including the ability to hide the letter frequencies of the plaintext, the ease of employing matrix multiplication and inversion for encryption and decryption, and the high speed and throughput. However, the Hill cipher has been proven to be vulnerable to cryptanalysis assaults. Because of its linear design, it is vulnerable to the known-plaintext attack, which allows an attacker to retrieve one or more plaintexts and matching ciphertexts². So various studies have been conducted to increase the security of the Hill cipher 3,4 .

Cryptography is a crucial application of linear algebra and number theory. It involves converting critical information to an unclear one. In this work, other technical is used without using the linear algebra or number theory to hide any sentence in topological spaces using non-classical sets, they are called permutation sets (B-sets), where for any permutation *B* in symmetric group S_n , we can consider (*B*-sets). There are many applications on permutation 5.6,7,8,9,10,11,12. In other side, some applications in recent years using non-classical sets are given in algebra and topology like, soft sets 13,14,15, 16,17,18,19,20,21,22, permutation sets 23,24,25,6,27,28,29, nano sets 30,31,32,33,34,35,36, fuzzy sets 37,38,39,40,41,42,43,44,45,46, and others 47,48,49,50,51,52.

In this work, a new technique is shown for converting any sentence in the English language to a permutation B in the symmetric group S_{26} and then investigating B-sets. Following that, a non-classical topological space (Ω, t^B) is created using B-sets, where, $\Omega = \{1, 2, ..., 26\}$, and $t^B = \{\lambda_i^B\}_{i=1}^{c(B)} \bigcup \{\phi, \Omega\}$. In addition, non-classical intersection \land and union \vee are defined on this topology. To define a subbasis in a classical topological space, consider the graph G(B) and create a family $D \subseteq G(B)$. The union of all subsets in the subbasis must equal the set G(B). Furthermore, on this topology, the classical intersection \cap and union \cup are defined. The topology T on G(B) is defined by taking all possible finite intersections of subsets from the subbasis. After that, the basis that defines topology T on G(B) by taking all possible finite intersections of subsets of the subbasis. Next, we take all possible unions of sets from the basis.

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2. PRELIMINARIES

In this section, we will present the fundamental concepts and facts needed for this investigation. **Definition 2.1:** 15

Let *B* be permutation in S_n over $\Omega = \{1, 2, ..., n\}$ and the cycle type of *B* is $\alpha(B) = (\alpha_1, \alpha_2, ..., \alpha_{c(B)})$, then *B* composite of pairwise disjoint cycles (DCs) $\{q_i\}_{i=1}^{c(B)}$ where $q_i = (r_1^i, r_2^i, ..., r_{\alpha_i}^i)$, $1 \le i \le c(B)$. If $q = (r_1, r_2, ..., r_m)$ is *m*-cycle, then $q^B = \{r_1, r_2, ..., r_m\}$ is said to be *B*-set of cycle *q*. Thus the *B*-sets of $\{q_i\}_{i=1}^{c(B)}$ are defined by $\{q_i^B = \{r_1^i, r_2^i, ..., r_{\alpha_i}^i\} | , 1 \le i \le c(B)\}$.

Definition 2.2: ¹⁵ Let q_i^B and q_j^B be *B* – sets in Ω , where $|q_i| = x$ and $|q_j| = y$. We say;

(1) They are disjoint iff
$$\sum_{k=1}^{x} r_k^i = \sum_{k=1}^{y} r_k^j$$
 and $\exists (1 \le s \le x)$ such that $r_s^i \ne r_z^j$, $\forall 1 \le z \le y$.

(2) They are equal iff $\forall (1 \le z \le y)$ such that $r_s^i = r_z^j$ for some $(1 \le s \le x)$.

(3) q_i^B is subset of q_j^B , iff $\sum_{k=1}^{\alpha_i} r_k^i < \sum_{k=1}^{\alpha_j} r_k^j$.

3. OUR ENGLISH ALPHABET TO GENERATE TOPOLOGICAL SPACES

In this work we consider English letters, as our alphabet consists of the 26 letters and added the blank space as \leftrightarrow_i to separate the word w_i of the next word w_{i+1} , its tacked the symbol o in that order.(In your computer work, however, employ the blank space itself rather than this special character!). When enciphering or decoding, the 26 characters in our English alphabet will be represented in order by the numbers 1,...,26, and o, as shown in Table 1, and the set of letters for the first word w_1 in the given sentence will be denoted by $L(w_1)$. Also, $L(w_k)$ is refer to the set of letters for the kth word in the our sentence.

