*Journal of Discrete Mathematical Sciences & Cryptography* ISSN 0972-0529 (Print), ISSN 2169-0065 (Online) Vol. 27 (2024), No. 5, pp. 1583–1590 DOI : 10.47974/JDMSC-1941

# Structure of a new system of information in a soft rough setting based on new groups

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# Abstract

Our goal in writing this article is to investigate roughness as it relates to soft sets on the cubic dihedral group. We distinguish between two types of sets in this paper: lower cubic dihedral sets and higher cubic dihedral sets that are soft. They are introduced on the other side by use of the typical soft cubic dihedral group that is associated with each parameter. We discuss a few key findings regarding our lessons over groups.

Subject Classification: Primary 20F29, 06F15, Secondary 05C38.

*Keywords:* Group homomorphism, Cubic dihedral sets, Rough sets, Cubic dihedral permutation groups, Normal soft groups.

#### 1. Introduction

Handling imprecision and uncertainties in gathered data is becoming increasingly common in modern culture, particularly in the disciplines of

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computer science, medical research, and environmental science. Fuzzy [1], soft [2] and rough [3] sets theory are some of the relevant mathematical theories offered by scientists and practitioners to cope with such difficulties. Pawlak [3], a computer scientist, introduced the notion of rough set theory (RST) in 1982, for order to transact with intricacy in (ISs). RST is a variant of set theory in which the membership function (MF) is replaced by two different sets. The lower approximation (LA) and upper approximation (UA) are the names given to these two sets. These approximations are used to extract meaningful information from large amounts of data. RST offers us with relatively easy techniques to characterize the initial items in an (IS) that have the same value of attributes. Molodtsov [2] in 1999, he presented the notion of soft set theory. Various researchers have given extensive studies on the basic features and operations of soft set theory [4,5]. Soft set is not classical set like permutation set [6,7], neutrosophic set [8,9], and others. Many academics have discovered soft set theory's applicability in many algebraic systems as a result of its theoretical enrichment. In this article, a novel type of soft rough set based on a normal soft cubic dihedral group is studied over cubic dihedral groups. The remainder of the paper is laid out as follows: The second section contains information on cubic dihedral sets, cubic dihedral groups, and (NSGs). Using the notion of (NSGs) in Sect. 3, to present the notions of soft rough cubic dihedral sets over groups. With the help of examples, certain significant characteristics of the soft lower and soft upper cubic dihedral groups are demonstrated.

#### 2. Preliminaries

In this section, the background information that we need in our work are recalled.

**Definition 2.1:** [10] Assume that (X,\*) is a monoid with  $e \in X$  is identity and let  $a, b, c \in X$  be three distinct elements. For any positive integer k, we say  $_k < a, b, c >_X^*$  is a cubic dihedral set if such that: (1)  $a^k = b^2 = c^2 = e$ , (2)  $ba = a^{k-1}b$  and  $ca = a^{k-1}c$ , (3) cb = bc, where  $_k < a, b, c >_X^* = \{r_i \mid 1 \le i \le 4k\} = \{e, a, a^2, \dots, a^{k-1}, b, ab, a^2b, \dots, a^{k-1}b, c, ac, a^2c, \dots, a^{k-1}c, abc, abc, a^2bc, \dots, a^{k-1}bc\}$ .

**Definition 2.2:** [10] Let  $_k < a, b, c >_X^*$  be cubic dihedral set and any  $r_i, r_j \in \{e, a, a^2, ..., a^{k-1}, b, ab, a^2b, ..., a^{k-1}b, ac, a^2c, ..., a^{k-1}c, bc, abc, a^2bc, ..., a^{k-1}bc$  are distinct whenever  $1 \le i \ne j \le 4$ . ( $_k < a, b, c >_X^*$ ,\*) with degree k and order 4k is referred to as a cubic dihedral group.

