

Two-phase analysis of heat transfer of nanofluid flow in a wavy channel heat exchanger: A numerical approach

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ABSTRACT

The heat transfer improvement by using CuO /water nanofluid (NF) in a wavy channel is evaluated numerically using the turbulent two-phase mixture and the $k-\varepsilon$ models. The numerical work is a 2-dimensional model created and analyzed in Gambit and Ansys Fluent software, respectively. The flow Reynolds (Re) numbers of 8000 – 40,000, wavelength ranging from 0 – 0.4 m, and solid volume fraction (SVF) of 0 % to 4 % are investigated. In all cases, a constant heat flux of $q''=5000 \text{ W/m}^2$ is applied on the outer surface of the heat exchanger. The heat transfer and fluid flow were analyzed by the flow visualization method and heat transfer evaluation indexes. The results show that by increasing the Re number, the vortices increase and more turbulence are generated in the vicinity of the waves near the channel inlet. As the amplitude of the channel waves increased, the velocity at the top of the wave increased, and the resulting pressure gradient behind the wave is intensified and the reverse flow is generated. The improving effect of using NF is more prominent where the effect of other enhancing factors is weak. For wall amplitude of 0.1 m, by increasing the SVF from 1 – 4 % the average Nu number (Nu_{avg}) increased by 35 % and 22 % in $Re = 8,000$ and 40,000, respectively.

1. Introduction

Industrial development relies heavily on the application of heat exchangers, and considering their large application in industries the study of their heat transfer improvement is a live subject. The heat transfer improvement could be made via both active and/or passive ways; Despite the problems of active methods, by their large benefits the passive techniques are the most popular and have drawn large attention through the years [1]; increasing the contact surface using the fins [2–4], employing the baffles [5–7] or vortex generators [8–13], inside the fluid flow channel and application of metallic porous foams are some of the most popular techniques. The application of porous materials extends the heat transfer area and increases the heat transfer rate [14]. The large area per unit volume of porous materials (such as metal foams)

has made them a good candidate for heat transfer enhancement. The application of porous materials beside the phase change materials ($PCMs$) makes use of both the heat storage and heat transfer potential simultaneously [15]. Among passive techniques, methods based on the deformation of the channel wall draw large attention; using the waveforms and corrugation on the wall for heat transfer improvement has been considered by many researchers [16–18]. The heat transfer enhancement resulting from the wall deformation is due to the increased mixing, the vortices, and rotational motion. When fluid flows through a corrugated channel, the flow is disturbed due to the growth of rotational areas close to the corrugated wall, which finally improves heat transfer [19]. Corrugated walls improve heat transfer by eliminating or reducing the thickness of the thermal boundary layer and increasing the flow mixing [20]. The waveform of the wall would enhance the transverse flows in the channel and therefore increase the heat transfer rate [21]. It

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Nomenclature			
<i>Abbreviation</i>		u, v	velocity components (m/s)
<i>LHS</i>	latent heat storage	U, V	velocity (m/s)
<i>NF</i>	nanofluid	w	width (m)
<i>NP</i>	nanoparticle	x	channel length (m)
<i>PCM</i>	phase change material	Y	channel height (m)
<i>PT</i>	plain tube	<i>Greek symbols</i>	
<i>QUICK</i>	quadratic upstream interpolation for convective kinetics differencing scheme	α	wave amplitude
<i>SIMPLE</i>	semi-implicit method for pressure-linked equations	Δ	difference
<i>SVF</i>	solid volume fraction (%)	ε	turbulent dissipated energy
<i>Symbols</i>		φ	nanofluid concentration (%)
a	amplitude (m)	κ	turbulent kinetic energy
c_p	specific heat (kJ/kg.K)	λ	wavelength
<i>CuO</i>	copper oxide	μ	dynamain viscosity (Pa.s)
<i>Da</i>	Darcy number	ρ	density (kg/m ³)
f	friction factor	τ	tension (Pa)
g	gravity acceleration (m/s ²)	<i>Subscripts</i>	
G	mass flow rate (kg/s)	<i>avg</i>	average value
h	local convective heat transfer coefficient (W/m ² . K)	b	bulk
J	heat transfer efficiency	f	base fluid
k	thermal conductivity (W/m.K)	e	entrance region
L	length (m)	<i>eff</i>	effective
Nu	Nusselt number	h	hydraulic
Pr	Prantdl number	i, in	inlet flow
q''	heat flux (W/m ²)	m	mixture
Ri	Richardson number	nf	nanofluid
Re	Reynolds number	p	particle
S	surface function (m)	pf	particle-base fluid
T	temperature (K)	s	smooth
		t	turbulence
		w	wall

should be noted that the heat transfer improvement is alongside the pressure drop increment. Numerical solvers are commonly used to calculate the pressure drop and heat transfer rate in corrugated channels [22–25]. Sui et al. [26] investigated the rate of heat transfer in a three-dimensional sinusoidal channel with a rectangular cross-section and water as the working fluid in a laminar flow regime. The results show that by passing the fluid through the curves in the channel, a symmetrical steady flow or Dean vortices is generated. Ferley and Ormiston [27] compared heat transfer in channels with sinusoidal walls in elliptical and v-shaped cross sections in the laminar flow regime. The results demonstrated that in terms of friction factor, heat transfer rate and Nu_{avg} , the sinusoidal wall channel performs better. Considering that increasing the heat transfer rate on one hand and decreasing the rate of pressure drop on the other hand in corrugated ducts are two desirable, albeit opposite goals, efforts were made to optimize the wall shape of these ducts [28]. As an important characteristic of internal flows, the entrance length takes effect from the wavy region. Mehta et al. [29] analyzed the hydrothermal properties of the laminar forced flow of water inside the wavy channel with different types of constant, increasing and decreasing amplitudes at different Re numbers and amplitude slopes (A). They found the independency of Nu number from the channel geometry at low Re and A numbers. As the other heat transfer improving factor, nanofluids (NFs) found wide application in heat exchangers [30] and solar systems [31]. Several reviews demonstrate the aspects of convective heat transfer improvement made by using NFs in different geometries [32,33]. Compared to other improving methods, NFs could be used as an inexpensive method to enlarge the heat transfer area and improve heat transfer. The NF flow as a two-phase medium could be modeled using two different approaches: single-phase and two-phase. The two-phase models were studied and their aspects for

modeling of NF flow have been investigated in different studies [34,35]. The two-phase mixture has predicted successfully the performance of NFs with solid volume fraction (SVF) < 10 % with the highest accuracy [19]. Hejazian [36] demonstrates that for the simulation of turbulent forced convection NF flow with SVF < 1 %, the results of two-phase modeling are more accurate than that of single-phase models. Dormhommadi et al. [37] built a numerical model to compute the entropy generation rate in a sinusoidal wavy-wall channel with the NF of Cu -water as the heat transfer fluid. The effects of SVF , Richardson (Ri) number, wave amplitude number and wavelength were studied. The showed optimal wave amplitude ratio is, for wavelengths 1 and 2. Nazari et al. [37] studied the heat transfer of water- CuO NF in a channel with a porous media in different Re and Darcy (Da) numbers. The results demonstrated the heat transfer improving effect of employing porous medium and NF . Kumar et al. [38] studied the thermo-flow behavior of non-Newtonian NF which flows inside the trapezoidal wavy channel. They reported the Nu number, pressure drop, and performance factor for different SVF values, Pr and Re numbers. They reported the growth of Nu number and pressure drop and reduction of performance factor by increasing the SVF , Re and Pr numbers. To improve thermal management in electronic systems, Mahsa Hajjalibabaei et al. [39] designed a novel wavy channel heat sink. To achieve better flow mixing and uniform temperature distribution, their design employed secondary branches with gradually decreasing spacing and height. They investigated the effect of flow rates on heat sink temperature and found that a 90-degree branch angle with a 1 mm width results to 5.27 % temperature reduction compared to the basic wavy channel heat sinks. To the best of the authors' knowledge and through the literature review, it was found that the two-phase turbulent flow of water/ CuO inside the wavy channel has not been investigated yet. In the present study, the turbulent

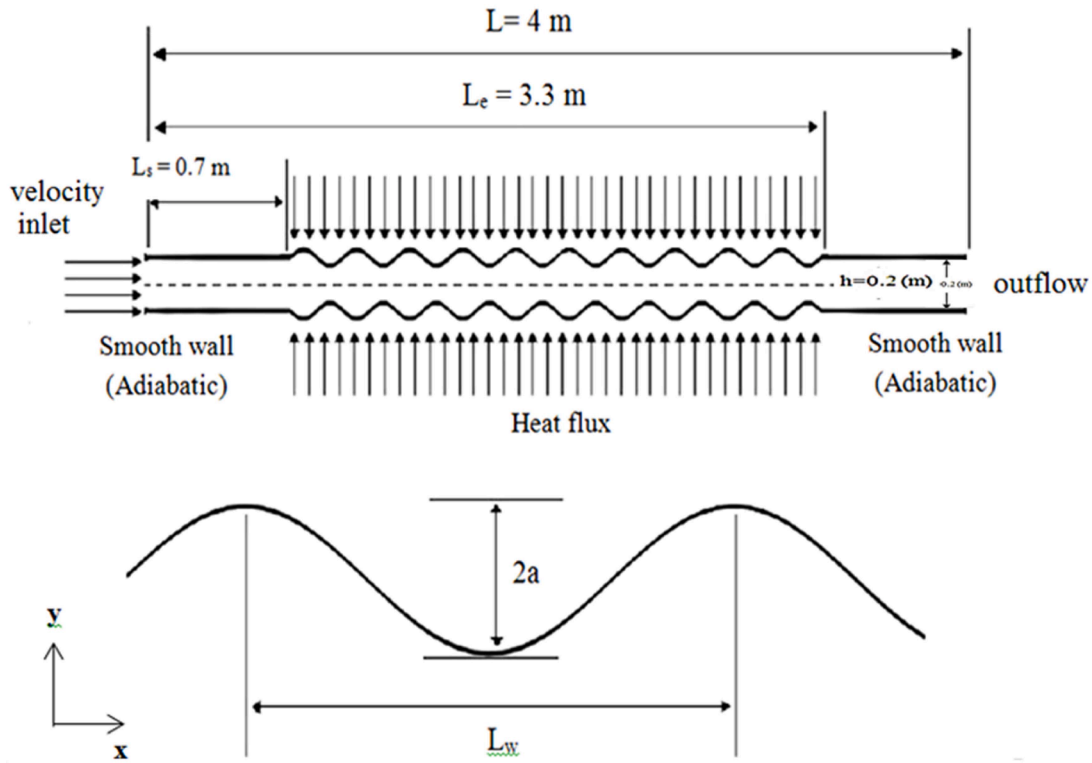


Fig. 1. The geometry of the problem, the boundary conditions and dimensions.

