

# Mobile Robot Navigation with Obstacles Avoidance by Witch of Agnesi Algorithm with Minimum Power

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## Abstract

*Obstacle avoidance in mobile robot path planning represents an exciting field of robotics systems. There are numerous algorithms available, each with its own set of features. In this paper a Witch of Agnesi curve algorithm is proposed to prevent a collision by the mobile robot's orientation beyond the obstacles which represents an important problem in path planning, further, to achieve a minimum arrival time by following the shortest path which leads to minimizing power loss. The proposed approach considers the mobile robot's platform equipped with the LIDAR 360o sensor to detect obstacle positions in any environment of the mobile robot. Obstacles detected in the sensing range of the mobile robot are dealt with by using the Witch of Agnesi curve algorithm, this establishes the obstacle's apparent vertices' virtual minimum bounding circle with minimum error. Several Scenarios are implemented and considered based on the identification of obstacles in the mobile robot environment. The proposed system has been simulated by the V-REP platform by designing several scenarios that emulate the behavior of the robot during the path planning model. The simulation and experimental results show the optimal performance of the mobile robot during navigation is obtained as compared to the other methods with minimum power loss and also with minimum error. It's given 96.3 percent in terms of the average of the total path while the Bezier algorithm gave 94.67 percent. While in experimental results the proposed algorithm gave 93.45 and the Bezier algorithm gave 92.19 percent.*

## Keywords

Mobile Robot Navigation, Obstacle Avoidance, Lidar Sensor, Path Planning, Witch of Agnesi, Minimum Power.

## I. INTRODUCTION

Path planning is a major area of interest within the field of robotics in which the planner determines the safest route from the source to the destination without colliding. It is frequently utilized in computer games, family services, and military applications. Path planning algorithms are grouped into two sorts based on the environment the robot will pass through: static obstacle algorithms and dynamic obstacle algorithms. Due to the robot using sensory input to determine the current location and direction of moving objects, dynamic route planning has more challenges than static path planning. A huge number of algorithms solve whole problems of path

planning in a dynamic environment, each with its own set of benefits and limitations. These algorithms are categorized based on various concerns such as safety, the length, and runtime of the path, and also based on accuracy, control, stability, uncertainty, and computing cost [1, 2]. The route-organizing approaches may be categorized into both local and global techniques based on the sensor knowledge available [3]. The robot designed its trajectory by global methods based on global environmental information [4]. This procedure ensures the path of the robot converges and directs whether the objective is achieved or not. On the other hand, planning approaches that focus on the beams of sensors have minimal (local) knowl-



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edge which does not promise global convergence to the target. Although these are easier to implement, with each control step, the robot utilizes its sensors to identify surrounding barriers and plan the next move to take. In the case of obstacles, various path mapping techniques use a Voronoi diagram under either the Global or the Local Knowledge Scenario [5–7] theory. Global intelligence assumes that each robot has full knowledge. The information state limits the data of each robot to the sensing spectrum of all obstacles in the system when in the local environment.

## II. LITERATURE REVIEW

In the presence of obstacles, several path-planning strategies use a Voronoi diagram depending on the assumption of either a global or a local knowledge condition. In techniques that are based on the Voronoi diagram, either a smooth or a piecewise linear smooth trajectory is the expected trajectory. In the first example, stopping at either end of the trajectory segments is acceptable for the robots, changing their direction according to the next section and restarting again afterward [8]. This kind of action is inconvenient and leads to extra energy loss. Additional methods for path planning include the artificial potential field (APF) and cell breakdown [9–13]. The theory of virtual forces, which affect the robot during navigation, is the foundation of the artificial potential field. The goal attracts the robot with an attractive force, while each obstacle repels it with a repulsive force. The combined effect of these two forces defines the robot's orientation [14, 15]. An enhanced APF was created to address the local minimum that occurs when the absolute values of the repulsive and attraction forces are equal, leaving the robot stranded. [16–21]. The evolutionary APF uses a genetic algorithm to develop APF in order to extract the robot from the trap utilizing an escape power [22–27]. Based on the idea of dividing empty space into cells, cell decomposition algorithms look for the shortest path in the resulting graph using the graph search method. A useful technique for cell breakdown is the grid approach, in which a grid represents the real environment [28, 29]. How to choose the ideal grid size is the main problem with this technique. The implementation of graphic search algorithms for the shortest path is another type of class of approaches for path designing. For such methods, the graph of feasible paths is first generated and an algorithm of the shortest path search is used. This visibility graph can be used, which is the visible visual obstacle graph (obstacles considered polygonal), in which a vertex (V) is identified as noticeable from the vertex (U) if there are no intersections of the obstacle edges in the environment in the VU segment. The computing time for shortest path estimation is a common restriction of techniques based on the graph. Therefore, simplifying the graph structure to be analyzed will increase the reliability of

