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The Alpha Star-Perfect Mapping in Topological Spaces

Abu Firas Al –Musawi [1], **Nadia Abbas**[2], **Shuker Khalil**[3]

[1]Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Iraq.

[2] Ministry of Education/ Directorate General of Education / Baghdad/ Al-Kark/3, Iraq.

[3]Department of Mathematics, College of Science, University of Basrah, Basrah 61004, Iraq.

[1]afalmusawi@gmail.com ; [2] mali.nadia@yahoo.com ; [3]shuker.khalil@uobasrah.edu.iq ;

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Abstract: In this paper, we introduce new types of α^*-continuity mappings by using α^*-open sets in topological spaces which is called α^*-perfect mapping (α^*-PM), and also study some properties of these types. Some definitions are given. This research shows that if $f: X \to Y$ is (α^*-PM). Then the restriction of f on the clopen set is also (α^*-PM). Next, for any mapping $f: X \to Y$ between two topological spaces with A, B are disjoint clopen sets, where $X = A \cup B$. This work shows that $f|_A$ and $f|_B$ are (α^*-PM) if and only if f is (α^*-PM). Finally, we prove that if $f: X \to Y$ and $g: U \to W$ are mappings with $f \times g: X \times U \to Y \times W$ is (α^*-PM), then each one of f and g is (α^*-PM).

Keywords: α - open sets, α^*- open sets, perfect mapping, continuity mapping, α^*-irresolute continuous mapping.

1. Introduction

One of the most significant concepts in mathematics, particularly in topology, is continuous mapping; there are numerous forms of it, one of which is known as a "perfect mapping". A mapping $f: X \to Y$ is considered perfect if it is continuous, closed, and contains compact fibres $f^{-1}(y)$, for each $y \in Y$. For more information, check ([1], [2]). In 1965, O. Njasted established the notion of $\alpha-$open [3]. In 2019, N. Ali and S. Khalil introduced the notion of α^*-open, which is an extension of the α-open set. For additional information, see [4]. A topological space (TS) is a quite broad term. Greater specificity is frequently desired. Many mathematicians have demonstrated certain research in (TS) and their extensions to non-classical (TS). In other side, some applications in recent years using non-classical sets are given in algebra and topology like fuzzy sets [5-19], soft sets ([20-33]), permutation sets ([34-55]), nano sets [56], neutrosophic sets ([57-64]) and others [65-72].

In this work, new type of α^*-continuity mappings by using α^*-open sets in topological spaces is introduced, which is called α^*-perfect mapping, also we study some properties of these types. We show that if $f: X \to Y$ is (α^*-PM). Then the restriction of f on clopen set is also (α^*-PM). Next, for any mapping $f: X \to Y$ between two topological spaces with A, B are disjoint clopen sets, where $X = A \cup B$. In this work, we show that $f|_A$ and $f|_B$ are (α^*-PM) if and only if f is (α^*-PM). Finally, we prove that if $f: X \to Y$ and $g: U \to W$ are mappings with $f \times g: X \times U \to Y \times W$ is (α^*-PM), then each one of f and g is (α^*-PM).

2. Some Basic Definition

Here we shall give some basic concepts which are used in this work

Definition (2.1) [4]
Let (X,T) be a topological space and $\emptyset \neq A \subseteq X$. The set A is called $\alpha-$open [resp., α^*-open] set if $A \subseteq int(cl(int(A)))$ [resp., $A \subseteq int_\alpha(cl(int_\alpha(A)))$]. Also, the complement of A is called $\alpha-$closed [resp., α^*-closed] set. The family of all $\alpha-$open (resp., $\alpha-$ closed, α^*- open , α^*- closed) set is denoted by $O_\alpha(X)$ [resp., $C_\alpha(X)$, $O_{\alpha^*}(X)$, $C_{\alpha^*}(X)$].

Remarks (2.2) [4]
(i) Every open (resp., $\alpha-$open, clopen) set is α^*-open set.
(ii) Every closed (resp., $\alpha-$closed) set is α^*-closed set.
(iii) X and $\emptyset$ are α^*-open sets in X.

Theorem (2.3) [4]
If $\{G_i\}_{i\in I}$ is a collection of α^*- open sets, then their union is also α^*- open set.