Table 1
Thermophysical properties of the base fluid and CuO nanoparticles [40].

	water	CuO
ρ (kg/m ³)	998	6350
C_p (J/kg.K)	4182	535.6
k (W/m.K)	0.597	69
μ (kg/m.s)	0.000993	-
Nanoparticle size (nm)	-	100

Table 2
Thermophysical properties of NF at different SVFs.

SVF	ρ_{nf} (kg/m ³)	C_{nf} (J/kg.K)	k_{nf} (W/m.K)	μ_{nf} (kg/m.s)
%1	1053.2	3956.87	0.62975	0.00108856
%2	1108.3	3753.74	0.646807	0.0011987856
%3	1163.254	3570.3	0.66418088	0.0013336891
%4	1218.272	3404.24	0.6818802	0.001492664

flow and heat transfer of the CuO-water NF in a corrugated channel is investigated. The outer wall of the channel is subjected to constant heat flux and the fluid flow is fully developed. The sensitivity analysis is made to reveal the effect of Re number and SVF on heat transfer of fluid flow. To model the fluid flow, the turbulent two-phase mixture and the $k-\epsilon$ models are considered. In the next section, the problem and the governing equations are described. After explaining the numerical procedures, the results are presented by discussing the diagrams and finally, the conclusions are drawn.

2. Problem description

Fig. 1 shows the geometry of the studied problem; also, the details of the wall can be seen. The fluid flow is between two corrugated sinusoidal plates with a total length of L and an inner width of h between the two

plates. The form of wall corrugation is according to Eq. (1). To achieve the fully developed condition, a smooth adiabatic length (L_s) is considered at the inlet of the channel; also, the outer wall is subjected to a fixed heat flux of 5000 W/m².

$$S = \pm \frac{w}{2} \pm \text{asin}\left[\frac{2\pi(x-L_s)}{L_w}\right] L_s \leq x \leq L_e \quad (1)$$

The base fluid and nanoparticle (NP) are water and CuO, respectively whose properties are listed in Table 1 [40]. The fluid flow is assumed incompressible, two dimensional, two-phase and axisymmetric. The previous studies reveal that the NF could exhibit both Newtonian and/or non-Newtonian behavior; however, in this work, the NF flow is considered Newtonian. The Re number is between 8000 and 40,000 and the fluid flow is turbulent. The SVF is 0%, 1%, 2%, 3% and 4% and the wall wavelengths are 0, 0.1, 0.2, 0.3 and 0.4 m. The length of the annulus is 4 m with its corrugated part of 2.5 m; also, the inlet temperature is 293 K.

2.1. Governing equations

The NF flow is considered steady and incompressible and the effect of viscous dissipation is neglected. The flow behaves Newtonian and its properties are assumed to be independent of the temperature. In addition, the relative velocity between the nanoparticles and base fluid is ignored. The equations of continuity, momentum and energy for the two-phase mixture model can be expressed as Eqs (2) – (5) [41]. In the following equations, the subscripts “m”, “f” and “p” stand for the mixture, base fluid and nanoparticles, respectively; also ϕ_k is the SVF of phase k .

$$\vec{\nabla} \cdot (\rho_m \cdot \vec{V}_m) = 0 \quad (2)$$

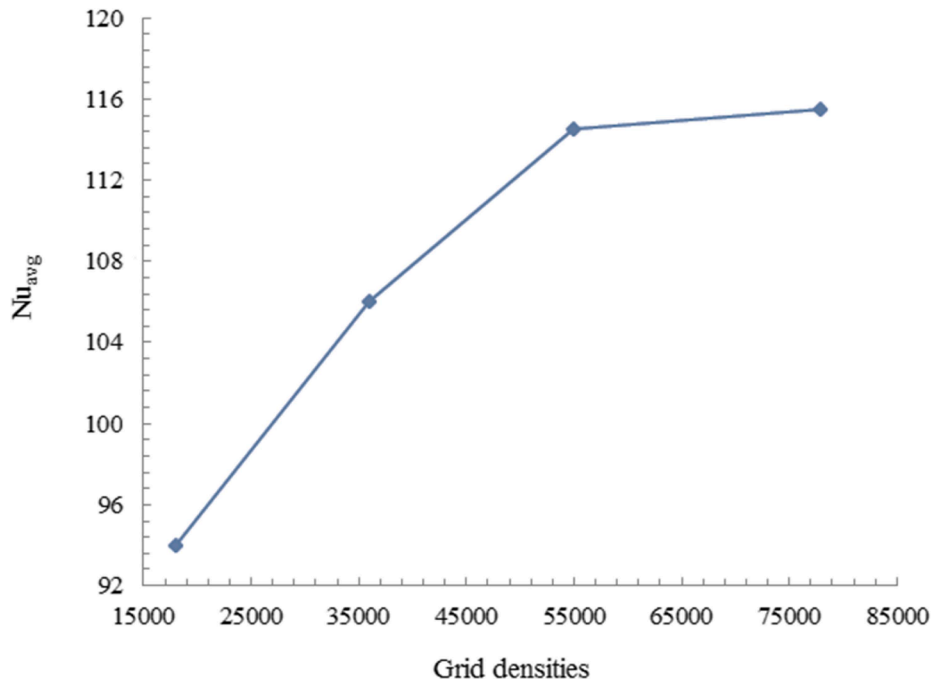


Fig. 2. The variation of Nu_{ave} for different grid densities.

Table 3
The Nu_{ave} for different grid sizes and the percent change at each step.

Mesh size	Nu_{avg}	Percent change
60 × 300	94	–
80 × 450	106	12.80%
550 × 100	114.5	8.00%
650 × 120	115.5	0.87%

$$\vec{\nabla} \cdot (\rho_m \vec{V}_m \vec{V}_m) = -\vec{\nabla} P_m + \vec{\nabla} \cdot ((\mu_m + \mu_{t,m}) \nabla \cdot \vec{V}_m) + \vec{\nabla} \cdot \left[\sum_k \varphi_k \rho_k \vec{V}_{dr,k} \vec{V}_{dr,k} \right] \quad (3)$$

$$\vec{\nabla} \cdot \left[\sum_k C_{pk} \varphi_k \vec{V}_k T \right] = \vec{\nabla} \cdot (k_{eff} \vec{\nabla} T) \quad (4)$$

$$\vec{\nabla} \cdot (\varphi_p \rho_p \vec{V}_m) = \vec{\nabla} \cdot (\varphi_p \rho_p \vec{V}_{dr,p}) \quad (5)$$

The mixture properties including density, viscosity, specific heat, thermal conductivity and mass average and drift velocities are defined in Eqs. (6) – (11) [41],

$$\rho_m = \sum_k \varphi_k \rho_k = (1 - \varphi) \rho_f + \varphi \rho_p \quad (6)$$

$$\mu_m = \sum_k \varphi_k \mu_k = (1 - \varphi) \mu_f + \varphi \mu_p \quad (7)$$

$$(\rho C_p)_m = (1 - \varphi) (\rho C_p)_f + \varphi (\rho C_p)_p \quad (8)$$

$$k_{eff} = k_f \left[\frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} \right] \quad (9)$$

$$\vec{V}_m = \frac{\sum_{k=1}^n \varphi_k \rho_k \vec{V}_k}{\rho_m} = \sum_{k=1}^n x_k \vec{V}_k \quad (10)$$

$$\vec{V}_{dr,k} = \vec{V}_{pf} - \vec{V}_m \quad (11)$$

In Eq. (10), x_k is determined using Eq. (12);

$$x_k = \frac{\varphi_k \rho_k}{\rho_m} \quad (12)$$

From Eqs (6) – (9) and the base and NP properties as given in Table 1, the NF properties in different SVFs could be determined which are listed in Table 2.

The relative velocity between the base fluid and nanoparticle is given

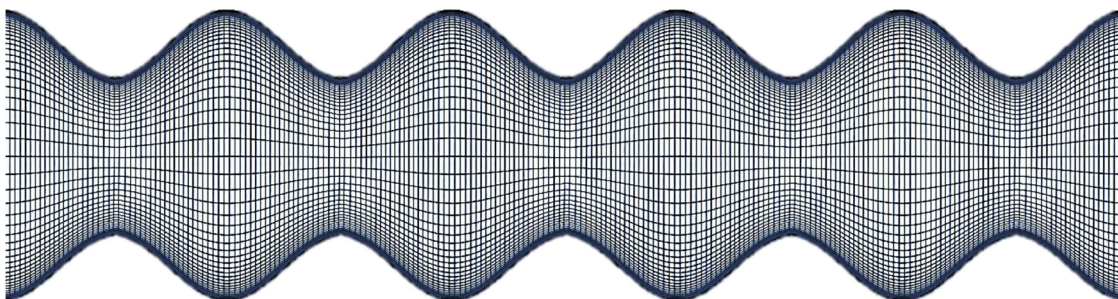


Fig. 3. A view of the employed mesh.