**Definition 2.3:** [10] Let  $\alpha$ ,  $\beta$ ,  $\gamma$  three possible combinations inside a symmetrical set( $S_n$ , o) and  $_k < \alpha$ ,  $\beta$ ,  $\gamma >_{S_n}^{\circ}$  be cubic dihedral set. We say ( $_k < \alpha$ ,  $\beta$ ,  $\gamma >_{S_n}^{\circ}$ , o) is a permutation group of cubic dihedrals of degree k and order 4k., whenever

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 $supp(\alpha) \cap supp(\beta) = \emptyset$ ,  $supp(\alpha) \cap supp(\gamma) = \emptyset$ ,  $supp(\beta) \cap supp(\gamma) = \emptyset$ and  $Ord(\alpha) = k$ .

**Definition 2.4:** [2] Initiate the universe using set W and define its parameters with set D. P(W) represents the set of all possible values for W. A subset of D is K. A soft set over W is defined as a pair ( $\Gamma$ ,K) where Fi is a multivalued function of K into P(W).

**Definition 2.5 [4]:** Assume that  $(J_1, H_1)$  and  $(J_2, H_2)$  are soft sets over W, where  $U_1, U_2 \subseteq D$ . We say  $(J_1, H_1)$  is a soft subset of  $(J_2, H_2)$ , [i.e,  $(J_1, H_1) \widetilde{\subseteq} (J_2, H_2)$ ] if: (1)  $H_1 \subseteq H_2 \& (2) J_1(d) \subseteq J_2(d), \forall d \in H_1$ .

**Definition 2.6 [11]:** Assume that  $(J_1, H_1)$  and  $(J_2, H_2)$  are soft sets over W, where  $J_1, J_2 \subseteq D$ . Then, their restricted union  $(J_1, H_1) \cap (J_2, H_2) = (I_1, H_3)$  and restricted intersection  $(J_1, H_1) \cap (J_2, H_2) = (I_2, H_3)$ , are defined as follows:  $(d) = J_1(d) \cup J_2(d)$  and  $I_2(d) = J_1(d) \cap J_2(d)$ ,  $\forall d \in H_3 = H_1 \cap H_2 \neq \emptyset$ .

**Definition 2.7[4]:** (J, H) is soft group (SG) over W, if J(d) is a subgroup of W,  $\forall d \in H$ .

**Definition 2.8 [4]:** We say (J, H) is a normal soft group (NSG) over W, if J(d) is a normal subgroup of W, for each  $d \in H$ .

**Definition 2.9 [4]:** Assume that  $(J_1, H_1)$  and  $(J_2, H_2)$  are soft sets over W. Then, their restricted soft product is referred as  $(J_1, H_1)^{\circ}(J_2, H_2) = (I, H_3)$ , and is defined by  $I(d) = J_1(d)J_2(d)$ , for each  $d \in H_3 = H_1 \cap H_2 \neq \emptyset$ .

**Lemma :2.10 [4]:** Let (J, H) and (h, M) be two soft groups over W. Then, (1)  $J(d)h(d) = h(d)J(d), \forall d \in H \cap M \neq \emptyset$ , and (2)  $J(d)J(d) = J(d), \forall d \in H$ .

# 3. Soft Lower (Upper) Cubic Dihedral Sets in Groups Via Normal Soft Cubic Dihedral Groups

The conception of soft rough set over cubic dihedral group is given in this section. Using a normal soft cubic dihedral group, we investigate the concept of soft lower (upper) cubic dihedral sets over groups and investigate several related results.

**Definition 3.1** Let  $W = {}_{k} < a, b, c >_{X}^{*}$  be a cubic dihedral group and (J, H) be a soft set. We say (J, H) is a soft cubic dihedral group (SCDG) if J(d) is a subgroup of cubic dihedral group W, for each  $d \in H$ . Also, (J, H) is said to be normal soft cubic dihedral group (NSCDG) if J(d) is a normal subgroup of cubic dihedral group W, for all  $d \in H$ . Assume that (J, H) is a (NSCDG) over W and let  $\emptyset \neq T \subseteq W$ . We say  $(\underline{J}_{T}, H)$  is a soft lower cubic dihedral set (SLCDS) and  $(\overline{J}^{T}, H)$  is the soft upper cubic dihedral set (SUCDS) of T in W with respect to J(d), where  $\underline{J}_{T}(d) = \{w \in W : wJ(d) \subseteq T\}$  and  $\overline{J}^{T}(r) = \{w \in W : wJ(d) \cap T \neq \emptyset\}, \forall d \in H$ . Note:  $J_T$  and  $\overline{J}^T$ :  $H \to P(W)$  are soft sets over W. Consider the following example to demonstrate this idea.