such approaches. This is why other graphs are introduced rather than the visibility graph. Many works have used the so-called tangent visibility graph. The visibility diagram is identified as the set of potential pathways extracted from the visibility graph by holding only the two-tangent edges of the graph [30–36]. The visibility graph is smaller than the visibility graph of vertices and sides. This results in the shortest path estimation method for the graph becoming less efficient. In the case of the algorithms for the shortest path, a complete visibility graph in [37] is the Dijkstra algorithm. The scope can be improved by the tangent graph as in [38, 39] and the visibility graph instead. To solve the problem of obstacle avoidance with the short distance between the robot's original position and the goal, [40] improves visibility by computing each obstacle's Virtual Minimum Bounding Circle from its Observable Vertices, and by the regular robot-to-go trajectory, of the binary tree algorithm operates in an area of polygonal obstacles. This path is made up of a circular arc, which shifts the robot's trajectory from the circular to the target and a tangent axis. After an obstacle crosses the robot's normal path, the inner and outer tangents between the robot's normal path and the circular collision obstacle are found. The paths from the tangent point to the target are then completed, and the direction to the target is selected, from the inner or outer tangent path, from the robot position. This operation continues until the target is met. The objective decrease power consumption and accurate robot navigation with minimal reactions, [41] proposed an obstacle avoidance algorithm with the shortest path to ensure smooth and safety in an unknown environment. The R-function is used to prevent a collision between the original location and destination and to identify the shortest path [42]. The R-functions are constant and vary to cause the mobile robot to escape obstacle collisions and hit its target in less time by the gradient feedback of the goal function. Numerous geometrical curves such as the three-dimensional continuum, multinomials, and the Bezier curve point to the problem of trying to find a new path from the mobile robot's starting point to the objective while the shorter path's positioning is uncertain [43–45]. In this paper, a simple low-cost, and high-precision path planning has been proposed for mobile robot navigation and performs obstacle avoidance, which is the Witch of Agnesi Algorithm (WAA). In this scenario, the mobile robot moves from its starting point to its destination while avoiding obstacles by assuming that both the obstacles and the robot are shaped like circles with a radius of R, depending on the information collected by the robot's RP LIDAR sensor. The proposed system is simulated by the V-rep platform, in which several scenarios have been designed to validate the proposed algorithm. The simulation results reveal that the shortest path to the target is achieved with no obstacles; furthermore, the proposed method is superior to



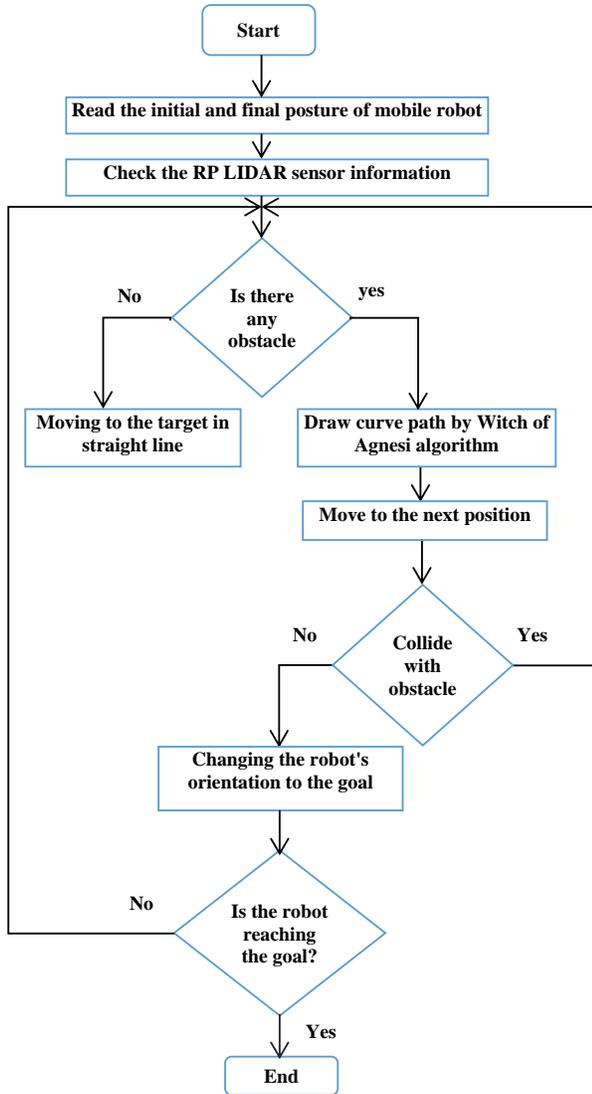


Fig. 2. The witch of Agnesi path planning algorithm’s proposed flow chart.