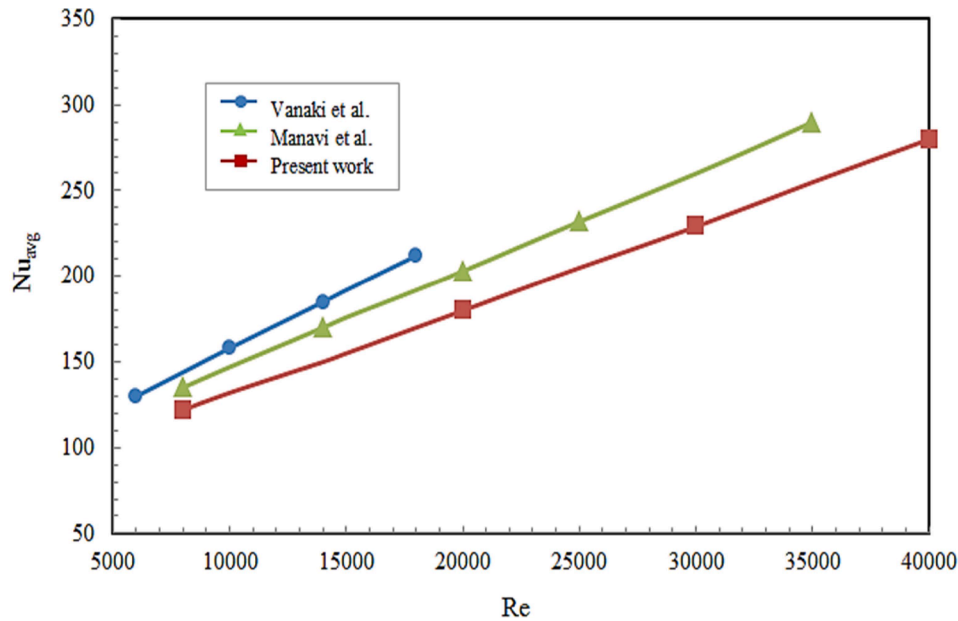


Fig. 4. The comparison of Nu_{avg} with that of Vanaki et al. [23] and Manavi et al. [44].

by Eq. (13) [42].

$$\vec{V}_{pf} = \vec{V}_p - \vec{V}_f = \frac{\rho_p d_p^2}{18\mu_f f_{drag}} \left(\frac{\rho_p - \rho_m}{\rho_p} \right) (g - (V_m \cdot \nabla) V_m) \quad (13)$$

The drag coefficient (f_{drag}) is specified by Eq. (14),

$$f_{drag} = \begin{cases} 1 + 0.15Re_p^{0.687} & Re_p \leq 1000 \\ 0.0183 & Re_p > 1000 \end{cases} \quad (14)$$

In this study, the κ - ϵ turbulent model is used in which the rate of turbulent dissipation (ϵ) and turbulent kinetic energy (κ) equations are expressed as Eqs. (15) and (16), respectively [43];

$$\nabla \cdot (\rho_m \vec{V}_m \kappa) = \text{div} \left(\frac{\mu_{t,m}}{\sigma_\kappa} \nabla \kappa \right) + G_{\kappa,m} - \rho_m \epsilon \quad (15)$$

$$\nabla \cdot (\rho_m \vec{V}_m \epsilon) = \nabla \cdot \left\{ \frac{\mu_{t,m}}{\sigma_\epsilon} \nabla \epsilon \right\} + C_1 \left(\frac{\epsilon}{\kappa} \right) G_{\kappa,m} - C_2 \rho_m \left(\frac{\epsilon^2}{\kappa} \right) \quad (16)$$

where \vec{V}_m is the time-averaged velocity of the fluid. The eddy viscosity $\mu_{t,m}$ and the rate of turbulent kinetic energy generation $G_{\kappa,m}$ were modeled by Eqs (17) and (18), respectively;

$$\mu_{t,m} = \frac{\rho C_\mu \kappa^2}{\epsilon} \quad (17)$$

$$G_{\kappa,m} = \mu_{t,m} (\nabla V_m + (\nabla V_m)^T) \quad (18)$$

In the above equations, C_1 , C_2 , C_μ , σ_κ and σ_ϵ are empirical constants whose values are given below [43];

$$C_1 = 1.44, C_2 = 1.92, C_\mu = 0.09, \sigma_\kappa = 1.0, \sigma_\epsilon = 1.3 \quad (19)$$

2.2. Boundary conditions

The governing equations are solved in conjunction with the following boundary conditions:

- On the wall boundaries, the velocity condition of no slip and the thermal condition of constant heat flux are exerted;

$$y = mx + a \sin(x/y) \Rightarrow q''(x, y) = q''_w, u = v = 0 \quad (20)$$

- The velocity and temperature at the inlet are known;

$$x = 0 \Rightarrow T = T_{in}, u = u_{in} \quad (21)$$

- At the outlet of the channel the fully developed conditions are applied;

$$x = L \Rightarrow \frac{\partial T}{\partial x} = 0, \frac{\partial u}{\partial x} = 0 \quad (22)$$

- The outflow boundary condition is exerted at the exit section.

2.3. Heat transfer and fluid flow evaluation indexes

To characterize the heat transfer rate from the inner wall to the flowing NF , the local convective heat transfer coefficient (h) is expressed as Eq. (23) [40].

$$h(x) = \frac{q''}{T_{w,x} - T_{b,x}} \quad (23)$$

In Eq. (23), $T_{w,x}$ and $T_{b,x}$ are the wall and fluid bulk temperatures, respectively. The average heat transfer coefficient (\bar{h}) is determined by integration of the local heat transfer coefficient along the heated length L as given by Eq. (24) [40]:

$$\bar{h} = \int_0^L h(x) dx \quad (24)$$

Having the average heat transfer coefficient, the Nu_{avg} over the channel surface is determined based on the hydraulic diameter (D_h) and the k_{eff} , according to Eq. (25) [40],

$$Nu_{ave} = \frac{\bar{h} D_h}{k_{eff}} \quad (25)$$

In Eq. (25), D_h is the hydraulic diameter of the conduit whose value is given by Eq. (26).

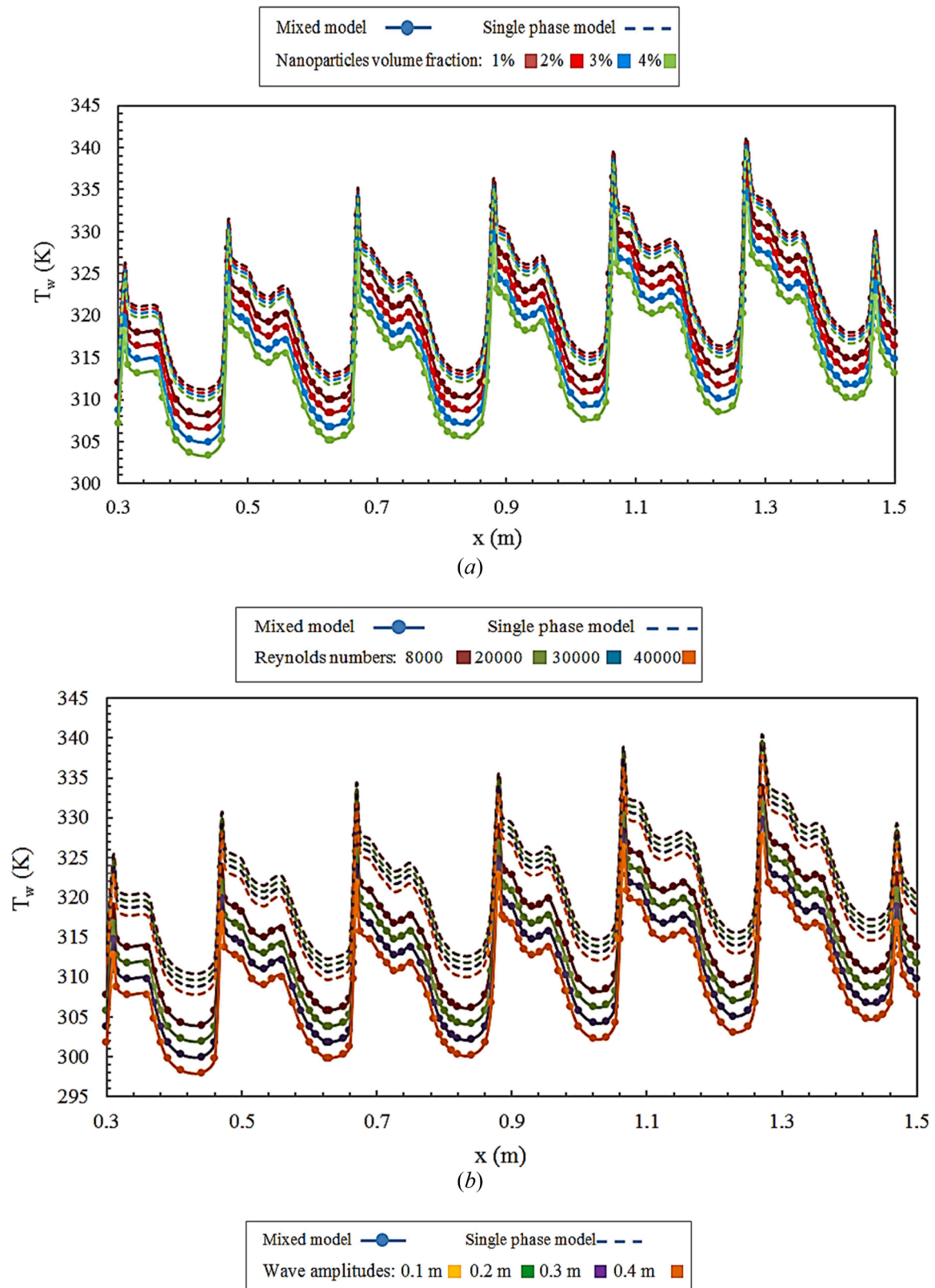


Fig. 5. The variation of wall temperature along the channel at different a) SVFs and $a = 0.3$ m & $Re = 8000$, b) Re numbers and $a = 0.3$ m & $SVF = 3\%$ and c) wall amplitudes numbers and $Re = 8000$ & $SVF = 3\%$ obtained by single phase and mixture models.