**Example 3.2:** Let  $W = \{e, n, t, nt\}$  and \* be a binary operation defined on W as Table 1:

| *  | е  | n  | t  | nt |  |  |  |  |  |  |
|----|----|----|----|----|--|--|--|--|--|--|
| е  | е  | n  | t  | nt |  |  |  |  |  |  |
| n  | n  | е  | nt | t  |  |  |  |  |  |  |
| t  | t  | nt | е  | n  |  |  |  |  |  |  |
| nt | nt | t  | n  | е  |  |  |  |  |  |  |

Table 1 (W,\*) is (CDG)

Then  $W = CD_1$  is a (CDG) of degree 1 and order 4. Let  $H = \{h_1, h_2\}$  and J:  $H \to P(W)$  be a multivalued function and defined by  $J(d) = \{e, n\}$  if  $d = h_1$ and  $(d) = \{e, nt\}$  if  $d = h_2$ . It is clear that (J, H) is a (NSCDG) over W. Let  $T = \{n, n\}$ *t*}. By Definition (3.1) of (SLCDS), we get:

 $\underline{J}_T(d) = \begin{cases} \emptyset, & \text{if } d = h_1 \\ \{n, t\}, & \text{if } d = h_2 \end{cases}, \quad \forall \ d \in H. \text{ Also, from Definition (3.1) of } \end{cases}$ (SUCDS), we get:  $\overline{J}^T(d) = \begin{cases} \emptyset, if \ d = h_1 \\ \{n, t\}, if \ d = h_2 \end{cases}$ , for all  $d \in H$ . Now, we can

consider information system (IS) for (NSCDG).

Table (2-a) Table (2-b) The (IS) for (J, H) with symbols The (IS) for (J,H) with  $\{0,1\}$ (I,H) $h_1$ h2 (I,H) $h_1$ h, 1 1 е  $\alpha_{11}$  $\alpha_{12}$ е 1 0 п п  $\alpha_{21}$  $\alpha_{22}$ t t 0 0  $\alpha_{31}$  $\alpha_{32}$ 0 1 nt nt  $\alpha_{41}$  $\alpha_{42}$ 

If  $\alpha_i \in J(h_i)$ , then  $\alpha_{ii} = 1$  otherwise  $\alpha_{ii} = 0$ , where  $\alpha_1 = e, \alpha_2 = n, \alpha_3 = n$ t, and  $\alpha_4 = nt$ . Then the (IS) for (J, H) is given in following Table 2.

By the same way we can consider the (ISs) for any (NSCDG), (SLCDS), and (SUCDS).

**Proposition 3.3:** Suppose that  $(J_1, H_1)$  and  $(J_2, H_2)$  are normal soft cubic dihedral groups (NSCDGs) over W and let  $\emptyset \neq T_1 \subseteq W$  and  $\emptyset \neq T_2 \subseteq W$ . Then (1)  $J_{\underline{1}_{T_1}}(r) \subseteq T_1 \subseteq \overline{J_1}^{T_1}(r)$ , for all  $r \in H_1$ , (2) if  $T_1 \subseteq T_2$ , then  $(\underline{J_1}_{T_1}, H_1) \cong$  $\underbrace{\left(J_{1}_{T^{2}}, H_{1}\right)}_{T_{1}}, \quad (3) \quad \text{if } T_{1} \subseteq T_{2}, \quad \text{then} \quad \left(\overline{J_{1}}^{T1}, H_{1}\right) \cong \left(\overline{J_{1}}^{T2}, H_{1}\right), \quad (4) \quad \left(\underline{J_{1}}_{T1}, H_{1}\right) \widehat{\cup} \\ \underbrace{\left(J_{1}_{T2}, H_{1}\right)}_{T_{1}}, \quad H_{1}\right) \cong \left(\underline{J_{1}}_{T1}, H_{1}\right) \cong \left(\overline{J_{1}}^{T1}, H_{1}\right) \cap \left(\overline{J_{1}}^{T2}, H_{1}\right),$ 