sensor detects an obstruction. The efficiency of the robot trajectory is improved under two conditions: firstly, To avoid colliding with the obstacles, the robot can smoothly circle around them. Second, in order to get there, the robot will take the quickest route possible. As seen in Fig.2, the suggested algorithm recommended path planning. To move the mobile robot from the preliminary point to the objective point, use the path optimization technique that is suggested in the following steps, which are as follows:

**Step 1:** Read the first parameters: the robot’s initial location  $(x_0, y_0)$ , the target position  $(x_T, y_T)$ , and the robot radius  $r$ .

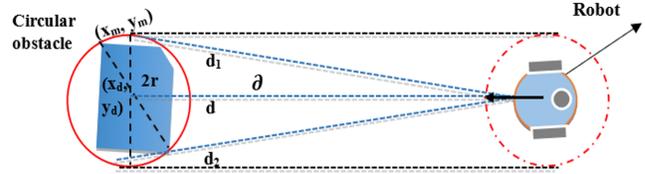


Fig. 3. Virtual circle obstacle with the correct orientation.

**Step 2:** Calculate the mobile robot’s next position: the robot is placed randomly in an unknown environment, therefore the next position and orientation are calculated according to the scanning information achieved from the RP LIDAR sensor equipped with the robot body. As shown in Fig. 2 if the RP LIDAR observes no obstacle, the mobile robot moves in a conventional path. Two points need to be situated in a straight line to study the value movement of the mobile robot. The next robot location, as indicated in Fig. 3, is determined by applying (10) to calculate the distance between the mobile robot’s beginning location and the goal point [20]:

$$L_i = \sqrt{(y_i - y_o)^2 + (x_i - x_o)^2} \tag{10}$$

1- Compute the angle of rotation  $\phi$  by:

$$\phi = \tan^{-1} \left( \frac{y_i - y_0}{x_i - x_0} \right) \tag{11}$$

2- The mobile robot’s straight path is drawn by using two points  $((x_1, y_1)$  and  $(x_2, y_2)$ ) which should be computed to determine the next position of the robot. By supposing that  $d$  is the extreme detecting range of the sensors, the points  $(x_1, y_1)$  and  $(x_2, y_2)$  can be computed from the following equations.

$$x_i = x_o + d \cdot \cos(\phi) \tag{12}$$

$$y_i = y_o + d \cdot \sin(\phi) \tag{13}$$

3- The steps from (1-3) are recurrent in anticipation the RP LIDAR sensor notices an obstacle or the robot influencing its

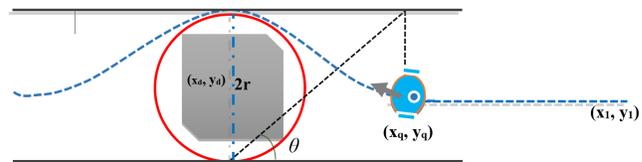


Fig. 4. Applying the Agnesi curve algorithm.

**Step 3:** Whenever an obstacle is detected by the RPLIDAR sensor, the mobile robot must determine the suitable direction to reach its target by using the Agnesi curve algorithm. For obstacle avoidance, two points should be measured to prevent reaching the obstacle. The first point: is the actual location of the robot  $(x_1, y_1)$  concerning obstacles to draw the witch of Agnesi curve. The second point: is a high curvature point at the point of tangency with the obstacle  $(x_2, y_2)$ . The following determines the next position of the robot:

- 1- The suggested approach avoids colliding with obstacles by virtually encircling them as in [8], where the new shapes of the obstacles will be represented by all of these virtual circles. The virtual circle will then be generated, but only for the obstacles that the robot's sensor has identified. The robot will move tangent to the virtual rings of the obstacles and look for the shortest path to its destination if the concealed edges are outside the generated circle, which may be estimated by measuring the diameter of the circle from the biggest distance between two distant vertices. The longest distance that could be measured between the two distant vertices is used to calculate the radius of the virtual circle.
- 2- The laser sensor of the robot intersects the obstacle at more than one point. Since each obstacle represents a virtual circle, the laser beam with the shortest distance gives a more accurate direction of the obstacle position which will be taken with a radius of the virtual circle. The obstacle location depends on the chosen laser beam whose distance and orientation are known. The orientation given by the sensor has  $\partial$  a degree from the correct direct orientation path between the robot attached to it and the obstacle. Also, an error in distance is obtained between them where  $d$  is the actual distance between the robot and the obstacle. To compute the position of the second point  $(x_q, y_q)$ , the actual orientation of the obstacle should be found first. The points  $d(x_d, y_d)$  and  $m(x_m, y_m)$  must be calculated by:

$$x_d = x_1 + (d + r) \cos(\theta + \partial) \quad (14)$$

$$y_d = y_1 + (d + r) \sin(\theta + \partial) \quad (15)$$

$$x_m = x_1 + (m + r) \cos(\theta) \quad (16)$$

$$y_m = y_1 + (m + r) \sin(\theta) \quad (17)$$

Where  $\theta$  is the rotation of the robot. Therefore, the direction of the obstacle edge is calculated by:

$$\phi = \tan^{-1} \left( \frac{y_m - y_d}{x_m - x_d} \right) \quad (18)$$

- 3- Apply the Witch of Agnesi curve algorithm to create the shortest free collision path by determining the second point  $q(x, y)$  by the following two equations:

$$x_q = 2r \cot(\phi) \quad (19)$$

$$y_q = 2r (1 - \cos^2(\phi)) \quad (20)$$

then as the robot increases its steps, the orientation of the obstacle is changed according to the angle information obtained by the RPLIDAR sensor until reaches its target.

- 4- Finally the energy model for a robot should consider the power consumption in mobile robot dynamics, DC motors, and electronics. The energy consumption model of a differential drive mobile robot [47] by using the equation below estimates approximately the energy consumed under the assumption that the output power of the battery is constant and the results will be shown in Table I.

$$E_{\text{robot}} = E_{\text{DC}} + E_K + E_f + E_{\text{elect}} \quad (21)$$

$E_K$  is the kinetic energy loss,

$E_f$  is the energy loss due to friction,

$E_{\text{elect}}$  is the energy loss in the electronics,

$E_{\text{robot}}$  is the energy of the battery used for the robot

**Step 4:** The robot rotates in the direction of the target when the RPLIDAR sensor is disabled as shown in Fig. 4. This method has to be achieved without an obstacle collision.

**Step 5: short distance path:** The robot searches for the quickest route between its current location and the goal in this stage by avoiding the obstacle by the following conditions after determining the starting point  $p_q(x_q, y_q)$  by using the Witch of Agnesi curve algorithm:

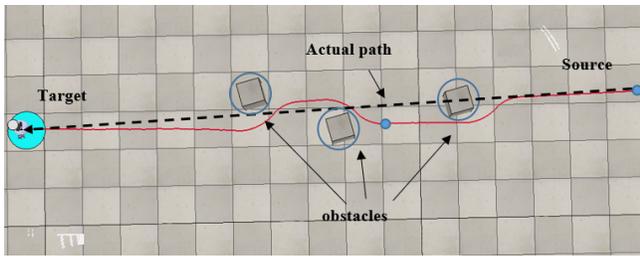
-IF  $p_q > 2r$  then

-  $j = j + 1$

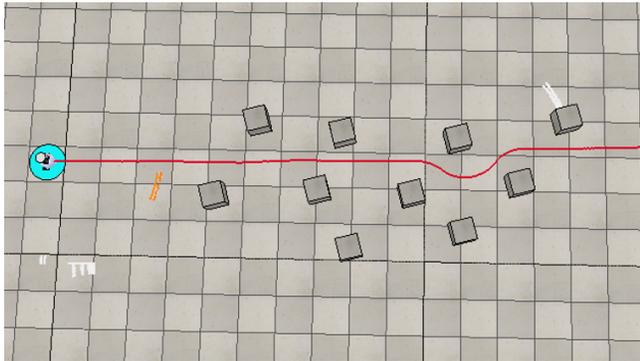
-Else

-  $j = j - 1$

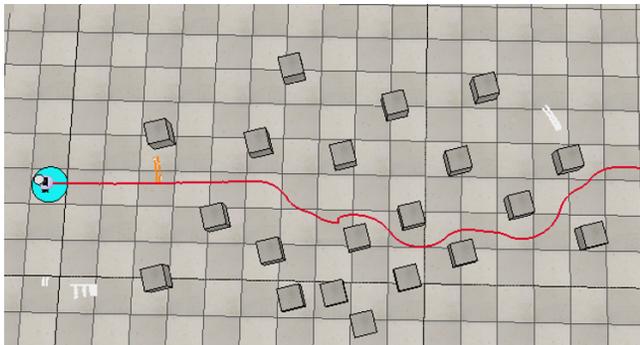
Where  $j$  is the number of steps taken by the robot to move the obstacle and reach its target.



