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Addressing stability challenges in linear descriptor systems: A unified approach to robust control

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A B S T R A C T

This paper focuses on studying the robust stability of linear descriptor systems with perturbations in the state matrix under rank-deficient and unmatched conditions. In fact, there are three main steps in this study. The first is constructing an equivalent optimal control to design a robust control. The second step is to solve the Riccati equation to find the optimal control solution. The third step is to implement the cost function of the equivalent optimal control to define the candidate Lyapunov function in order to verify the stability conditions. The proposed control method is verified through simulations. Our results demonstrate that the proposed approach can guarantee robust stability under uncertain and perturbed conditions.

1. Introduction

Recently, control theory has played a fundamental role in life science [\[1–](#page-7-0)[3](#page-7-1)]; in particular, descriptor control systems are widely used in various fields, including control engineering, signal processing, and system modelling [\[4\]](#page-7-2). Meanwhile, stability theory is a powerful tool to investigate the behaviour of dynamical systems [\[5](#page-7-3)[,6\]](#page-7-4). In real-life problems, perturbations and randomness are common occurrences. Perturbations refer to small disturbances or changes that can affect a system or process. Randomness, on the other hand, refers to the inherent unpredictability or variability of certain phenomena. can be compromised under uncertain and perturbed conditions [[7](#page-7-5)[–10](#page-7-6)]. However, the state matrix perturbations with deficient rank and unmatched conditions can significantly affect the system's stability. Also, the role of initial values in determining the behaviour of the perturbations system can be significant, as small changes in the initial conditions can lead to large differences in the behaviour of the system over time $[11-15]$ $[11-15]$ $[11-15]$.

System behaviour through feedback loops is crucial in many behavioural disciplines, including those dealing with light and sound, nanotechnology and materials science, quantum technologies, scanning, and sensing. By employing closed-loop operation, it is possible to improve the system's stability, speed up transient operations, and reduce the impact of disturbances [[16\]](#page-7-9). The generic formula for the optimal control problem (OCP) consists of the performance index and the state space equations [[17](#page-7-10)[,18](#page-7-11)]. The analysis and synthesis of robustly stable systems under conditions of uncertainty are one of the most pressing issues in the fields of system and control theory [\[19](#page-7-12)[–27](#page-7-13)]. Preserving stability is often the most challenging aspect of control. The stability criteria for the equilibrium point of an ordinary differential equations are derived using the well-known Lyapunov direct approach. Also, previous research has investigated the stability of linear descriptor systems under various perturbations. For example, Yen et al. [[28\]](#page-7-14). Recent research has focused on the use of advanced control techniques, such as adaptive and robust control, to address the challenges posed by disturbances in input matrices. For example, the authors proposed a novel approach to adaptive control that enables a robot arm to maintain precision control despite disturbances in its input matrix Huang [[29–](#page-7-15)[31\]](#page-7-16). In another study, Zhao [\[32](#page-7-17)], has developed a robust control strategy to mitigate the impact of disturbances in an input matrix for a microgrid system. Furthermore, recent studies

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have explored the use of machine learning and artificial intelligence techniques to improve control performance in dynamic systems affected by disturbances. Also, a hybrid control strategy combining model predictive control and deep reinforcement learning to address the challenge of disturbance in input matrices for a wind turbine system has been developed in [[33\]](#page-7-18).

The novelty of this paper is that it offers several advantages over existing methods. First, it provides a unified framework for handling uncertain and perturbed systems with deficient rank and unmatched conditions. Second, it leverages the equivalence principle to derive efficient, systematic solutions that guarantee stability. Finally, it offers a practical and effective approach to designing robust controllers for real-world applications.

This paper is organized as follows: the fundamental concepts and problem formulation are given in Section [2.](#page-1-0) In Section [3,](#page-1-1) we will discuss the robust descriptor control problem (RDCP). Section [4](#page-2-0) proposes a new approach to construct an optimal control equivalent problem (OCEP). Finally, some concluding remarks are given in the last section.