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 $(6)\left(\overline{J_{1}}^{T1\cup T2},H_{1}\right) \cong \left(\overline{J_{1}}^{T1},H_{1}\right) \bigcirc \left(\overline{J_{1}}^{T2},H_{1}\right), \quad (8)\left(\underline{J_{1}}_{T1\cap T2},H_{1}\right) \cong \left(\underline{J_{1}}_{T1},H_{1}\right) \cap \left(\underline{J_{1}}_{T2},H_{1}\right), \quad (9) \quad \text{if} \quad (J_{1},H_{1}) \cong (J_{2},H_{2}), \quad \text{then} \quad \left(\overline{J_{1}}^{T1},H_{1}\right) \cong \left(\overline{J_{2}}^{T1},H_{2}\right), \quad (10) \quad \text{if} \quad (J_{1},H_{1}) \cong (J_{2},H_{2}), \quad \text{then} \quad \left(\underline{J2}_{T1},H_{2}\right) \cong \left(\underline{J_{1}}_{T1},H_{1}\right), \quad (11) \quad \underline{J_{2}}_{T1},H_{3}) \cong \left(\underline{J_{1}}_{T1},H_{1}\right) \cap \left(\underline{J_{2}}_{T1},H_{2}\right).$ 

**Proof:** (1) Let  $w \in \underline{J_1}_{T_1}(r)$ , then  $wJ_1(r) \subseteq T_1$  and hence  $w \in T_1$ , thus  $\underline{J_1}_{T_1}(r) \subseteq T_1$ . Also, since  $wJ_1(r) \subseteq T_1$ , then  $wJ_1(r) \cap T_1 = wJ_1(r) \neq \emptyset$ , thus  $w \in \overline{J_1}^{T_1}(r)$ . Hence  $\underline{J_1}_{T_1}(r) \subseteq T_1 \subseteq \overline{J_1}^{T_1}(r)$ ,  $\forall r \in H_1$ . Also, [(2)-(11)] the proof is indisputable. The following examples show how Proposition (3.3)'s inclusions in (4), (5), (10) and (11) are strict:

**Example 3.4** Let  $\alpha = (1 \ 3)(5 \ 8), \beta = (2 \ 7), \gamma = (4 \ 6)$  be three permutations in  $(S_8, 0)$ . Take k = 2. Then we consider the following :  $(1)\beta\alpha = (2 \ 7)(1 \ 3)(5 \ 8) = (1 \ 3)(5 \ 8)(2 \ 7) = \alpha^{k-1}\beta$ ,  $(2)\gamma\alpha = (4 \ 6)(1 \ 3)(5 \ 8) = (1 \ 3)(5 \ 8)(4 \ 6) = \alpha^{k-1}\gamma$ ,  $(3) \ \beta\gamma = (4 \ 6)(2 \ 7) = (2 \ 7)(4 \ 6) = \gamma\beta$ . Hence  $CD_2 = _2 < (1 \ 3)(5 \ 8), (2 \ 7), (4 \ 6) >_{S_8}^o$  is an 8-order, 2-degree (CDPG). Also,  $(CD_2, o)$  is a subgroup of  $(S_8, o)$ . See Table 3.

|     |     |     |     |     | 0   |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0   | е   | α   | β   | αβ  | γ   | αγ  | βγ  | αβγ |
| е   | е   | α   | β   | αβ  | γ   | αγ  | βγ  | αβγ |
| α   | α   | е   | αβ  | β   | αγ  | γ   | αβγ | βγ  |
| β   | β   | αβ  | е   | α   | βγ  | αβγ | γ   | αγ  |
| αβ  | αβ  | β   | α   | е   | αβγ | βγ  | αγ  | γ   |
| γ   | γ   | αγ  | βγ  | αβγ | Ε   | α   | β   | αβ  |
| αγ  | αγ  | γ   | αβγ | βγ  | α   | е   | αβ  | β   |
| βγ  | βγ  | αβγ | γ   | αγ  | β   | αβ  | е   | α   |
| αβγ | αβγ | βγ  | αγ  | γ   | αβ  | β   | α   | е   |