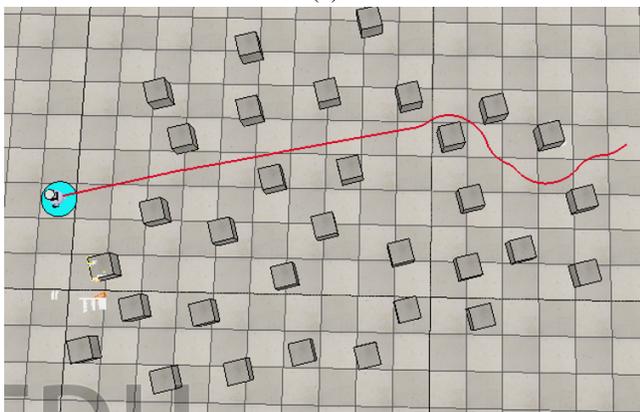
(a)



(b)

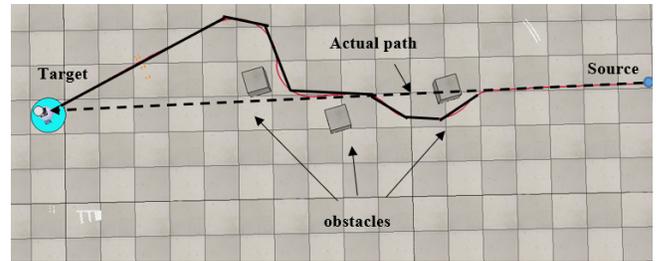


(c)

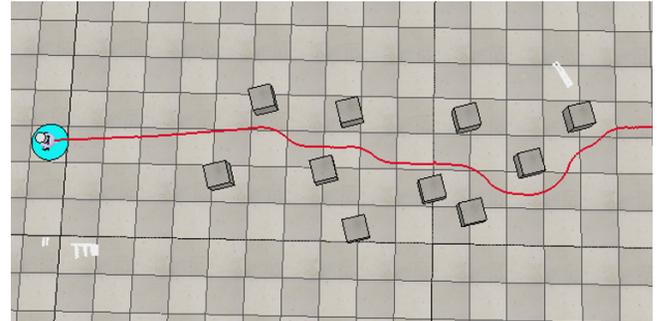


(d)

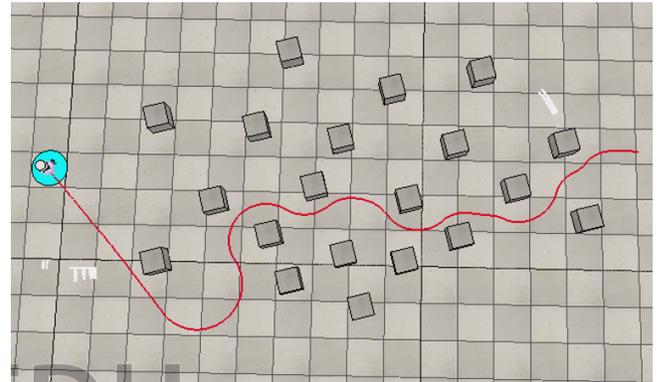
Fig. 5. The simulation of path planning trajectories when the witch of Agnesi Curve Algorithm is used.



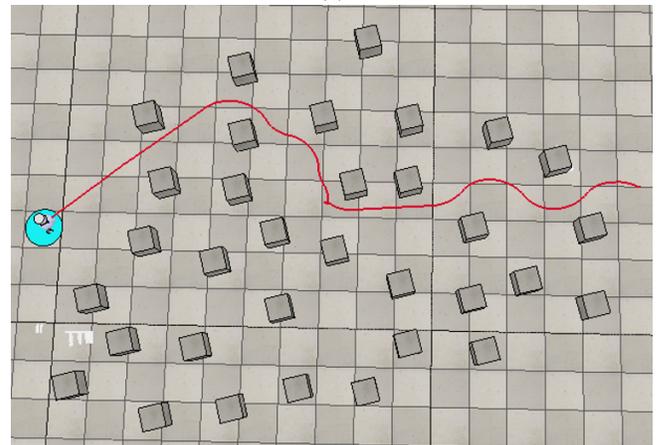
(a)



(b)



(c)



(d)

Fig. 6. The simulation of path planning trajectories when witch of Bezier Algorithm is used.

