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Simultaneous Approximation by Modification of Classical Szãsz Operators with Parameter *s*

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Abstract. This paper is approving the simultaneous approximation order by a modification family of the classical sequence of Szãsz operators. This modification depends on a real parameter $s > -1/2$. First, we show that this family (or its derivatives) converges to the function (or the corresponding derivatives) being approximated. Next, a Voronovski-type asymptotic formula for this family is given. The order of this approximation is improved from the order $O(n^{-1})$ (the order of the classical sequence) to the order $O\left(n^{-(2s+1)}\right)$ whenever $s > 0$. Finally, the study supports by some numerical examples and discusses the results obtained.

1. Introduction

In [2] Bernstein defined a sequence of positive linear operators as:

$$B_n(f;x) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right), \qquad x \in [0,\infty) \tag{1.1}$$

where $f \in C[0,1]$.
In [9] Szãsz generalized the Bernstein sequence to the unbounded interval $[0,\infty)$ as

$$S_n(f(t);x) = \sum_{k=0}^{\infty} q_{n,k}(x)\, f\left(\frac{k}{n}\right), \tag{1.2}$$

where $q_{n,k}(x) = \frac{(nx)^k}{k!e^{nx}}$, $x \in [0,\infty)$ and $f \in C_\alpha[0,\infty) = \{f \in C[0,\infty): |f(t)| \le Ae^{\alpha t} \text{for some } A, \alpha > 0\}$.

The sequence $S_n(f(t);x)$ is said to be the classical Szãsz sequence.
Several modifications of the Szãsz sequence were constructed and studied. All of them have the same order $O(n^{-1})$, here we refer to [1], [7], [10] and [11].
Some techniques got a better approximation order by the sequences of positive linear operators introduced and studied [6] and [3].

In [8], Pallini modified the Bernstein sequence by using the parameter s, where $s > -1/2$, as follows