Table 3(CD2, 0) is (CDG) of degree 2

Let  $H = \{h_1, h_2\}$  and  $J: H \to P(CD_2)$  be a multivalued function defined by  $J(d) = \begin{cases} \{e, \alpha\}, & \text{if } d = h_1 \\ \{e, \beta\gamma\}, & \text{if } d = h_2 \end{cases}$ , for all  $d \in H$ . Hence (J, H) is a (NSCDG) over  $CD_2$ . Assume that  $T_1 = \{\gamma, \beta\gamma, \alpha, \beta\}$  and  $T_2 = \{e, \alpha\beta, \alpha\gamma, \alpha\beta\gamma\}$ , then  $T_1 \cup T_2 = CD_2$ . We have arrived at this conclusion by simple computations:  $J_{T1} (h_1) = \emptyset, J_1$  $T_2 (h_1) = \{\alpha\}$  and  $J_{T1} \cup T_2 (h_1) = CD_2$ . Then we have,  $J_{T1\cup T2}(h_1) \nsubseteq J_{T1}(h_1) \cup J_{T2}(h_1)$ . Let  $T_3 = \{\gamma, \alpha\beta\gamma, \alpha\beta, \beta\gamma\}$  and  $T_4 = \{\alpha, \beta\gamma, \beta, \alpha\gamma\}$  we obtain:  $\overline{J}^{T3}(h_2) = \overline{J}^{T4}(h_2) = CD_2$  and  $\overline{J}^{T3\cap T4}(h_2) = \{e, \beta\gamma\}$ . It shows that  $\overline{J}^{T3}(h_2) \cap \overline{J}^{T4}(h_2) \nsubseteq J_2$ . **Theorem 3.6:** Suppose that T is a subgroup of (CDG) W satisfies  $\underline{J}_T(d) \neq \emptyset$ , for some  $d \in H$ . Then,  $\underline{J}_T(d) = T = \overline{J}^T(d)$ ,  $\forall d \in H$ .

**Proof:** We claim that  $y \in \underline{J}_T(d)$ ,  $d \in H$ . By assumption on  $\underline{J}_T(d)$ , there exists  $z \in \underline{J}_T(d)$  and hence  $z \in T$  [From Proposition 3.3- (1)]. Also, by hypothesis we get:  $\overline{J}(d) = z^{-1}.zJ(d) \subseteq T.T = T \dots (3.1)$ . Thus,  $y \in \underline{J}_T(d)$  and this satisfies the claim. In other side,  $\underline{J}_T(d) \subseteq T \subseteq \overline{J}^T(d)$ ,  $\forall d \in H$ . (see Proposition (3.3)- (1)). Let  $s \in \overline{J}^T(d)$ , where  $d \in H$ . There exists  $w \in W$  such that  $w \in sJ(d) \cap T$ . Then wJ(d) = sJ(d) and  $w \in T$ . We claim that  $wJ(d) \subseteq T$ . From Equation (3.1), we have  $J(d) \subseteq T$ . Then  $wJ(d) \subseteq wT = T$ . This proves the claim. Hence,  $s \in \underline{J}_T(d)$ ,  $\forall d \in H$ . Therefore,  $\underline{J}_T(d) = T = \overline{J}^T(d)$ ,  $\forall d \in H$ .

**Theorem 3.7:** Suppose that D and F are normal subgroups of (CDG) W, if  $\underline{F}_D \neq \emptyset$ . Then,  $\underline{F}_D = D = \overline{F}^D$ , where  $\underline{F}_D = \{c \in W | cF \subseteq D\}$  and  $\overline{F}^D = \{c \in W | cF \cap D \neq \emptyset\}$ .