Theorem (2.4) [4]
(i) If A is α^*- open and B is open, then $A \cap B$ is also α^*- open set.
The finite intersection of α^*- closed set is also α^*- closed set.
(ii) The finite intersection of α^*- closed sets is α^*- closed set.

Theorem (2.5) [4]
Let X and Y be two topological spaces, $A \subseteq X$, and $B \subseteq Y$, then A, B are α^*- open (resp., α^*- closed)sets iff $A\times B$ is α^*- open (resp., α^*- closed) in $X \times Y$.

Lemma (2.6) [4]
Let (Y, t_Y) be subspace of (X, τ) with $A \subseteq Y \subseteq X$. Then;
(i) If $A \in O_{\alpha^*}(X)[\ resp., C_{\alpha^*}(X)]$, then $A \in O_{\alpha^*}(Y)[\ resp., C_{\alpha^*}(Y)]$.
(ii) If $A \in O_{\alpha^*}(Y)[\ resp., C_{\alpha^*}(Y)]$, then $A \in O_{\alpha^*}(X)[\ resp., C_{\alpha^*}(X)]$, where Y is closed subspace of X.

Theorem (2.7) [4]
$A \in O_{\alpha^*}(X)$ iff $B \in O_{\alpha^*}(X)$ with $B \subseteq A \subseteq int_\alpha(cl(B))$.

Corollary (2.8) [4]
$A \in C_{\alpha^*}(X)$ iff there exists $F \in C_{\alpha^*}(X)$ with $cl((int(F) \subseteq A \subseteq F$.

Theorem (2.9) [4]
If $A \in O_{\alpha^*}(X)$ with $A \subseteq B \subseteq int_\alpha(cl(A))$, then $B \in O_{\alpha^*}(X)$.

Theorem (2.10) [24]
Let X and Y be two topological spaces and $f: X \to Y$ be any mapping, we say f is:
(i) α^*- continuous mapping if $f^{-1}(A)$ is α^*-open set in X, for any open set in Y and it is denoted by α^*-CM.
(ii) α^*-irresolute continuous mapping if $f^{-1}(A)$ is α^*-open set in X, for any α^*-open set A in Y, and it is denoted by $I_{\alpha^*}-CM$.
(iii) Strong α^*- continuous if $f^{-1}(A)$ is open set in X, for any α^*-open set A in Y, and it is denoted by $S_{\alpha^*}-CM$.

Theorem (2.11) [24]

(i) Every $\alpha^* - CM$ is $I_{\alpha^*} - CM$.
(ii) Every $I_{\alpha^*} - CM$ is $S_{\alpha^*} - CM$.

Theorem (2.12) [24]

Let (X, t_1) and (Y, t_2) be two topological spaces and $f : X \to Y$ be mapping, then:

(i) $f|_F: F \to Y$ is $\alpha^* - CM$, where F is open set in X, if f is $\alpha^* - CM$.
(ii) $f|_F: F \to Y$ is $I_{\alpha^*} - CM$, where F is open set in X, if f is $I_{\alpha^*} - CM$
(iii) $f|_F: F \to Y$ is $S_{\alpha^*} - CM$, where F is α^* −open set in X, if f is $S_{\alpha^*} - CM$.

Theorem (2.13) [24]

Let (X, t_1) and (Y, t_2) be two topological spaces and $f : X \to Y$ be mapping, and A, B are disjoint sets in X with $X = A \cup B$. We say :

(i) f is $\alpha^* - CM$ iff $f|_A$ and $f|_B$ are also , where A, B are $\alpha -$ open sets.
(ii) f is $I_{\alpha^*} - CM$iff $f|_A$ and $f|_B$ are also , where A, B are open sets.
(iii) f is $S_{\alpha^*} - CM$ iff $f|_A$ and $f|_B$ are also $\alpha^* - CM$, where A, B are $\alpha^* -$ open sets.

Theorem (2.14) [24]

Let (X, t_1) and (Y, t_2) be two topological spaces and $f : X \to Y$ be mapping, and define $f_A : f^{-1}(A) \to A$ by $f_A(x) = f(x)$, for any set A in Y and $x \in f^{-1}(A)$. Then;

(i) f_A is $\alpha^* - CM$ if f is also , where A is open set in Y.

(ii) f_A is $I_{\alpha^*} - CM$ (resp., $S_{\alpha^*} - CM$) if f is also , where A is closed set in Y.