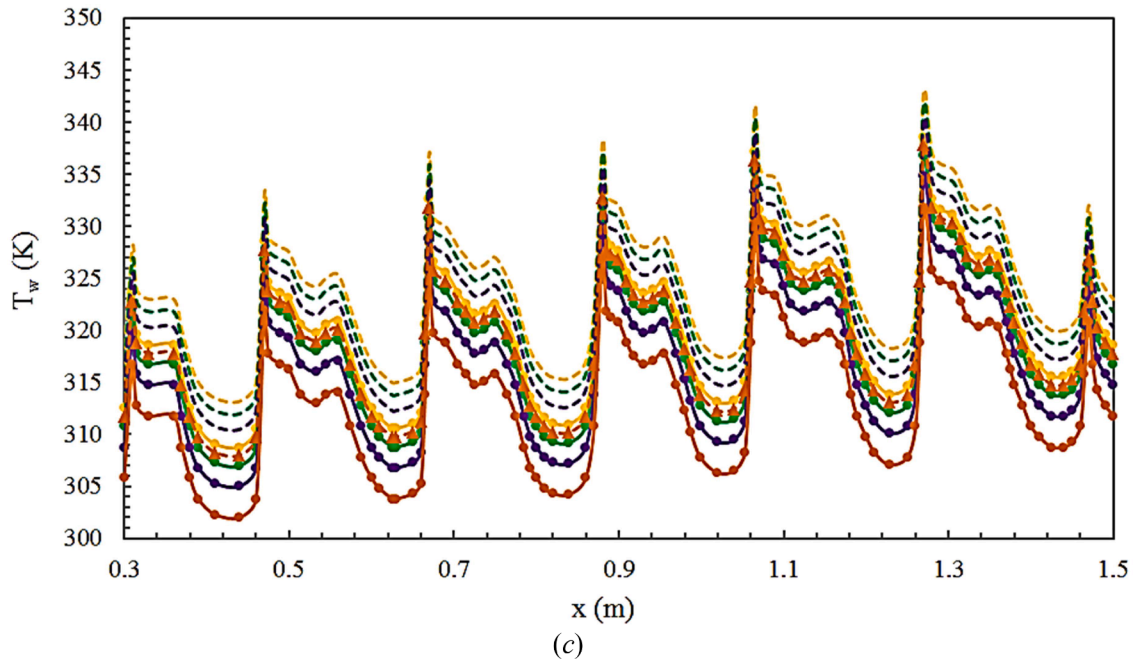


Fig. 5. (continued).

$$D_h = 4 \frac{A}{P} = \frac{2wh}{(w+h)} \cong 2h \quad (26)$$

The presence of *NPs* affects both the heat transfer and fluid flow performance which could be described by heat transfer efficiency, or more accurately Colburn factor, and friction factor as denoted by Eqs. (27) and (28), respectively [41].

$$J = \frac{Nu}{Re Pr^3} \quad (27)$$

$$f = \frac{\tau}{\rho_m \frac{u_m^2}{2}} \quad (28)$$

3. Numerical method and validation

The governing equations alongside the corresponding boundary conditions are solved using the finite volume approach. The convective and diffusive terms of the governing equations are discretized using the second-order upwind and second-order central schemes, respectively. Considering the incompressible assumption made for modeling the *NF* flow, the *SIMPLE* algorithm is used for the pressure-velocity coupling. The structure is the *2D* squared structured type which for getting higher accuracy has been refined in high gradient areas near the walls. To ensure the independency of the obtained result from the employed mesh, the grid study is performed. Fig. 2 illustrates Nu_{avg} for different grid densities in $SVF = 1\%$. In addition, the numerical values of the Nu_{avg} for different grid sizes are shown in Table 3. It can be concluded that the grid with 550×100 cells is sufficiently fine to get independent results from the generated grid. A view of the applied mesh is shown in Fig. 3. The meshing process was done in Gambit and the solving procedure was conducted using Ansys Fluent.

To confirm the validity of the numerical method, Nu_{ave} for fully developed turbulent flow of *CuO*-water *NF* in the corrugated channel is compared against the results of Vanaki et al. [23] and Manavi et al. [44] (Fig. 4). Manavi et al. [44] investigated the thermal and flow performance of Al_2O_3 *NF* inside a wavy channel in a wide range of *Re* numbers using the two-phase model. Moreover, Vanaki et al. [23] studied the thermo-flow behavior of different *NFs* inside the channels with sinusoidal walls and with different *Re* numbers of 5000 – 18,000 and

constant wall temperature. The figure shows a good agreement between the results of the present numerical model with that of the previous experimental data with a maximum deviation of 7%. From the above comparisons, it is concluded that the employed *CFD* code is reliable and could be used to investigate the hydrodynamics and thermal behavior of the turbulent flow of the *NF*.

4. Results and discussion

By using the validated numerical code, the fluid flow and heat transfer are simulated in the channel. The system characteristics were evaluated using the wall and bulk flow temperatures, velocity profile, average and local *Nu* number in different *Re* numbers, *SVFs*, and wall amplitude numbers.

4.1. The variation of system indexes vs. the channel length

The wall temperature distribution along the channel axis versus the *SVF*, *Re* number and wavelength calculated by the mixture and single-phase modeling approaches and are shown in Fig. 5(a), (b) and (c), respectively. As shown, by increasing the *SVF* and due to the increased thermal conductivity of *NF*, the rate of heat transfer from the wall increased and the wall temperature decreased. In the mixture model, the wall temperature difference between different *SVFs* is more distinguishable than that in the single-phase model; in addition, the higher the *SVF*, the lower the wall temperature would be. Compared to single-phase modeling, the mixture model predicts a higher thermal conductivity for the fluid which results in a lower wall temperature. Another point that could be inferred from Fig. 5 is the overall increment of the wall temperature through the flow path which is the result of constant heat flux to the channel wall and despite the heat removal, would be augmented inside the wall. This increasing trend also follows the geometry of the channel; in the areas near the top of the wave and due to velocity increment, the wall temperature decreased and vice versa, near the concave areas, increased. Between the study cases, the lowest wall temperature is for the highest *SVF*, highest *Re* numbers and highest wave amplitudes which are due to the highest *Nu* numbers in these cases (Fig. 5).

Fig. 6 shows the bulk fluid temperature along the channel axis versus

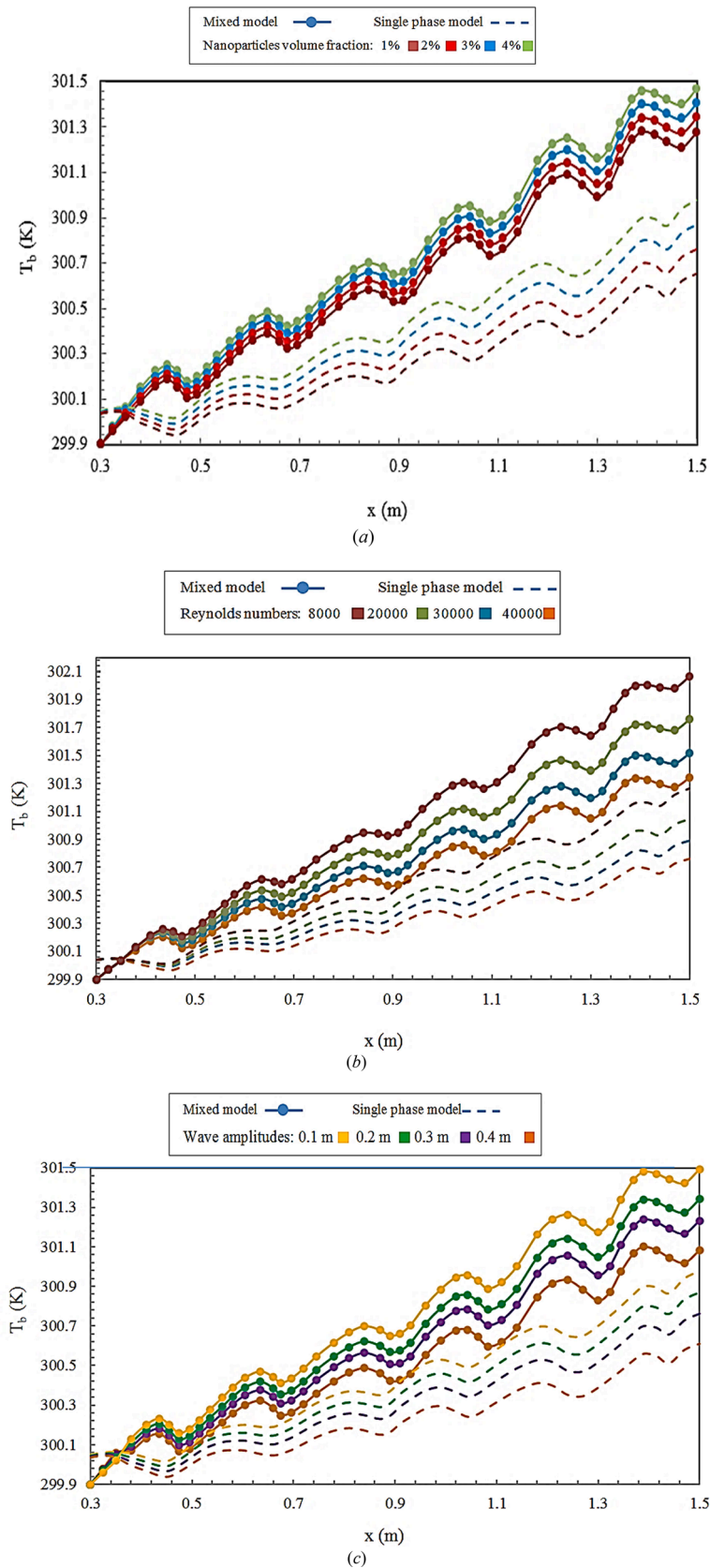


Fig. 6. The variation of bulk fluid temperature along the channel length at different a) SVFs and $a = 0.3$ m & $Re=8000$, b) Re numbers and $a = 0.3$ m & $SVF=3$ % and c) wall amplitude numbers and $Re=8000$ & $SVF=3$ % obtained by the mixture and single-phase modeling.