2. Preliminaries and problem formulation

Consider the linear following descriptor systems:

$$
E\dot{z} = A(q)z + Bu, \quad \forall q \in [a, b], \quad \text{and} \quad z(t_0) = z_0,\tag{2.1}
$$

where $E, A(q) \in R^{m \times m}$, rank $\{E\} = m_1, 0 < m_1 \leq m$, $z \in R^m$, $u \in R^r$, and $B \in R^{m \times r}$. In fact, A(q) represent the uncertainty of the system and has the following form:

$$
A(q) = A(q_0) + (1 - B_1 B_1^T) \Delta A(q),
$$
 for some nominal $q_0 \in [a, b]$, (2.2)

and satisfy the following upper bound condition:

$$
\alpha^{-2}B_1^T\Delta A(q)^T\Delta A(q)B_1 \leq h,\tag{2.3}
$$

where $\alpha > 0$ is the design parameter.

Definition 2.1 ([[34\]](#page-7-19)). The pencil of the system [\(2.1](#page-1-2)) is regular if $|sE - A(q)| \neq 0$, for some $s \in C$

Lemma 2.1. *If the system* [\(2.1\)](#page-1-2) *is regular, then it is equivalent to the following system:*

$$
\dot{z}_1 = A_1(q)z_1 + A_2(q)z_2 + B_1u \tag{2.4}
$$

$$
0 = A_3(q)z_1 + A_4(q)z_2 + B_2u,\tag{2.5}
$$

where $z_1 \in R^{m_1}$ *, and* $z_2 \in R^{m_2}$ *.*

Assumption 2.1 ($[35]$ $[35]$). Assume there exist on open set \hbar , $\subset D$ such that for all

 $z_1 \in \hbar_z$ it is possible to solve $A_3(q)z_1 + A_4(q)z_2 = 0$ for z_2 , we define the corresponding solution manifold as

$$
\hbar = \{ z_1 \in \hbar_z, z_2 \in R^{m-m_1}, \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} \in \aleph \left(A_3(q) A_4(q) \right), t \ge 0 \}
$$
\n(2.6)

also consider the manifold $M \subseteq R^m$ determined by $M = \aleph \left[A_3(q) A_4(q) \right]$, where $\aleph(.)$ denotes the kernel.

Lemma 2.2 ([\[36\]](#page-7-21)). If $A_4(q)$ is of the rank deficient matrix i.e, rank $A_4(q) < (m-m_1)$, then there exist a matrix L of dimension $(m-m_1)\times m_1$ *such that the system* [\(2.4\)](#page-1-3)*–*([2.5\)](#page-1-4) *will be in the reduced form.*

Definition 2.2 ([\[37](#page-7-22)]). For any nonlinear system $\frac{dx}{dt} = f(x(t))$, $x \in \mathbb{R}^n$, the equilibrium x_e is stable if, and only if, for any $\epsilon > 0$ there exists a $\delta(t, \epsilon)$ such that $||x(0) - x_e|| < \delta$, then for any $t > 0$ we have $||x(t) - x_e|| < \epsilon$. Moreover, x_e is asymptotically stable: if it is stable and it is locally attractive.

Definition 2.3 ([\[37](#page-7-22)]). For any nonlinear system $\frac{dx}{dt} = f(x(t))$, $x \in \mathbb{R}^n$, the equilibrium x_e is globally asymptotically stable if, and only if, there exist a real value function $V(x) : R^n \rightarrow R$ satisfy the following:

- $V(x_e) = 0$
- $V(x) > 0, \forall x \in R^n \{x_e\}$
- $\dot{V}(x) < 0, \forall x \in R^n$
- $\dot{V}(x) < 0$ does not equal to zero along any $x \in \mathbb{R}^n$, other than the trivial solution $x(t) = 0$

3. Robust descriptor control problem

The aim of this section is to find a feedback control $u = -Kz$ such that the closed-loop system of

$$
\dot{z}_1 = (A_1(q) + A_2(q)L)z_1 + B_1u
$$

u, (3.1)

$$
z_2 = Lz_1,\tag{3.2}
$$

is asymptotically stable.