Theorem (2.15) [24]

Let (X, t_1) and (Y, t_2) be two topological spaces and $f : X \to Y$ be mapping. Then f is called;

(i) α^* −open mapping ($\alpha^* - OM$) if $f(A)$ is α^* −open set in Y, for any open set A in X.
(ii) Strongly α^* −open mapping ($S\alpha^* - OM$) if $f(A)$ is α^* −open set in Y, for any α^* −open set A in X.
(iii) α^* −closed mapping if $f(A)$ is α^* −closed set in Y, for any closed set A in X.
(iv) Strongly α^* −closed mapping if $f(A)$ is $\alpha^* -$ closed set in Y, for any $\alpha^* -$ closed set A in X.

Theorem (2.16) [73]

Let $A \subseteq Y \subseteq X$. then A is compact set in X iff A is compact in Y.

3. $\alpha^* -$ Perfect Mappings:

In this section the concept of α^* −open set will be used to define some new types of α^* −continuity which is called α^* −perfect.

Definition (3.1)

Let (X, t_1) and (Y, t_2) be two topological spaces and $f : X \to Y$ be surjective mapping. We say f is α^* −perfect mapping ($\alpha^* - PM$) if f is ($\alpha^* - CM$), ($\alpha^* -$ closed), and all fibers $f^{-1}\{y\}$ is compact set in ,$\forall y \in Y$.

To illustrate this concept see the following Example.

Example (3.2): Let (X, t_1) and (Y, t_2) be two topological spaces, where $X = Y = \{a, b, c\}$, $t_1 = \{\emptyset, X, \{a\}, \{b, c\}\}$, $t_2 = \{\emptyset, Y, \{c\}, \{a, b\}\}$, and define $f : X \to Y$ by $f(a) = c$, $f(b) = a$, and $f(c) = b$. We see f is surjective and all fibers $f^{-1}\{y\}$ is compact set ,$\forall y \in Y$. Also, f is $(\alpha^* - CM)$, because $f^{-1}(Y) = X$ and $f^{-1}(\emptyset) = \emptyset$, where X and $\emptyset$ are $\alpha^* -$ open sets, see remark (2.2). Also, $f^{-1}(\{a, b\}) = \{b, c\}, f^{-1}(\{c\}) = \{a\}$ are clopen sets so they are $\alpha^* -$ open sets in X, see remark (2.2). Thus f is $(\alpha^* - CM)$. Moreover, $f(\{b, c\}) = \{a, b\}, f(\{a\}) = \{c\}$ are closed sets and hence they are $\alpha^* -$ closed sets. then f is $\alpha^* -$ closed mapping , because $f(F)$ is $\alpha^* -$ closed, for any closed set F in X, Hence f is $(\alpha^* - PM)$.

Example (3.3): Let (X, t_1) and (Y, t_2) be two topological spaces, where $X = Y = \{1,2,3,4\}$, $t_1 = \{\emptyset, X, \{1\}, \{1,2\}\}$, $t_2 = \{\emptyset, Y, \{1,2,3\}\}$, and define $f : X \to Y$ by $f(1) = 4$, $f(2) = 1$, $f(3) = 2$, and $f(4) = 3$. We observe f is surjective. Also, $f^{-1}(Y) = X \in O_{\alpha^*}(X)$ & $f^{-1}(\{1,2,3\}) = \{2,3,4\} \in O_{\alpha^*}(X)$, since $\{2,3,4\} \subseteq int_\alpha(cl(int_\alpha(\{2,3,4\})))$ That means $f^{-1}(A) \in O_{\alpha^*}(X)$, for any open set A in Y, so f is $(\alpha^* - CM)$. But f is not $\alpha^* -$ closed since $\{3,4\}$is closed set in X but $f(\{3,4\}) = \{2,3\}$ which is not $\alpha^* -$ closed set in Y since $\{2,3\}$ is not contains $cl_\alpha(int_\alpha(cl_\alpha(\{2,3\}))) = Y$ (i.e., $Y \subseteq \{2,3\}$). Thus f is not $(\alpha^* - PM)$.

Theorem (3.4)

Let $f : X \to Y$ be $(\alpha^* - PM)$. Then the restriction of f on clopen set is also $(\alpha^* - PM)$.