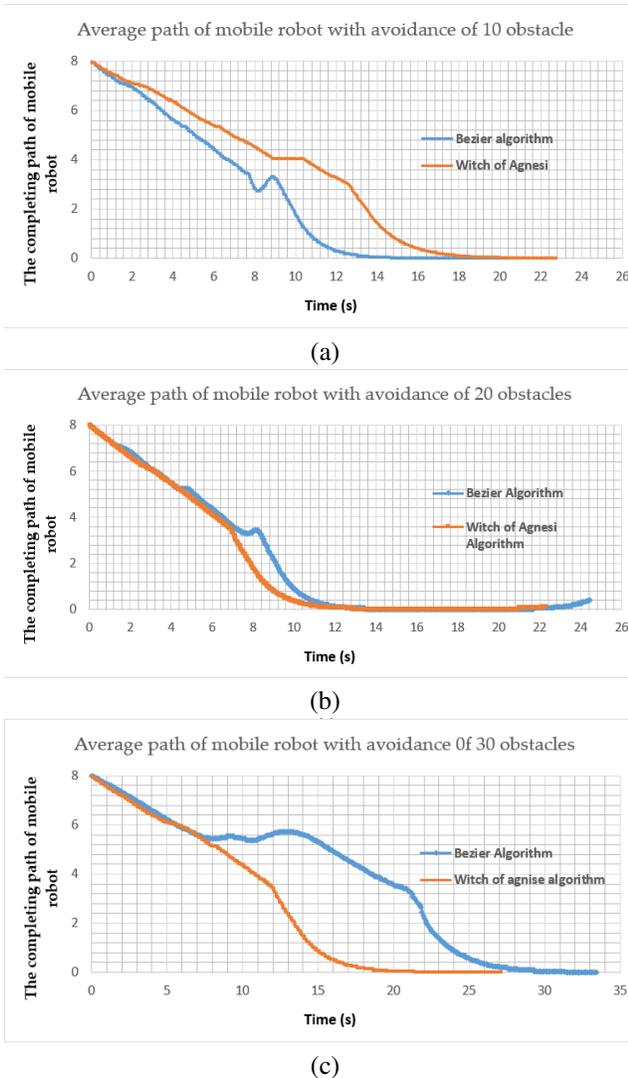


Fig. 7. The average of completing path by Witch of Agnesi curve algorithm and Bezier Algorithm with the different number of obstacles.

## V. SIMULATION RESULTS

The mobile robot navigation, applying a witch of Agnesi arc, has been implemented by utilizing the V-REP platform in an area of (10 m\*10 m) and examined with an Intel Core i5 CPU running at 2.53 GHz in a Windows environment. The sensing ranges of the sensor is expected to be unlimited due to the using an RP LIDAR sensor which has 360° orientation and large distance measurement. As a result, the mobile robot can identify obstacles in any sensing area. In simulations, a simple small differential-drive robot is assumed. With two cylindrical working wheels—the right and left—and a single spherical wheel in the back, the robot's body measures 26 cm

in diameter and weighs 1,029 kg. The suggested technique's efficiency was confirmed by comparing the acquired results to the Bezier curve algorithm. To evaluate how well a mobile robot moves to its destination while avoiding obstacles, several trials are conducted. In these tests, the robots are supposed to use the Witch of Agnesi curve method to change direction with the least amount of error to avoid colliding with the obstacle. Multiple tests in a linear monitoring scenario, where the mobile robot travels in a straight line and the area used in this scenario has several obstacles, will be conducted to confirm the effectiveness of the mobile robot by the suggested model. In Fig. 5 several depictions of the trajectories produced by the witch from the Agnesi algorithm. A shorter trajectory compared to the results of the other two algorithms is shown in Fig. 6, have been provided for qualitative analysis. Because it aims to maintain algorithm security while the other algorithms urge the robot into courses that might be closer to obstacles, the Voronoi algorithm develops the longest trajectory. In this instance, simulations with potential barriers ranging from 0 to 30 have been repeated. For every value of the number of obstacles, different realizations were considered since for each realization, a specific distribution of the obstacles on the plane was considered. A situation with a variety of distinct obstacles to form is considered for example purposes. Since the RP LIDAR reads at 360 degrees, there are expected to be unlimited sensor ranges. The mobile robot is thus capable of recognizing obstacles in all directions. The results obtained were compared to the Bezier curve algorithm to validate the efficacy of the proposed algorithm. Various trajectory snapshots were taken from the proposed curve algorithm Witch of Agnesi. The simulation is repeated at different locations. Table 1 shows the shortest path and the average time for our algorithm and the Bezier curve algorithm. The results show that when a Witch of Agnesi curve algorithm is extended to a path length that is shorter than that of the path algorithm with the Bezier curve algorithm. The average value obtained of the distance moved between the basis and the goal is measured and displayed in Fig. 7 which represents the average of completing path by the Witch of Agnesi curve algorithm and Bezier Algorithm with the different number of obstacles. For each navigation algorithm in the proposed algorithm takes less time to reach the target than the Bezier curve algorithm. Results illustrate the efficient target of the introduced algorithm compared with the Bezier curve algorithm as shown in Fig.8 in which our algorithm takes less error as compared with the Bezier curve algorithm. and also, with minimum error as in Fig. 9 which represents the mean value for a mobile robot to reach its target with the standard deviation.