Now, if we substitute (3.2) (3.2) in (3.1) (3.1) (3.1) , we have

$$
\dot{z}_1 = (A_1(q) + A_2(q)L - B_1K)z_1
$$
\n(3.3)

The system (3.1) can be written in an equivalent form

$$
\dot{z}_1 = (A_1(q_0) + A_2(q_0)L)z_1 + B_1u + \alpha(1 - B_1B_1^T)v + (1 - B_1B_1^T)\Delta A(q)
$$
\n(3.4)

Finding an appropriate matrix K is necessary to ensure that the feedback is linear and dynamical Eq. ([3.3](#page-2-2)). Our approach is to translate Eq. [\(3.3](#page-2-2)) into the following problem.

4. Optimal control equivalent problem

The OCP becomes the following linear quadratic regulator problem (LQR). For the nominal system

$$
\dot{z}_1 = (A_1(q_0) + A_2(q_0)L)z_1 + B_1u + \alpha(1 - B_1B_1^T)v
$$
\n(4.1)

Find a feedback control low $u = -Kz$, $v = -lz$ that minimizes the following cost function where $H = hI$,

$$
J(u(.)) = \int_{o}^{\infty} (\rho^2 z_1^T H z_1 + u^T R u + \rho^2 v^T W v + \beta^2 z_1^T Q_1 z_1) dt
$$

The necessary condition for optimality for the equivalent OCP. when, the necessary condition for the existence of optimal control (OC) is that $J(z_1)$ must satisfy the Hamilton–Jacobi–Bellman (HJB) equation [\[38](#page-7-23)].

Theorem 4.1. *The solution of Robust Control Problem (RCP) Eq.* [\(2.1](#page-1-2)) *is the solution OCP if one can choose parameter design* α , ρ and β such that the solution to the OCP denoted by $(u(z), v(z))$, exists and the condition is satisfied $\rho^2 ||v||_w^2 \leq \beta^2 ||z_1||_{Q_1}^2$, for all $z_1 \in h_z$ and *for some* $\hat{\beta}$ *such that* $\|\hat{\beta}\| \le \|\beta\|$ *. Then u-component of the solution to the OCP is the solution of the RCP Eq.* [\(2.1\)](#page-1-2)*.*

Proof. The OCP finds the minimum of the cost function. Define

$$
J(z_1) = \int_0^\infty (\rho^2 z_1^T H z_1 + u^T R u + \rho^2 v^T W v + \rho^2 z_1^T Q_1 z_1) dt
$$
\n(4.2)

Which is subject to the system (4.1) (4.1) , since $J(z_1)$ satisfied HJB then:

$$
0 = \min_{u,v \in \triangle} (\rho^2 z_1^T H z_1 + u^T R u + \rho^2 v^T W v + \beta^2 z_1^T Q_1 z_1) + J^T(z_1) [(A_1(q_0) + A_2(q_0)L)z_1 + B_1 u + \alpha(1 - B_1 B_1^T) v]
$$
\n(4.3)

Where, $J_{z_1} = \partial J / \partial z_1$

∞

Therefore, if $u = -Kz$, $v = -iz$ (with $K^T K$, $i^T l$ are negative definite) are the solution of the OCP then

$$
0 = (\rho^2 z_1^T H z_1 + u^T R u + \rho^2 v^T W v + \beta^2 z_1^T Q_1 z_1) + J(z_1)^T [(A_1(q_0)
$$

$$
+ A_2(q_0)L)z_1 + B_1u + \alpha(1 - B_1B_1^T)v
$$
\n(4.4)

$$
\partial J/\partial u = 2u^T R + J(z_1)^T B_1, \ \partial J/\partial u = 0 \tag{4.5}
$$

$$
\partial J/\partial v = 2\rho^2 v^T W + J(z_1)^T \alpha (1 - B_1 B_1^T), \ \partial J/\partial v = 0 \tag{4.6}
$$