Proof:

We need to show that $f|_A : A \to Y$ is $(\alpha^* - PM)$, for any clopen set A in X. Since A is clopen, thus A is open and hence $f|_A : A \to Y$ is $(\alpha^* - CM)$ [From Theorem (2.12)]. Now, let B be closed set in A, also A is closed set in X [since A is clopen set in X]. Thus B is closed in X and hence $f(B)$ is $\alpha^* -$ closed in Y [since f is α^* −closed]. But $f|_A(B) = f(B)$, thus $f|_A$ is α^* −closed mapping. Also, $f|_A$ is surjective since f is surjective. Moreover, we have ${f|_A}^{-1}(\{y\}) = A \cap f^{-1}\{y\}$, where $f^{-1}\{y\}$ is compact set in X for all $y \in Y$, then $f^{-1}\{y\}$ is compact set in A, see theorem (2.16). Hence $f|_A : A \to Y$ is $(\alpha^* - PM)$.

Theorem (3.5)

Let X and Y be a topological spaces, $f : X \to Y$ be any mapping, and let A, B be disjoint clopen sets, where $X = A \cup B$. Then $f|_A$ and $f|_B$ are $(\alpha^* - PM)$ if and only if f is $(\alpha^* - PM)$.

Proof: Assume that f is $(\alpha^* - PM)$, then $f|_A$ and $f|_B$ are also [From Theorem (3.4)]. Now, let each one of $f|_A$ and $f|_B$ is $(\alpha^* - PM)$. Thus f is $(\alpha^* - CM)$ by [Theorem (2.13)]. Also, let M be closed set in X, we get $f(M) = f(M \cap X) = f(M \cap (A \cup B) = f([(M \cap A) \cap A] \cup [(M \cap B) \cap B]) = f|_A\, (M \cap A) \cup f|_B\, (M \cap B)$, where $(M \cap A)$ and $(M \cap B)$ are closed sets in A and B, respectively. Since $f|_A$ and $f|_B$ are $\alpha^* -$ closed mappings, thus $f|_A\, (M \cap A)$ and $f|_B\, (M \cap B)$ are $\alpha^* -$ closed sets in Y. So, their union is also $\alpha^* -$ closed set in Y. Then $f(B)$ is $\alpha^* -$ closed set in Y. Now, since each one of $f|_A$ and $f|_B$ is $(\alpha^* - PM)$, we get ${f|_A}^{-1}(\{y\})$ and ${f|_B}^{-1}(\{y\})$ are compact sets in in A and B, respectively. Then we get ${f|_A}^{-1}(\{y\})$ and ${f|_B}^{-1}(\{y\})$ are compact in X by Theorem (2.16). So, their union is also compact in X [Since the union of finite compact sets is a compact set]. Now, it is obvious that f is onto. Therefore, we get f is $(\alpha^* - PM)$.

Theorem (3.6)

Let $f: X \to Y$ and $g: U \to W$ be mappings. If $f \times g: X \times U \to Y \times W$ is $(\alpha^* - PM)$, then each one of f and g is $(\alpha^* - PM)$.

Proof:

We shall prove only f is $(\alpha^* - PM)$. In the first, we have to prove f is $(\alpha^* - CM)$. Let A be an open sets in Y. Thus $A \times W$ is open set in $Y \times W$, thus $(f \times g)^{-1}(A \times W) = f^{-1}(A) \times g^{-1}(W) = f^{-1}(A) \times U$, then by Theorem (2.5) we obtain $f^{-1}(A)$ is $\alpha^* -$ open set in X. Therefore $f: X \to Y$ is $(\alpha^* - CM)$. Now, let B closed set in X, thus $B \times U$ is closed in $X \times U$. So $(f \times g)(B \times U)$ is $\alpha^* -$ closed in $Y \times W$, where $(f \times g)(B \times U) = f(B) \times g(U)$. Then $f(B)$ is $\alpha^* -$ closed in Y. On the other side, since $f \times g$ is surjective mapping, then f and g are surjective mappings. Now, to prove the fourth condition, we have to show that $f^{-1}(\{y\})$ is compact set in $X, \forall y \in Y$. Since $(f \times g)^{-1}(y, w) = f^{-1}(y) \times g^{-1}(w)$ is compact set in $X \times U, \forall (y, w) \in Y \times W$. Hence $f: X \to Y$ is $(\alpha^* - PM)$, In similar way, we show that g is $(\alpha^* - PM)$.

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