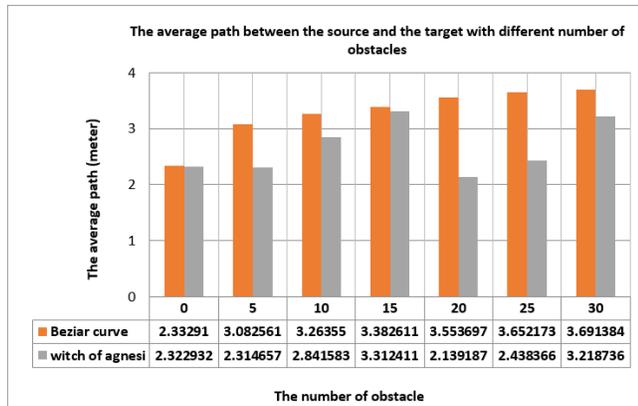


Fig. 8. The Performance comparison of the motion prediction of mobile robot in a unknown environment on a straight line trajectory with different number of obstacles.

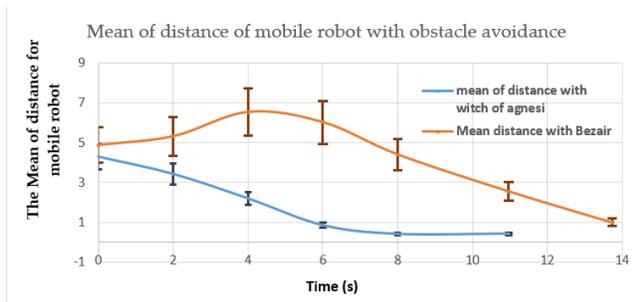


Fig. 9. The mean value and standard deviation of the distance between the robot's positions and its target for two algorithms.

## VI. EXPERIMENTAL RESULTS

In this experiment, a differential-drive mobile robot with two adjustable steering wheels and one balancing wheel was used. To move the robot in an unfamiliar setting, two variables must be controlled: the robot's linear velocity  $v(t)$  and its angle of orientation. It is necessary to develop a model of the mobile robot to design an appropriate control system for navigating it in an unknown environment. To create this structure, the following assumptions are made:

1. The mobile robot moves forward at a constant speed, except when it approaches an obstacle.
2. The rotation of a mobile robot around a small axis.
3. The robot's wheels do not slip up, and the environment surface appears to be flat.

This study developed and implemented a practical autonomous differential drive mobile robot. We use the constructed robot to complete our path-tracking situations. A robot is polygonal

in shape and measures 24 cm in radius and 10 cm in height. Two separate continuous servo motors power the two active wheels. For instance, the DS04-NFS continuous DC motors are specified as follows: The speed is  $(0.22s/60at4.8V)$ , the torque is  $(5.5kg/cm at 4.8V)$ , the weight is 38 g, and the dimensions are 7.4 cm x 1.7 cm. Three batteries (1.5 V AA) are implemented to power the robot. Using two rotary shaft encoders, the left and right wheels' current speeds is computed by an AVR mega 2560 microcontroller. The microcontroller provides the proper PWM signals for controlling the value speed of the motors. In addition, the RP LIDAR sensor is mounted on the autonomous mobile robot's upper half. Numerous experiments are conducted to validate the suggested algorithm on a practical level. Moreover, regular shapes of obstacles are built in the laboratory environment. Experiments are categorized into additional situations based on the target's location and the number of obstacles present in the experimental setting. The robot begins at  $(0, 0)$ , and the viewpoint of inclination between the robot's beginning position and the target's initial location is zero, with a distance of  $L = 200$  cm between the robot's initial location and the target. The robot is surrounded by three obstacles, based on the RP LIDAR sensor's initial readings of 43 cm. The track of navigation of the mobile robot in the testing area is depicted in Fig. 10. The robot's task is separated into two sections in this illustration. The first half of the task is avoiding obstacles; the second portion is completing the mission. The witch of the Agnesi algorithm is used to accomplish the task. The experimental results in Fig. 11 demonstrate that when the suggested algorithm is used, The robot completes the task by taking the shortest path to reach the target and avoiding any obstacles in its path with a minimum time of 93.45 percent as compared with the Bezier algorithm gave 92.19 percent.