**Proof:** Since  $\underline{F}_D \subseteq D \subseteq \overline{F}^D$ . From hypothesis  $\underline{F}_D \neq \emptyset$ , thus  $q \in \underline{F}_D$ , for some  $b \in W$ , we have  $b \in D$ . Hence,  $F = eF = (b.b^{-1})F = (bN).(b^{-1}N) \subseteq D.D \subseteq D...(3.2)$ . Let  $s \in \overline{F}^D$ . Hence  $t \in sF \cap D$  for some  $t \in W$ . By Eq. 3.2, we consider that  $tF = (t.e)F = tF.eF \subseteq yF.D \subseteq D$ . However, tF = sF. Thus  $sF \subseteq D$ . Therefore,  $s \in \underline{F}_D$ . The proof is now complete. The following results are presented to explain the relationship between soft cubic dihedral sets in lower and upper setting:

**Proposition 3.8:** If W is a (CDG) with  $\emptyset \neq T_1 \subseteq W$  and  $\emptyset \neq T_2 \subseteq W$ . Then, (1)  $(\overline{J}^{T1T^2}, H) \cong (\overline{J}^{T1}, H)^{\delta} (\overline{J}^{T^2}, H)$ , (2)  $(\underline{J}_{T1}, H)^{\delta} (\underline{J}_{T2}, H) \cong (\underline{J}_{T1T2}, H)$ .

**Proof:** (1) Let  $s \in \overline{J}^{T1}(d) \cdot \overline{J}^{T2}(d)$  and  $d \in H$ . Then s = tq, for some  $t \in \overline{J}^{T1}(d)$  and  $q \in \overline{J}^{T2}(d)$ . There exist  $s_1, s_2 \in W$  such that  $s_1 \in tJ(d) \cap T_1$  and  $s_2 \in qJ(d) \cap T_2$ . It yields that  $s_1s_2 \in tqJ(d)$  and  $s_1s_2 \in T_1T_2$ . Thus  $s_1s_2 \in tqJ(d) \cap T_1T_2$ . Therefore,  $s = tq \in \overline{J}^{T1T2}(d)$ ,  $\forall d \in H$ . Conversely, assume that  $y \in \overline{J}^{T1T2}(d)$  with  $d \in H$ . Thus  $b \in yJ(d) \cap T_1.T_2$ , for some  $b \in W$ . Since (J, H) is a (NSCDG) over W, as a consequence  $y \in bJ(d)$  and  $b = m_1m_2$ , for some  $m_1 \in T_1$  and  $m_2 \in T_2$ . Thus,  $y \in (m_1m_2)J(d) = (m_1J(d))(m_2J(d))$ . Let  $y = c_1 c_2$ , for some  $c_1 \in m_1J(d)$  and  $c_2 \in m_2 J(d)$ . Then,  $m_1 \in c_1 J(d)$  and  $m_2 \in c_2 J(d)$ . Thus,  $m_1 \in c_1 J(d) \cap T_1$  and  $m_2 \in c_2 J(d) \cap T_2$ , which yields that  $c_1 \in \overline{J}^{T1}(d)$  and  $c_2 \in \overline{J}^{T2}(d)$ . Hence,  $y = c_1 c_2 \in \overline{J}^{T1}(d) \cdot \overline{J}^{T2}(d)$ ,  $\forall d \in H$ . (2) Let  $y \in \underline{J}T_1(d) \cdot \underline{J}T_2(d)$ , where  $d \in H$ . Then y = tq, for some  $t \in \underline{J}T_1(d)$  and

 $q \in \underline{J} T_2$  (d). Hence,  $tJ(d) \subseteq T_1$  and  $qJ(d) \subseteq T_2$ . It implies that  $(tq)J(d) = (tJ(d))(qJ(d)) \subseteq T_1T_2$ . This proves that  $y = tq \in \underline{J}T_1.T_2(d), \forall d \in H$ .