The goal of the OCP is to steer z_1 to 0 in Eq. [\(3.4\)](#page-2-4) is globally asymptotically stable for all admissible uncertain to do so, we show that $J(z_1)$ is a Lyapunov function of the system ([3.4](#page-2-4)). Since $u(0) = 0$, $v(0) = 0$. and $z_1^TQ_1z_1 > 0$, $u^TRu > 0$, $v^TWv > 0$ $\forall z = (z_1, Lz_1) \neq (0, 0)$, then $J(z_1) > 0$, $\forall z_1 \neq 0$

$$
\begin{aligned} \dot{J}(z_1) &= J_{z_1}^T \dot{z}_1 \\ &= J_{z_1}^T [(A_1(q_0) + A_2(q_0)L)z_1 + B_1 u + (1 - B_1 B_1^T) \Delta A(q) z_1] \\ &= J_{z_1}^T [(A_1(q_0) + A_2(q_0)L)z_1 + B_1 u + \alpha (1 - B_1 B_1^T) v - \alpha (1 - B_1 B_1^T) v \\ &\quad + (1 - B_1 B_1^T) \Delta A(q) z_1] \end{aligned}
$$

Substitution Eqs. (4.4) , (4.5) and (4.6) (4.6) (4.6) , yields Then

$$
= -\rho^2 z_1^T H z_1 - \beta^2 z_1^T Q_1 z_1 - u^T u + \rho^2 v^T W v - 2\alpha^{-1} \rho^2 v^T W \Delta A(q) B_1 z_1
$$

Then

$$
\begin{split} \dot{J}(z_1) &= -\rho^2 z_1^T H z_1 - \beta^2 \|z_1\|_{Q_1}^2 - \|u\|_R^2 + \rho^2 \|v\|_w^2 + \rho^2 \|v\|_w^2 \\ &\quad + \alpha^{-2} \rho^2 \| \Delta A(q) B_1 z_1\|_w^2 \end{split}
$$

̇(¹

$$
= (-\rho^2 z_1^T H z_1 - \alpha^{-2} ||\Delta A(q) B_1 z_1||_w^2) - ||u||_R^2 + 2\rho^2 ||v||_w^2
$$

$$
- \beta^2 ||z_1||_{Q_1}^2
$$

$$
f(z_1) \le ||z_1||_{Q_1}^2 < 0
$$

5. Illustrated example and dissections

In this section we serve as a bridge between theoretical insights and real-world applications. They enhance understanding, facilitate analysis, and promote active learning.

Example 5.1. Consider the robust descriptor system

$$
E\dot{z} = A(q)z + Bu, \quad \forall q \in [-6, 6]
$$

where

$$
z = \begin{pmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_3(t) \end{pmatrix}, E = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, A(q) = \begin{pmatrix} 2+2q & 1 & 2q & 0 \\ 1 & 0 & 1 & 0 \\ q & 0 & q & q \\ q & 0 & q & q \end{pmatrix}, and B = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
$$

Now, applying [Lemma](#page-1-6) [2.1](#page-1-6) for the system above, we have

$$
\dot{z}_1 = A_1(q)z_1 + A_2(q)z_2 + B_1u,
$$

$$
0 = A_3(q)z_1 + A_4(q)z_2.
$$

By using [Assumption](#page-1-7) [2.1,](#page-1-7) and [Lemma](#page-1-8) [2.2,](#page-1-8) we find a nonsingular matrix L such that $z_2 = Lz_1$ with $A_3(q) + A_4(q)L = 0$. Here, we find $L =$ $\begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}$. So, the reduced system is

$$
\dot{y}_1 = \begin{pmatrix} 1/2 + q & 1 + q \\ 1 & 1 \end{pmatrix} y_1 + B_1 u
$$