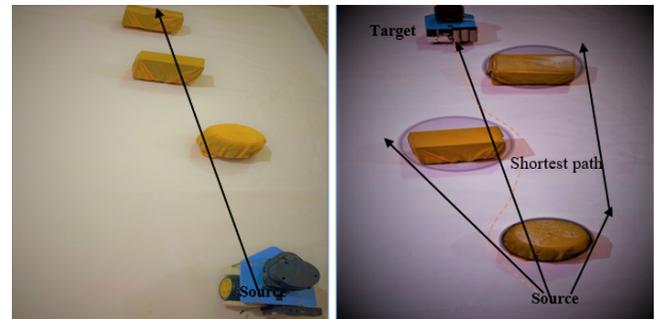


Fig. 10. The practical implementation of Witch of Agnesi algorithm for obstacle avoidance.

## VII. CONCLUSIONS

A new algorithm is proposed for avoiding obstacles by using the Agnesi curve approach for the navigation of the mobile

TABLE I.  
ACHIEVEMENT ON A SCENARIO WITH THE DIFFERENT NUMBER OF OBSTACLES FOR BOTH THE WITCH OF AGNESI AND THE BEZIER CURVE ALGORITHMS

Target position	Bezier algorithm			Witch of Agnesi algorithm		
	Shortest path	Average Time of arrival (s)	Average Power estimation (w)	Shortest path	Average Time of arrival (s)	Average Power estimation (w)
(-2.73, 3.77)	8.9147311	13.7	0.976065	7.163459	11.23	0.956829
(6.17, 4.06)	8.977723	24.18	0.556931	7.453456	21.63	0.516883
(5.97, 4.69)	9.235561	23.50	0.589504	7.894478	19.03	0.622266
(5.99, 3.92)	9.310771	33.41	0.418023	8.102398	27.67	0.439234

robot in an unknown environment. For the mobile robot, one RP LIDAR sensor with 360 degree is attached to detect the obstacle in the robot's path. This algorithm is designed to create a path consisting of many components, each of which has a circular trajectory to a void of obstacle collisions, and to give the mobile robot a secure and smooth path. By utilizing V rep software, different environment scenarios are analyzed and replicated. In all scenarios, to prevent any collision with obstacles, the Agnesi curve procedure is used to steer the mobile robot. The Bezier Algorithm is used for comparative purposes in the navigation of mobile robots. According to the simulation and experimental results, the suggested approach performs better in terms of time and average error than the Bezier curve algorithm while offering a shorter path to the destination. It's given 96.3 percent in terms of the average of the total path while the Bezier algorithm gave 94.67 percent while in practical results it's given 93.45 and the Bezier algorithm gave 92.19 percent. The suggested method is put to the test using a self-sufficient differential drive mobile robot. Five distinct investigational contexts are investigated. The acquired results demonstrate the validity and assessment of the Witch of Agnesi procedure for robot navigation in an unknown environment.

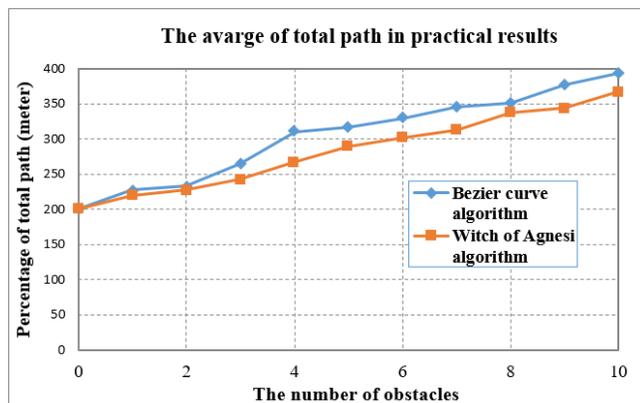


Fig. 11. The practical result of the Witch of Agnesi algorithm.

## CONFLICT OF INTEREST

The authors have declared no conflict of interest.

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