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Numerical study of the time-fractional partial differential equations by using quartic B-spline method

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ABSTRACT

This paper utilizes the quartic B-spline method for the numerical resolution of time-fractional partial differential equations. The fractional Caputo derivative is employed to depict anomalous diffusion processes influenced by memory effects. The proposed numerical method utilizes quartic B-spline functions for spatial discretization and employs a finite difference method to address the time-fractional derivative. Based on the Fourier method, its stability has been evaluated to demonstrate its efficacy in addressing fractional-order models. Numerical experiments, encompassing both linear and nonlinear scenarios, are performed to illustrate the method's effectiveness and accuracy. The results obtained are compared with exact solutions and alternative numerical methods, demonstrating improved performance in convergence and computational efficiency.

1. Introduction

Over the past two decades, fractional calculus has acquired significant relevance owing to its numerous applications in fields such as biomedical engineering, hydrology, probability theory, finance, and electrochemistry. Fractional calculus represents an advancement of traditional calculus.[1–6] The ability of non-integer-order derivatives to encapsulate memory or hereditary characteristics of phenomena is their principal advantage. Moreover, these classifications of derivatives and integrals exhibit a higher degree of freedom than their classical counterparts. Thus, fractional calculus can replicate a wide range of engineering and physical phenomena. Additionally, fractional differential equation models have been utilized across diverse domains, including epidemiological models,[7–15] economics,[16] electrical engineering,[17,18] automatic control,[19] diffusive transport,[20,21] plasma physics,[22] thermal modeling,[23–25] rheology,[26] electrochemistry,[27] networks,[28] calculus of variations,[10,29–31] among others.

In recent years, time-fractional partial differential equations (TFPDEs) have garnered considerable interest for their ability to model various physical and engineering systems that display anomalous diffusion and memory effects. Unlike classical partial differential equations, TFPDEs incorporate a fractional derivative in time, typically defined in the Caputo sense, which introduces nonlocality and a memory effect into the model. These properties improve the accuracy of fractional models in representing diverse phenomena. However, fractional diffusion wave equations have attracted considerable interest in physics, especially in electromagnetism. These equations are categorized as a form of fractional partial differential model.[32–34] In fact, analytical solutions for TFPDEs are generally challenging to obtain due to the complex nature of fractional derivatives. Therefore, numerical methods have become vital tools for solving these types of equations. Many approaches have been modified to be suitable for solving TFPDEs, including finite difference methods,[35] finite element methods,[36] B-spline method,[37–39] spectral methods,[40] Legendre wavelets,[41] and spline-based methods.[42,43]

However, the fractional diffusion wave equation is obtained from the second-order partial differential equation by replacing the term of the second time derivative with a real number ω where $\omega \in (1,2]$.[32] In this paper, we will concentrate on the time fractional diffusion wave equation.

$$\frac{\partial^{\omega} u(r,s)}{\partial s^{\omega}} + \eta \frac{\partial u(r,s)}{\partial s} + \lambda u(r,s) - \frac{\partial^2 u(r,s)}{\partial r^2} = G(u(r,s)) + f(r,s), \quad (r,s) \in \Omega \times (0,S]. \tag{1.1}$$

$$u(r,0) = \zeta_0(r), \quad \frac{\partial u(r,0)}{\partial s} = \zeta_1(r), \qquad r \in \Omega,$$

$$u(a,s) = \varsigma_0(s) \quad and \quad u(b,s) = \varsigma_1(s),$$

where $r \in [a,b]$ and $s \in [0,S]$. while, $f(r,s) : [a,b] \times [0,S] \longrightarrow \mathbb{R}$, $G(u(r,s)) : \mathbb{R} \longrightarrow \mathbb{R}$ where G is nonlinear with respect $u(r,s)$, $\varsigma_0(s), \varsigma_1(s) : [0,S] \longrightarrow \mathbb{R}$ $\zeta_0(r), \zeta_1(r) : [a,b] \longrightarrow \mathbb{R}$, In fact, all of the above functions must be given. We should point out that what we mean

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