This sign  $\cong$  (soft inclusion) cannot take the place of  $\cong$  (soft equality) in Proposition 3.8 as follow:

**Example 3.9:** Take  $\alpha = (4 \ 1 \ 2)(1 \ 4 \ 2 \ 6)$ ,  $\beta = (5 \ 9)$ ,  $\gamma = (3 \ 8)$  three permutations in  $(S_9, 0)$  and  $H = \{m, n\}$ . Then  $(_2 < \alpha, \beta, \gamma >_{S_9}^o, o)$  is a (CDPG). Let (J, H) be a (NSCDG) over  $W = _2 < \alpha, \beta, \gamma >_{S_9}^o$  and defined by:  $J(d) = _2 < \alpha, \beta, \gamma >_{S_9}^o$ , if  $d = m \& J(d) = \{e, (5 \ 9)\}$ , if d = n.

For all  $d \in H$ . Assume that  $T_1 = \{e, \alpha, \gamma\}$  and  $T_2 = \{\beta\gamma, \alpha\beta, \gamma\}$ . Hence,  $T_1T_2 = \{e, \gamma, \beta\gamma, \alpha\beta, \beta, \alpha\gamma, \alpha\beta\gamma\}$ . Then by the definition of (SLCDS), we consider the following:  $\underline{J}T_1T_2(n) = \{e, \gamma, \beta\gamma, \beta, \alpha\gamma, \alpha\beta\gamma\}$ ,  $\underline{J}T_1(n) = \emptyset$  and  $\underline{J}T_2(n) = \{\gamma, \beta\gamma\}$ . This shows that  $\underline{J}T_1(n) \cdot \underline{J}T_2(n) \not\supseteq \underline{J}T_1T_2(n)$ .

## 4. Conclusions

We distinguish between two types of sets in this paper: lower cubic dihedral sets and higher cubic dihedral sets that are soft. Then, the (NSCDGs) are used to insert them. For future research the members of symmetric group permutations will utilize to examine cubic dihedral permutation d/BCK –algebra. Furthermore, certain results will present to investigate the types of even and odd permutations provided by the structure of their permutations.

## References

- [1] L.A., Zadeh, Fuzzy sets. Inf Control, 8(3), 338-353 (1965).
- [2] D. Molodtsov, Soft set theory first results. *Comput Math Appl* 37(4-5), 19-31 (1999).
- [3] Z. Pawlak, Rough sets. Int J Comput Inf Sci 11(5), 341-356 (1982).
- [4] H. Aktas, N. Cagman, Soft sets and soft groups. *Inf Sci*, 177(13), 2726-2735 (2007).
- [5] A. H. Mohanad, M. A. Nadia, K. Shuker, On soft α\* open sets and soft contra α\* - continuous mappings in soft topological spaces, *Journal of Interdisciplinary Mathematics*, 24(3), 729-734 (2021). DOI: 10.1080/09720502.2020.1861786
- [6] S. Alsalem, A. F. Al-Musawi, and E. Suleiman,"On Permutation Bcenter and Derived Permutation B-algebras, (MCSI), Athens, Greece, 46-50 (2022), doi: 10.1109/MCSI55933.2022.00014.

- [7] S. Alsalem, A. F. Al-Musawi, and E. Suleiman, "On maximal permutation BH-ideals of Permutation BH-Algebras," (MCSI), Athens, Greece, 40-45 (2022), doi:10.1109/MCSI55933.2022.00013.
- [8] A. R.Nivetha, M. Vigneshwaran, N. M. Abbas and S. Khalil, On Ng<sup>\*</sup> α- continuous in topological spaces of neutrosophy, J. Interdiscip. Math., 24(3), 677-685 (2021).
- [9] N. Ali, S. Khalil and M. Vigneshwaran, "The Neutrosophic Strongly Open Maps in Neutrosophic Bi-Topological Spaces", J. Interdiscip. Math., 24(3), 667-675 (2021). Doi: 10.1080/09720502.2020.1860287
- [10] S. Mahmood, S., Fakher, H. T., The Cubic Dihedral Permutation Groups of Order 4k, ECS Transaction, 107(1), 3179–3186 (2022). doi:10.1149/10701.3179ecst.
- [11] F. Feng, Y. B. Jun, X. Zhao, Soft semirings. Comput Math Appl, 56(10), 2621-2628 (2008).

Received November, 2023