|--|

Α	В	С	D	Ε	F	G	Н	Ι	J	K	L	М	N
1	2	3	4	5	6	7	8	9	10	11	12	13	14
	0	Р	Q	R	S	Т	U	V	W	X	Y	Ζ	\leftrightarrow_i
	15	16	17	18	19	20	21	22	23	24	25	26	0

In this work, new method is given, this method help us to generate the structure of topology for each sentence. This our approach depends on permutation *B* in symmetric group S_{26} . Let $W = \{A, B, C, ..., Z, \leftrightarrow_i\}$ be the set of all English letters with spaces. Define $h: W \to \{1, 2, ..., 26, o\}$ by table 1. Let $S = \{L(w_1), \leftrightarrow_1, L(w_2), \leftrightarrow_2, ..., L(w_\alpha)\}$ be the set of all English letters of words with spaces between them in given sentence. For any $L(w_1) = \{a_1, a_2, ..., a_m\} \in S$ define $f: S \to S_{26} \cup \{o\}$ by $f(L(w_1)) = (h(a_1), h(a_2), ..., h(a_m))$ and $f(\leftrightarrow_i) = h(\leftrightarrow_i) = o$. Also, Let $T(S) = f(L(w_1))f(\leftrightarrow_1)f(L(w_2))f(\leftrightarrow_2) \dots f(\leftrightarrow_{\alpha-1})f(L(w_\alpha)) = B$, where *B* is a permutation in symmetric group S_{26} . Therefore, for any sentence we can find permutation in S_{26} . Now, since for any permutation *B* can be written as product of disjoint cycles (DCs). Thus $B = (r_1^1, r_2^1, ..., r_{\alpha_1}^1)$ $(r_1^2, r_2^2, ..., r_{\alpha_2}^2) \dots (r_1^{c(B)}, r_2^{c(B)}, ..., r_{\alpha_{c(B)}}^{c(B)})$, where for

any $i \neq j$ we get $\{r_1^i, r_2^i, ..., r_{\alpha_i}^i\} \cap \{r_1^j, r_2^j, ..., r_{\alpha_j}^j\} = \phi$. So we can write *B* as $q_1q_2...q_{c(B)}$. With q_i (DCs) of length $|q_i| = \alpha_i$ and c(B) is the number of (DC) factors including the 1-cycle of *B*. The partition $\alpha = \alpha(B) = (\alpha_1(B), \alpha_2(B), ..., \alpha_{c(B)}(B)) = (\alpha_1, \alpha_2, ..., \alpha_{c(B)})$ is called a cycle type (CT) of *B*. For the symmetric group S_{26} on the set $\Omega = \{1, 2, ..., 26\}$, let $B \in S_{26}$, and $\alpha(B) = (\alpha_1, \alpha_2, ..., \alpha_{c(\beta)})$ be the (CT) of *B* then *B* composite of pairwise (DCs) $\{q_i\}_{i=1}^{c(\beta)}$ where $q_i = (r_1^i, r_2^i, ..., r_{\alpha_i}^i), 1 \leq i \leq c(B)$. Now, we will introduce two methods to generate Non-classical and Classical topological spaces using permutation *B* in S_{26} . **3.1 Non-classical Topological Spaces**

For any m-cycle $q = (r_1, r_2, ..., r_m)$ in S_{26} we define B-set as $q^B = \{r_1, r_2, ..., r_m\}$ and is called B-set of cycle q. So the B-sets of $\{q_i\}_{i=1}^{c(B)}$ are defined by $\{q_i^B = \{r_1^i, r_2^i, ..., r_{\alpha_i}^i\} | , 1 \le i \le c(B)\}$. For example, let (S_{26}, o) be a symmetric group and B be a permutation in S_{26} , where B =

$$3 + 5 + 10 = \sum_{k=1}^{3} r_{k}^{5} = 2 + 8 + 9. \text{ The operations } \wedge \text{ and } \vee \text{ are defined on } B \text{ -sets in } \Omega \text{ as followers:}$$

$$\lambda_{i}^{\beta} \wedge \lambda_{j}^{\beta} = \begin{cases} \lambda_{i}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} < \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{j}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} > \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{i}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} > \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{i}^{\beta}, \text{ if } \lambda_{i}^{\beta} = \lambda_{j}^{\beta} = \lambda^{\beta} \\ \phi, \text{ if } \lambda_{i}^{\beta} & \& \lambda_{j}^{\beta} \text{ are disjoint} \end{cases} \text{ and } \lambda_{i}^{\beta} \vee \lambda_{j}^{\beta} = \begin{cases} \lambda_{i}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} > \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{i}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} < \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{i}^{\beta}, \text{ if } \sum_{k=1}^{\sigma} r_{k}^{i} < \sum_{k=1}^{\nu} r_{k}^{j} \\ \lambda_{i}^{\beta}, \text{ if } \lambda_{i}^{\beta} = \lambda_{j}^{\beta} = \lambda^{\beta} \\ \Omega, \text{ if } \lambda_{i}^{\beta} & \& \lambda_{j}^{\beta} \text{ are disjoint} \end{cases}$$

Then (Ω, t^B) is a topological space generated by permutation *B* where $\Omega = \{1, 2, ..., 26\}$, and $t^B = \{\lambda_i^B\}_{i=1}^{c(B)} \bigcup \{\phi, \Omega\}$. Also, \wedge and \vee are non-classical intersection and union that defined on this topology.