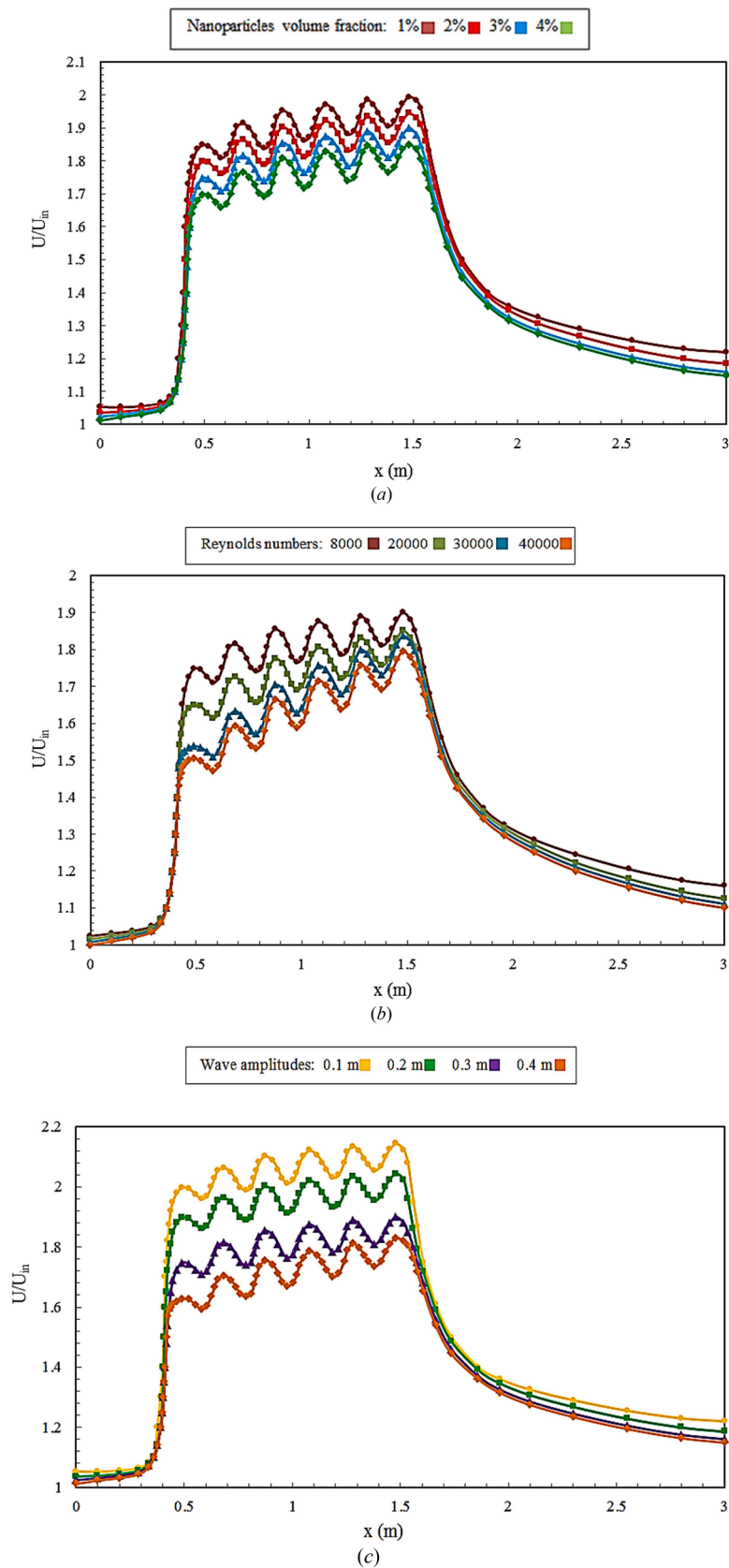


Fig. 7. The variation of dimensionless axial velocity along the channel centerline in different a) SVFs and $a = 0.3$ m & $Re=8000$, b) Re numbers and $a = 0.3$ m & $SVF = 3\%$ and c) Wall amplitude numbers and $Re=8000$ & $SVF = 3\%$.

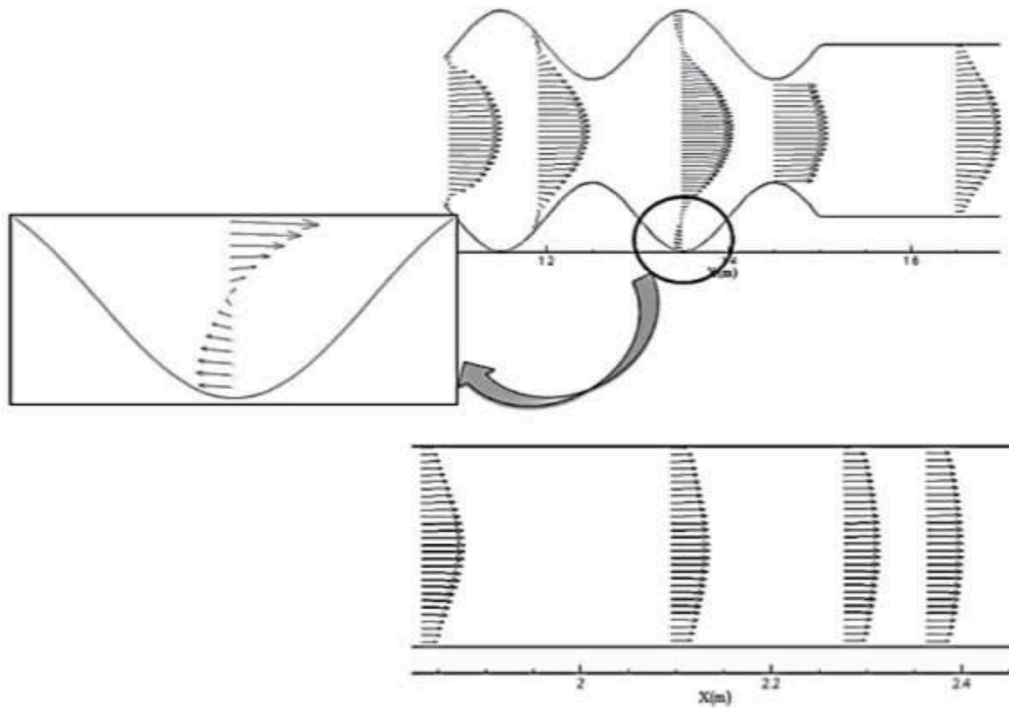


Fig. 8. Velocity profile in different channel sections at $Re=20,000$, $SVF=0.03$ and $a = 0.3$ m.

the parameters of SVF (a), Re number (b), and wavelength (c). As could be seen, like the wall temperature the overall bulk fluid temperature along the channel length is increasing. Comparing the bulk fluid temperature obtained by the mixture and single-phase models, the former model shows the higher temperature; also, by increasing the SVF in both models and due to the increased thermal conductivity of NF , the fluid temperature would be increased. It could also be observed that the temperature difference between different study cases increased through the channel length; at $x = 1.5$, the bulk temperature difference is 0.2, 0.76 and 0.44 between SVF of 0 and 4 %, Re of 8000 and 40,000 and wave amplitudes of 0.1 and 0.4, respectively, where the corresponding values are zero at the entrance ($x = 0$). This shows that the heat transfer improving effect of increasing Re number, SVF , and wave amplitude becomes more significant by distancing from the entrance region; in other words by departing from the entrance region, the effect of heat transfer improving factors augments which increases the bulk fluid temperature more. In addition, in the flow path, the difference between the results of the mixture and single-phase models increased and the bulk temperature in the mixture model is approximately 5 % higher than in the single-phase model; this shows that by increasing flow time, the accuracy of the single-phase model decreased.

Fig. 7 shows the centerline axial velocity distribution in dimensionless form along the channel length versus the SVF (a), Re number (b), and wavelength (c). By inspecting the figure, an increasing trend could be observed vs the $SVFs$, Re numbers, and wave amplitudes increment. The overall increment of centerline velocity through the flow path which is the result of the formation and growth of the vortices in the flow path has an intermittent nature; these fluctuations are the result of the formation of vortices in the recesses of the waves and the formation of backflow. As could be seen in Fig. 7(a), increasing the SVF , decreases the centerline velocity, particularly in wavy regions of the channel; by increasing the SVF from 1 % – 4 % the dimensionless velocity decreased by 0.16 in the corrugated area and remains nearly unchanged in the smooth zone. The higher effect of NP increment on decreasing the axial velocity in the corrugated zone comes back to the existence of vortices and therefore, the higher effect of fluid viscosity on velocity reduction. As shown in Fig. 7(b), after $x = 2.8$ and for all Re numbers a fully

developed region could be observed, which is the result of continuous flow in the smooth channel. By the Re number increment, the beginning of the flow development moves downstream because the increased axial velocity transmits the generated perturbation in the flow direction, the center line velocity is also reduced, which is the result of a more uniform velocity profile at higher Re numbers.