Let $\rho = 1$, $\beta = 1$, $\alpha = 0.04$ then the corresponding OCP and let $q_0 = 0$ be the nominal uncertainties in the [−6, 6] then we get from Eq. ([4.1](#page-2-3))

$$
\dot{y_1} = \begin{pmatrix} 1/2 & 1 \\ 1 & 1 \end{pmatrix} y_1 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 0.04 & 0 \\ 0 & 0 \end{pmatrix} v
$$

Find the FBC Low $u = -kz$ and $v = -lz$ that minimize the cost function

$$
\int_0^\infty (Y_1^T \left[\begin{pmatrix} 22500 & 0 \\ 0 & 22500 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] Y_1 + u^T u + v^T v) dt
$$

Solve Riccati equation in M atlab we get $S =$ $\begin{pmatrix} 4.0203e + 03 & 26.7210 \\ 26.7210 & 151.1809 \end{pmatrix}$ and

u = −(26.7210 151.1809)y, v = −(160.8128 1.0688) y Stability is being checked by using the eigenvalues of the coefficient matrix (−6.1034–150.0100) Therefore, the system is stable.

Now, we draw some dissection in this example. Here, the uncertainty range is [−6*,* 6] and the solution behaviour is illustrated in [Figs.](#page-5-0) [4](#page-5-0), and [5](#page-6-0) which are not asymptotically stable because of the small change in the initial values, while [Figs.](#page-4-0) [1,](#page-4-0) [2](#page-4-1) and [3](#page-5-1) show the robust and optimal solution with consistent initial conditions where the uncertain parameters are in the given range [−6*,* 6]. [Fig.](#page-6-1) [6](#page-6-1) represents the non-smoothness and unconvergence of the robust control within the class of consistent initial conditions and uncertain parameters when the uncertain parameters are not in the given range [−6*,* 6].

6. Conclusions

This paper introduces a mathematical analysis, including the theoretical framework, design methodology, and implementation procedures. In this line, we design robust controllers for linear descriptor systems under state matrix perturbations with deficient rank and unmatched conditions. Indeed, the approach offers several advantages over existing methods, including a unified framework for handling uncertain and perturbed systems, leveraging the equivalence principle to derive efficient and systematic solutions, and providing a practical and effective approach for real-world applications. Additionally, extensive simulation results have been provided to validate the effectiveness of the approach. The findings obtained have helped to understand the function of consistent requirements in maintaining the stability of the system. Even when the eigenvalues in the reality portion are negative, the system is not asymptotically stable because of the small change in the initial values.

Fig. 1. Robust solution asymptotically stable q=1 ∈ [−6*,* 6] $(y_{1,0}, y_{2,0}, y_{3,0}, y_{4,0}) = (1, 2, 1, -3) \in P_k.$

Fig. 2. Robust solution asymptotically stable q=-6 ∈ [−6*,* 6] $(y_{1,0}, y_{2,0}, y_{3,0}, y_{4,0}) = (1, 2, 1, -3) \in P_k.$

Declaration of competing interest

We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

Data availability

No data was used for the research described in the article.

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Fig. 3. Robust solution asymptotically stable q=6 ∈ [−6*,* 6] $(y_{1,0}, y_{2,0}, y_{3,0}, y_{4,0}) = (1, 2, 1, -3) \in P_k.$

Fig. 4. solution not asymptotically stable q=6 ∈ [−6*,* 6] $(y_{1,0}, y_{2,0}, y_{3,0}, y_{4,0}) = (3, 2, 1, -3) \notin P_k.$

Fig. 5. Robust solution not asymptotically stable q=6 ∈ [−6*,* 6] $(y_{1,0}, y_{2,0}, y_{3,0}, y_{4,0}) = (1, 2, 0, -4) \notin P_k.$

Fig. 6. Robust control(unstable) q=10 ∉ [−6*,* 6] $(y_{1,0}, y_{2,0}y_{3,0}, y_{4,0}) = (1, 2, -1, -3) \in P_k.$

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