Example 3.1.1: Find the non-classical topological space for this sentence (I play tense with Jack). **Solution:** Since our sentence contains 5 words, then $S = \{L(w_1) = \{I\}, \leftrightarrow_1, L(w_2) = \{p, l, a, y\}, \leftrightarrow_2, L(w_3) = \{t, e, n, s\}, \leftrightarrow_3, L(w_4) = \{w, i, t, h\}, \leftrightarrow_4, L(w_4) = \{j, a, c, k\}\} \Rightarrow T(S) = f(L(w_1))f(\leftrightarrow_1) f(L(w_2))f(\leftrightarrow_2) f(L(w_3))$ $f(\leftrightarrow_3)f(L(w_4)) = (9) o (16 \ 12 \ 1 \ 25) o (20 \ 5 \ 14 \ 19) o (23 \ 9 \ 20 \ 8) o (10 \ 1 \ 3 \ 11) = B$, where $B = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25 \ 26)$ $(3 \ 2 \ 11 \ 4 \ 14 \ 6 \ 7 \ 23 \ 5 \ 25 \ 10 \ 1 \ 13 \ 19 \ 15 \ 12 \ 17 \ 18 \ 20 \ 8 \ 21 \ 22 \ 9 \ 24 \ 16 \ 26)$ is a permutation in S_{26} . Then B can be written as product of (DCs). So, $B = (1 \ 3 \ 11 \ 10 \ 25 \ 16 \ 12) o (2) o (4) o (5 \ 14 \ 19 \ 20 \ 8 \ 23 \ 9) o (6) o (7) o (13) o (18) o (21) o (22) o (24) o (26).$ Therefore (Ω, t^B) is a topological space generated by permutation B, where $\Omega = \{1, 2, ..., 26\}$, and $t^B = \{\lambda_i^B\}_{i=1}^{12} \bigcup \{\phi, \Omega\}$.

3.2 Classical Topological Spaces

For any sentence we can find permutation in S_{26} , and for any permutation *B* we can find *B*-sets $\{q_i^B = \{r_1^i, r_2^i, ..., r_{\alpha_i}^i\} | 1 \le i \le c(B)\}$. Now, we can consider graph G(B) contains c(B) of layers, where layer k is

B-set q_k^B , also for each pair $q_i^B = \{r_1^i, r_2^i, ..., r_{\alpha_i}^i\}$ and $q_{i+1}^B = \{r_1^{i+1}, r_2^{i+1}, ..., r_{\alpha_{i+1}}^{i+1}\}$ there are edges

 $\{E_{m,n}^{i,i+1} \mid 1 \le m \le \alpha_i; 1 \le n \le \alpha_{i+1}; 1 \le i \le c(B) - 1\}, \text{ where } E_{m,n}^{i,i+1} \text{ start from } r_m^i \in q_i^B \text{ and end at } r_n^{i+1} \in q_{i+1}^B \text{ . For } r_n^{i+1} \in q_i^B \text{ and end at }$

example, let *B* be a permutation in S_{26} , where B =

 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 \\ 3 & 8 & 5 & 12 & 10 & 18 & 6 & 9 & 2 & 1 & 14 & 26 & 20 & 21 & 24 & 23 & 19 & 25 & 22 & 16 & 15 & 17 & 13 & 11 & 7 & 4 \\ Then <math>B = (1 & 3 & 5 & 10)o(7 & 6 & 18 & 25)o(4 & 12 & 26)o(17 & 19 & 22)o(2 & 8 & 9)o(16 & 23 & 13 & 20)o(15 & 24 & 11 & 14 & 21). \\ So, we have <math>X = \{q_i^B\}_{i=1}^{7} = \{\{1,3,5,10\}, \{6,7,18,25\}, \{4,12,26\}, \{17,19,22\}, \{2,8,9\}, \{13,16,20,23\}, \\ \{11,14,15,24,21\}\}. Hence, we consider <math>G(B)$ contains seven layers and there are 16 edges between first layer $q_1^B = \{1,3,5,10\}$ and second layer $q_2^B = \{6,7,18,25\}$, 12 edges between second layer $q_2^B = \{6,7,18,25\}$ and third layer $q_3^B = \{4,12,26\}, 9$ edges between third layer $q_3^B = \{4,12,26\}$ and fourth layer $q_4^B = \{17,19,22\}, 9$ edges between third layer $q_6^B = \{13,16,20,23\}$ and eventh layer $q_7^B = \{11,14,15,24,21\}$. Then $G(B) = \{E_{1,2}^{1,2}, E_{1,2}^{1,2}, E_{2,2}^{1,2}, E_{2,2}^{1,2}, E_{2,3}^{2,3}, E_{2,3}^{2,3}, E_{2,3}^{2,3}, E_{2,3}^{2,3}, E_{2,3}^{3,4}, E_{2,3}^{3,4},$