For better visualizing the hydrodynamics of fluid, the velocity profile in different channel sections at $Re=20,000$, $SVF=0.03$ and $a = 0.3$ m is depicted in Fig. 8. As seen, the fluid has its maximum velocity in the converging section through the channel way. In each section, the maximum velocity is in the centerline and the minimum is in the vicinity of walls; a collision of fluid particles to wall dissipates its energy and the reverse flow is created. In the center of the depression zone, the velocity becomes nearly zero and negative toward the wall. The reverse and main flow collision generates vortices near the wall. By adding NPs and increasing their concentration in NF , the fluid velocity increases in both forward and backward directions which increase the vortex strength which enhances the wall-fluid heat transfer. At the end part, by eliminating the wavy shape, the vortex generation stopped and the flow get gradually developed and a nearly uniform profile is generated.

Fig. 9 shows the variation of Nu_{avg} versus the phase lag in different SVF (a), Re number (b), and amplitude values (c). The increment of the Nu_{avg} for angles of less than 180° is due to the shrinkage of vortices where an improvement of 7.5 % was observed. Changing the phase difference of the wave changes the behavior of the flow in such a way that the vortices will not have enough time to grow. These vortices trap some of the fluid inside, preventing the cold fluid from coming into contact with the hot wall. Therefore, their shrinkage increases the temperature gradient and, consequently, increases the Nu number and heat transfer. It could also be seen that, between the study cases the highest Nu number is for the highest SVF ($SVF=4$ %), Re number ($Re=80,000$), and wall amplitude ($a = 0.4$ m).

The distribution of the Nu number along the channel length in different SVF , Re numbers, and wavelengths, is depicted in Fig. 10(a), (b), and (c), respectively. As could be seen, for different study cases, the overall variation of local Nu number vs. the channel length is decreasing with a small declination rate. This downward trend is due to get heating

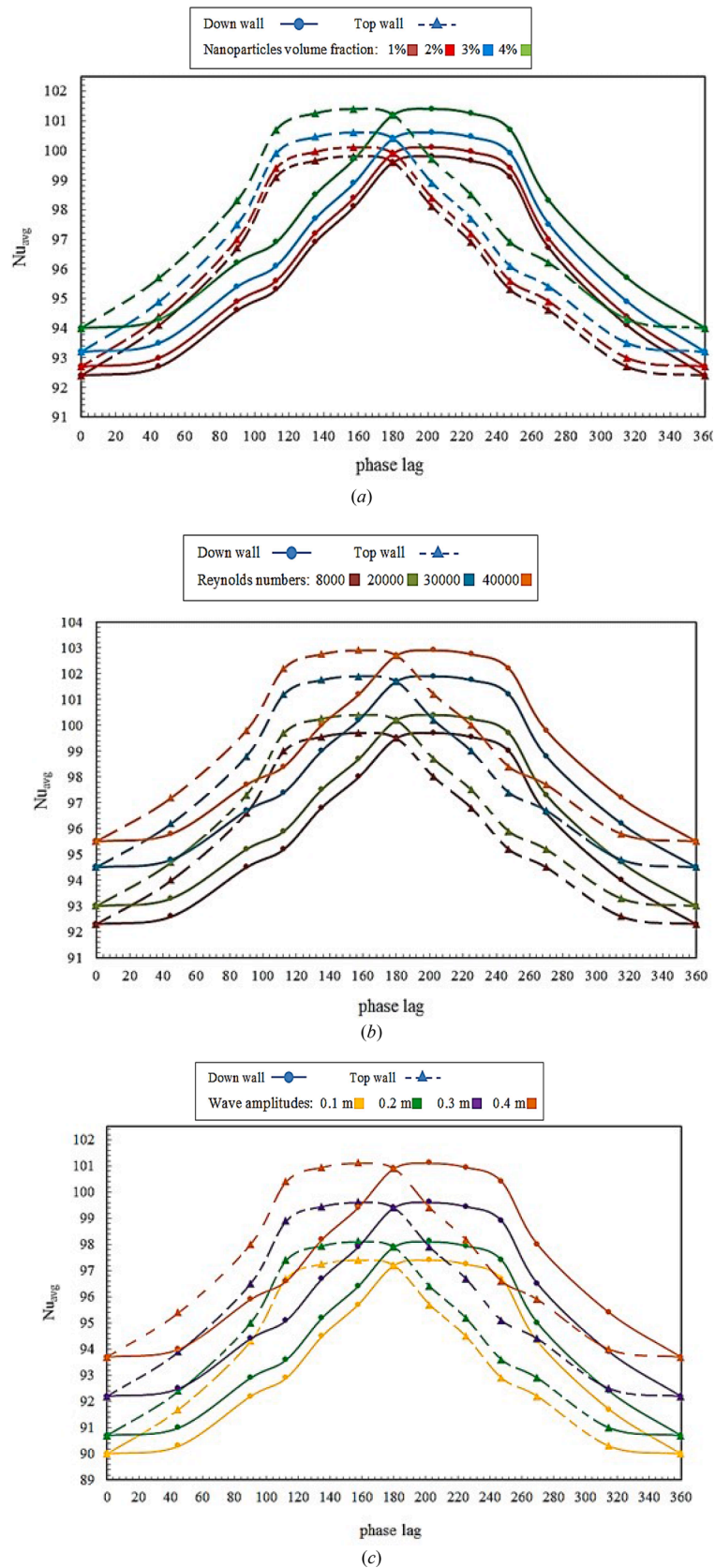


Fig. 9. The variation Nu_{ave} vs the phase lag in different a) SVFs and $a = 0.3 \text{ m}$ & $Re=8000$, b) Re numbers and $a = 0.3 \text{ m}$ & $SVF = 3 \%$ and c) Wall amplitude numbers and $Re=8000$ & $SVF = 3 \%$.

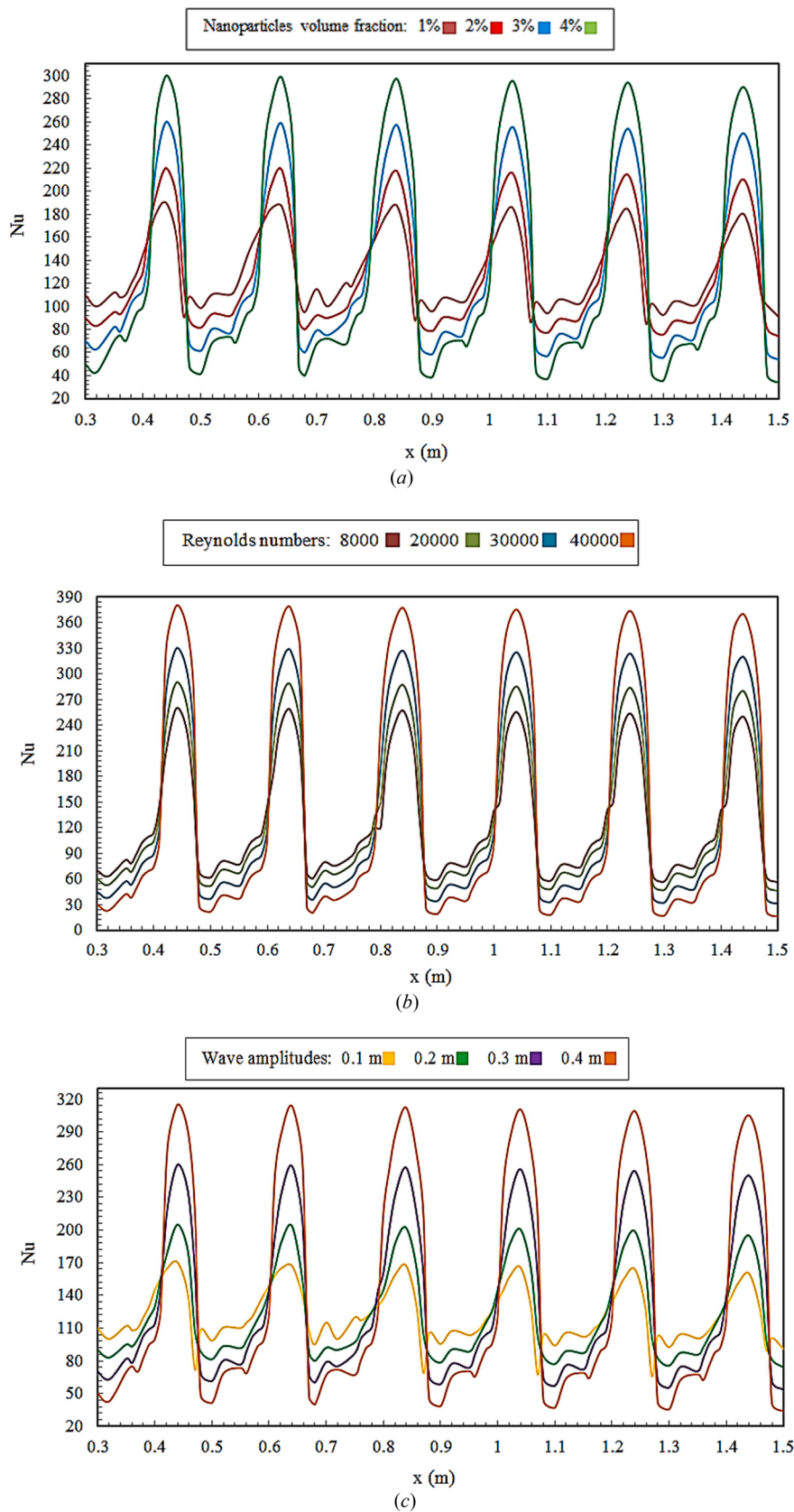


Fig. 10. The variation Nu number vs the channel length in different a) SVFs and $a = 0.3$ m & $Re=8000$, b) Re numbers and $a = 0.3$ m & $SVF = 3\%$ and c) Wall amplitude numbers and $Re=8000$ & $SVF = 3\%$.

