The Operator $_r\Phi_s$ and the Polynomials K_n

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Abstract

Based on basic hypergeometric series, a new generalized q-operator $_r\Phi_s$ has been constructed and obtained some operator identities. Also, a new polynomial $K_n(a_1,\dots,a_r,b_1,\dots,b_s,c;a;q)$ is introduced. The generating function and its extension, Mehler's formula and its extension and the Rogers formula for the polynomials $K_n(a_1,\dots,a_r,b_1,\dots,b_s,c;a;q)$ have been achieved by using the operator $_r\Phi_s$. In fact, this work can be considered as a generalization of Liu work's by imposing some special values of the parameters in our results. Therefore the q^{-1}-Rogers-Szegö polynomials $h_n(a,b|q^{-1})$ can be deduced directly.

Keywords: q-operator, generating function, Mehler's formula, Rogers formula, the q^{-1}-Rogers-Szegö polynomials.

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1. Introduction

Throught this paper, the notations in [2] will be used here and assuming that $|q| < 1$.

Definition 1.1. [2] . *Let a be a complex variable. The q-shifted factorial is defined by*

$$(a;q)_0 = 1, \qquad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), \qquad (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

The compact notation for the multiple q-shifted factorial will be adopted here

$$(a_1,\dots,a_r;q)_n = (a_1;q)_n \dots (a_r;q)_n,$$

where n is an integer or ∞.

Definition 1.2. [2] . *The basic hypergeometric series $_r\phi_s$ is defined by*