Figure (1): Graph G(B) with seven layers

After, consider G(B), then we can define subbasis by given a family $D \subseteq G(B)$ that define a subbasis on G(B), i.e., D satisfied the following two conditions: For any two subsets, F_1 and F_2 , from the subbasis D, then the intersection of F_1 and F_2 can be expressed as a union of finite intersections of subsets of the subbasis. The union of all subsets in the subbasis must be equal to the set G(B). Moreover, \bigcap and \bigcup are classical intersection and union that defined on this topology. After that, the basis that defines a topology T on G(B) by taking all possible finite intersections of subsets of the subbasis. Next, we take all possible unions of sets from the basis. Finally, we need to ensure that the constructed topology satisfies the three axioms of a topology: T_{1}

- The empty set \emptyset and the entire set G(B) are in the set T.
- The union of any members in *T* is also in *T*.
- The intersection of any finite number of members in T is also in T.

Remarks 3.2.1:

(1) In general we can find the family $D \subseteq G(B)$ that define a subbasis for any graph G(B) by put; $D = \{\{E_{m,n}^{i,i+1}, E_{n,x}^{i+1,i+2}\} | 1 \le m \le \alpha_i; 1 \le n \le \alpha_{i+1}; 1 \le x \le \alpha_{i+2}; 1 \le i \le c(B) - 2\}.$

(2) We have three possible non-empty intersections among the elements of D as follows:

(a)
$$\{E_{m,n}^{i,i+1}, E_{n,x}^{i+1,i+2}\} \cap \{E_{n,x}^{i+1,i+2}, E_{x,y}^{i+2,i+3}\} = \{E_{n,x}^{i+1,i+2}\}.$$

 $E_{m,n}^{i,i+1} \qquad E_{n,x}^{i+1,i+2} \qquad E_{x,y}^{i+2,i+3}$
 $\bullet r_{m}^{i+1} \rightarrow \bullet r_{n}^{i+1} \rightarrow \bullet r_{x}^{i+2} \rightarrow \bullet r_{y}^{i+3}$

Figure (2): The sub-sets of D with common middle edge.



Figure (3): The sub-sets of D with common first edge.



Figure (4): The sub-sets of D with common last edge.

(3) We consider the basis V for graph G(B) will be as $V = \emptyset \cup D \cup (\bigcup_{i=1}^{c(B)-1} \left(\bigcup_{m=1}^{\alpha_i} \left(\bigcup_{n=1}^{\alpha_{i+1}} \left\{ E_{m,n}^{i,i+1} \right\} \right) \right))$.

(4) We consider (G(B), T) is a classical topological space, where T is a family of all possible unions of sets from the basis V.

permutation in	Then can be written as prod	luct of (DCs). So, (1 20 5)	o (6 9 19 8) o (7) o (
) (18) (21) },{14},{	(22) (26). So, we have },{21},{	},{26}}. Hence, we consider	,{7},{
Step 1	$\begin{bmatrix} 2\\ 8\\ 7,E \\ 15\\ 24\\ 24\\ \end{bmatrix}$	в, ^{8,} 15, Е	3 7, 12 18,
Step 2	2 6, }, },{	2 9, }, 19, }, },{	},
	12 18,	15,	15 24,
Step 3	2, },{ },{	² ₉ , },{ ₁₉ , },{	28,
	},{ 12,E 18,E	},{ 15,	},
	$\langle E \rangle \langle E \rangle, \{E $	$\{E\},\$	},{

Hence (G, T) is a classical topological space generated by permutation

4. CONCLUSION

This paper introduces a new technique for generating non-classical and classical topology from any sentence in English. In other words, the statement is buried in this paper. Consequently, in future work, we shall extract the sentences from both non-classical and classical topology.

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