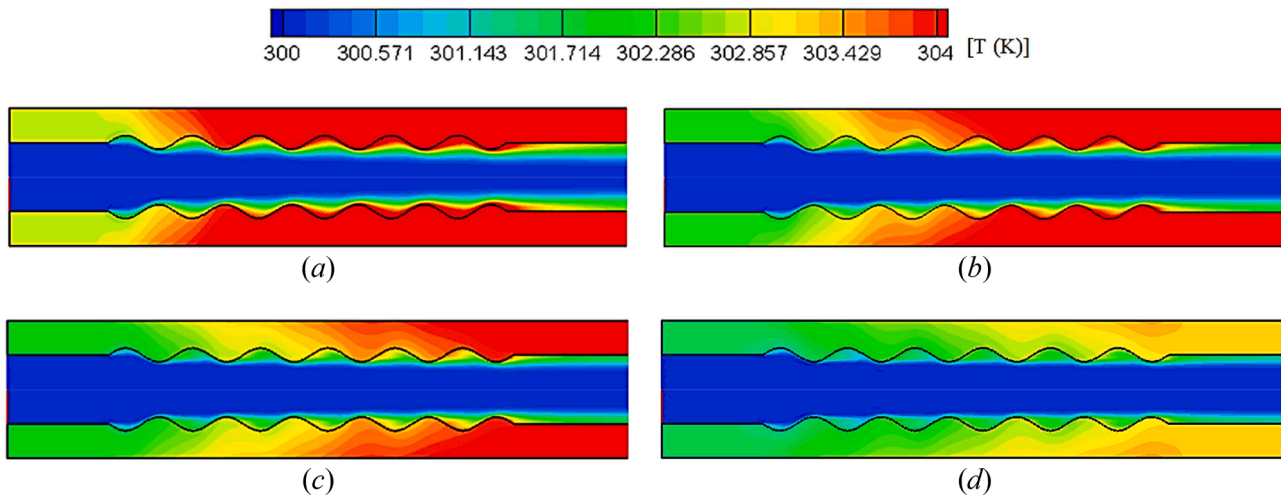


Fig. 11. Temperature contours of the wall and fluid flow in different Re numbers of a) 8000, b) 20,000, c) 30,000 and d) 40,000.

of the fluid and the temperature gradient decrement along the channel length. Another point that can be seen in the figure is the waveform variation of the local Nu number along the channel; at the ridges of the wall and due to the velocity increment, the local Nu number is the highest, behind it the Nu number decreased due to flow separation. In the zero amplitude section, a decreasing trend could be seen. At higher wavelengths, the local Nu number in indentations has a sharp decline in such a way that it becomes less than Nu_{avg} . This phenomenon which is due to the formation of vortices is in contrast to the situation in the protrusions. In protrusions the Nu number has a sharp increment due to the highest temperature gradient there which, increases Nu_{avg} ; for wall amplitude number of $a = 0.3$ m, $SVF=0.03$ and the $Re=8000$, the Nu_{avg} is 2.3 times than that in the smooth wall.

4.2. The influence of Re number

The temperature contour of fluid flow for $SVF=3\%$ in different Re numbers is shown in Fig. 11. As can be seen, by exerting the heat flux to the channel wall, the fluid temperature increased along the channel length and, with increasing the Re number, and due to continuous and periodic reconstructions of the thermal boundary layer, the rate of heat transfer increased rapidly. By increasing the Re number and the resulting flow velocity and turbulence, the penetration of cold fluid into the channel wall increases and reduces the wall temperature, and as shown in Fig. 11, from the $Re=8000-40,000$, the internal wall temperature at the end of the channel dropped from 304 K to approximately 303 K. The Re number increment which is associated with velocity increase has a positive effect on cooling the wall; moreover, decreases the stationary zones inside the wall depression and prevents the possibility of the formation of hot spots in these regions. The temperature reduction of the channel wall by the Re number increase confirms the growth of the heat transfer rate.

To justify the temperature contour inside the channel, the flow streamlines at different Re numbers are depicted in Fig. 12. As seen, the vortex flows are weak at initial lengths and become more and stronger along the way. Also, all cavities are filled with vortices and by the Re number increment, the vortex number and their intensity increase too, which improves the heat transfer rate.

The variation of Nu_{avg} vs the Re number at Re numbers of 8000, 20,000, 30,000, and 40,000 and $SVF=0.01, 0.02, 0.03$, and 0.04 and different wavelengths shown in Fig. 13. As the Re number increased, the Nu_{avg} shows an upward trend. As shown in Fig. 13, the mixture model outputs a higher Nu_{avg} than the single-phase model; for example at $Re=40,000$ and $SVF=0.03$ and wavelength $\alpha = 0.3$ m, Nu_{avg} is approximately 1.1 times of the single-phase model. This approves the higher accuracy

of the mixture model to predict the performance of NF flow. By inspecting Fig. 13 it is revealed that the Nu_{avg} increased by increasing the SVF , Re number and wall amplitude increment. The improvement of Nu_{avg} resulting from the SVF increment is mainly due to the increase of the effective thermal conductivity of the NF compared to the base fluid; this would reduce the temperature difference between wall and fluid which eventually increases the heat transfer coefficient and Nu number under constant heat flux input. As the Re number increased, the axial velocity increased and the heat penetration from the channel wall into the central region of the fluid flow increased, resulting in the growth of the heat transfer rate. It is worth noting that SVF increment does not work the same in different Re numbers and has a higher improving effect in lower Re numbers; in the case of wall amplitude of 0.1 m, by increasing the NF concentration from 1–4% the Nu_{avg} increases by 35% and 22% in $Re=8000$ and 40,000, respectively. This behavior, which is also observed in other wall amplitudes, suggests the higher effect of using NF where the other enhancing factors are absent. A similar situation exists in the case of wave amplitude increment; the Nu_{avg} improvement resulted from increasing the SVF from 1%–4% ceases by increasing the wave amplitude value and is 35%, 25%, 18%, and 14% for amplitudes of 0.1, 0.2, 0.3 and 0.4 m, respectively. This shows that the enhancing effect of using NF is more significant where the effect of other enhancing factors (such as the Re number or SVF increment) is weak.

4.3. The effect of wave amplitude number

To reveal the effect of wave amplitude number, the temperature contour of the channel wall (thickness of 0.1 m) and fluid flow for different wall amplitudes are illustrated in Fig. 14. As can be seen, increasing the wall amplitude disrupts the behavior of the fluid and increases the penetration of heat into the fluid flow. The heat penetration by the wave amplitude increase could be seen in thinning the cold center line. As the amplitude of the wave increased, the separation rate behind the sinusoidal convex portion became more intense (Fig. 8) which had a significant effect on the flow. At higher amplitudes, the velocity at the top of the wave increases which enhances the pressure gradient behind the wave. Increasing the velocity along with increasing the turbulence and mixing of the flow will increase the heat transfer rate. The increased heat transfer rate has also a positive effect on cooling the wall and could postpone its overheating to later. To reveal the effect of wave amplitude on temperature contour, the flow streamlines at different amplitude numbers and $Re=20,000$ is shown in Fig. 15.

As seen in Fig. 15, by increasing the wave amplitude, the separation region behind the convex region becomes larger. At the smallest

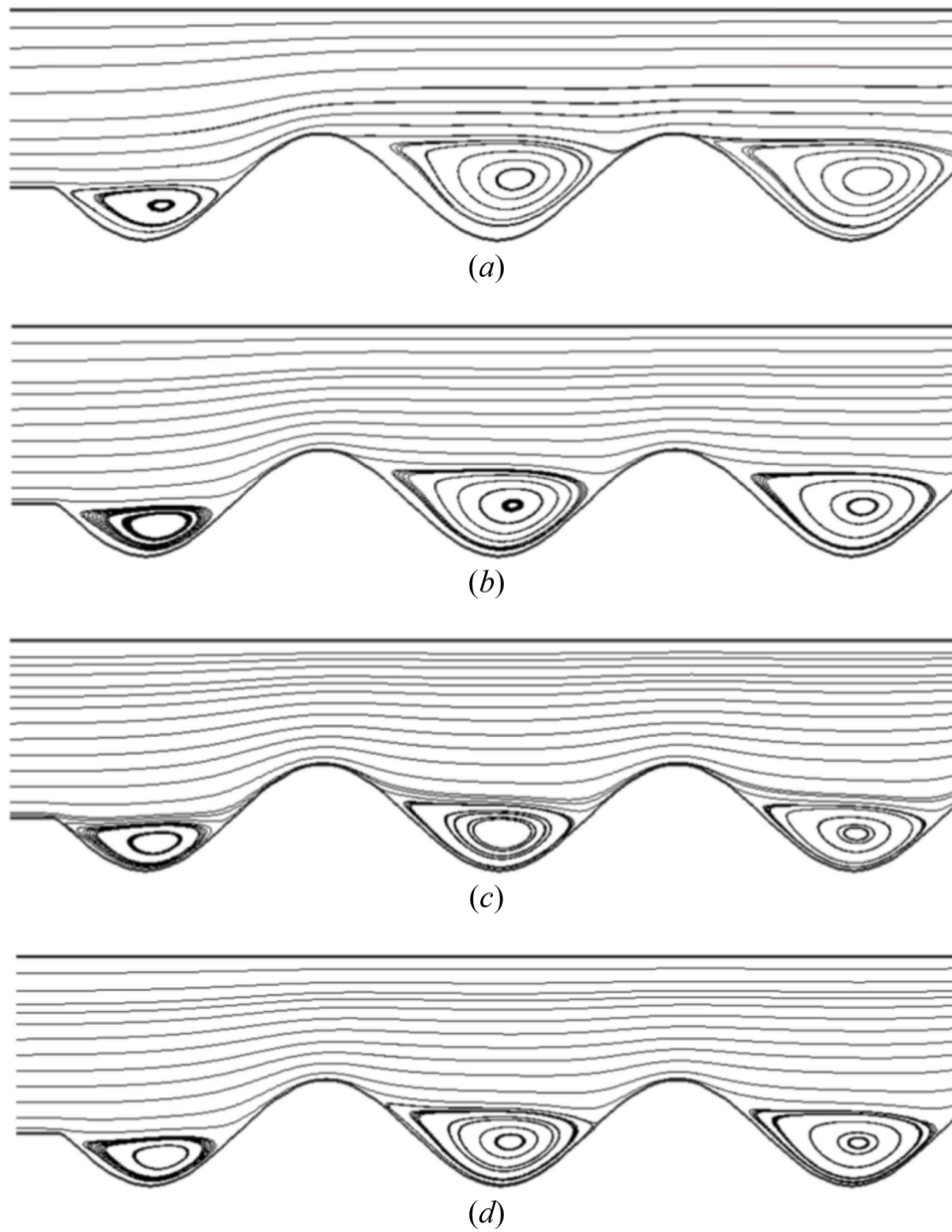


Fig. 12. Flow streamlines at different Re numbers of a) 8000, b) 20,000, c) 30,000 and d) 40,000 for $a = 0.3$ m and $SVF=0.03$.

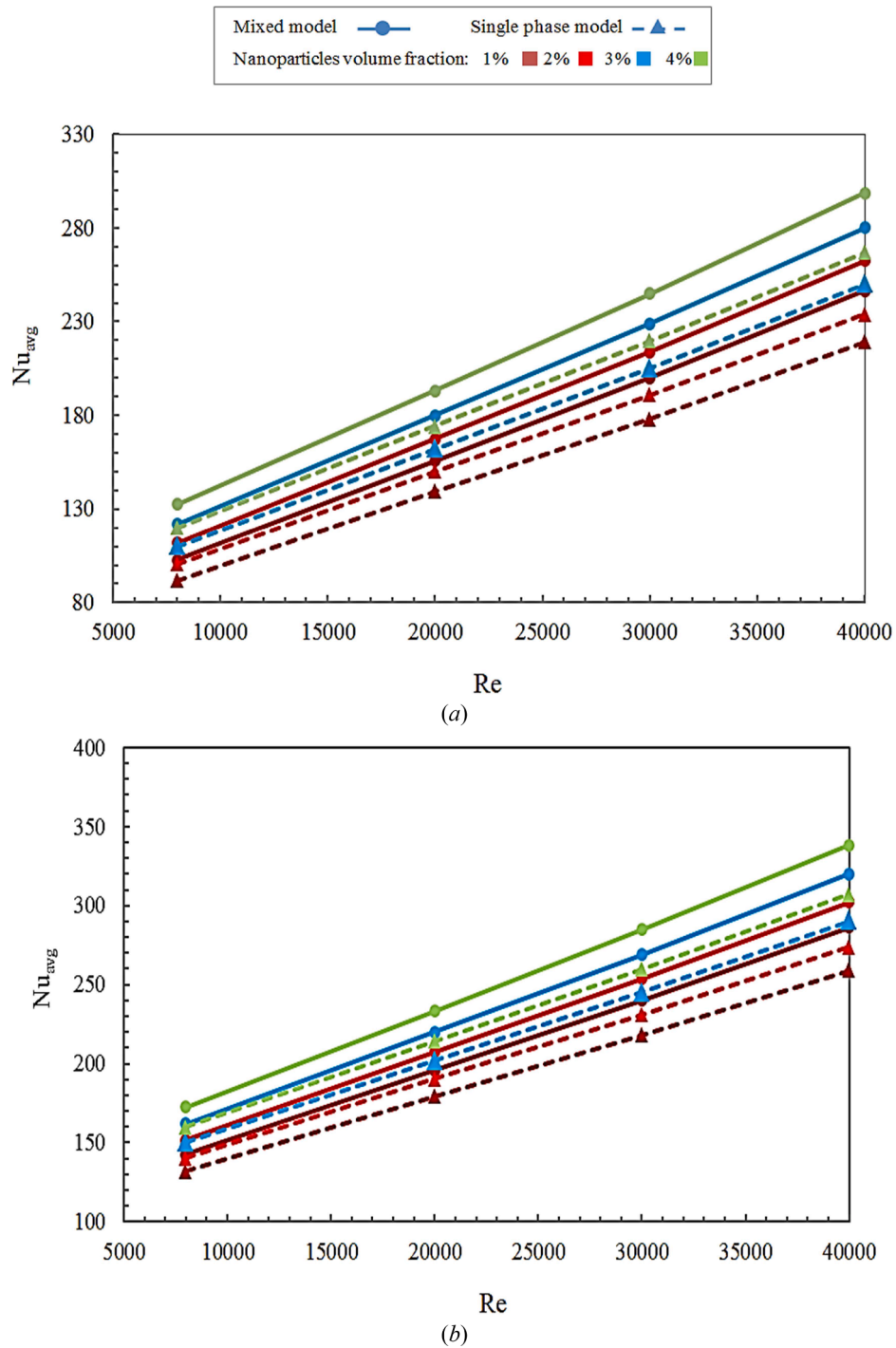


Fig. 13. Comparison of the Nu number in single-phase and mixture models at different wavelengths of a) 0.1 m, b) 0.2 m, c) 0.3 m and d) 0.4 m.

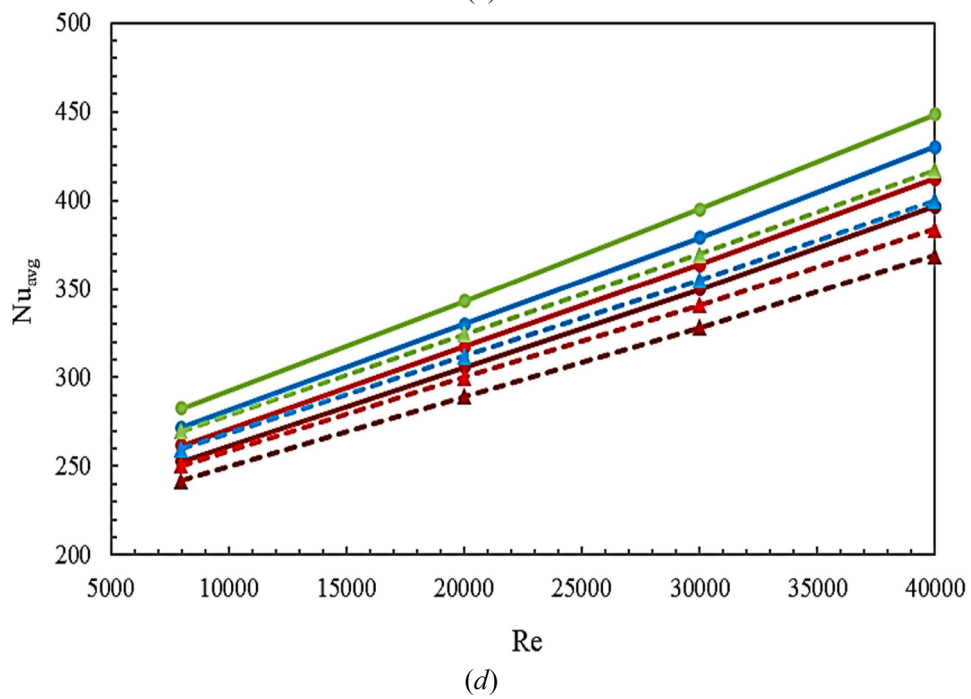
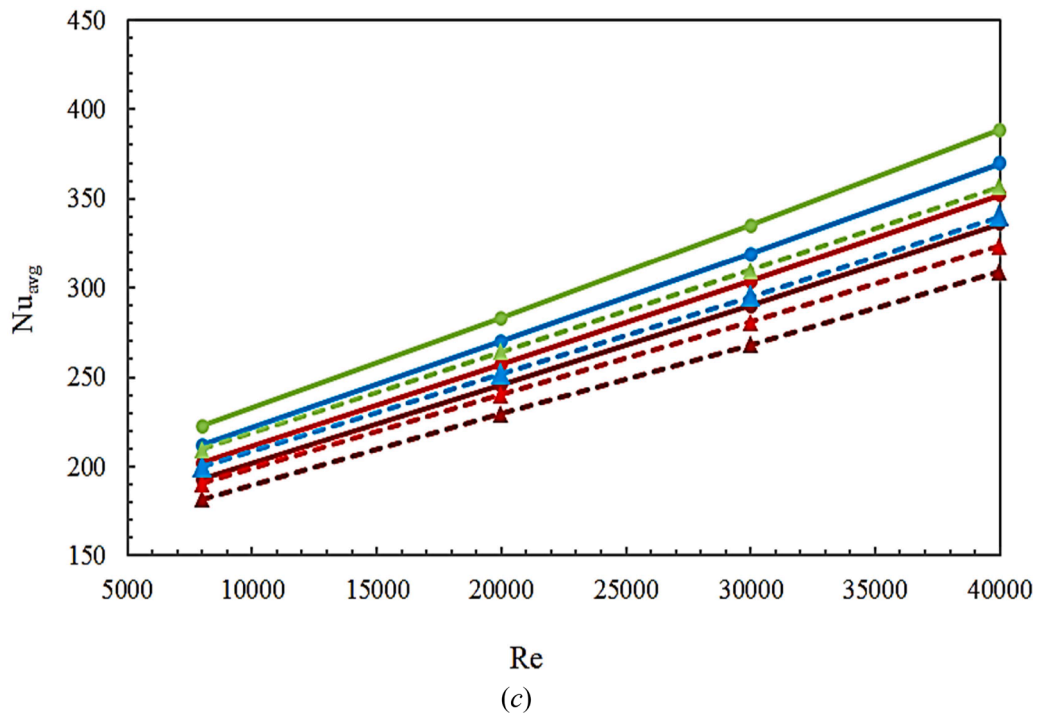


Fig. 13. (continued).

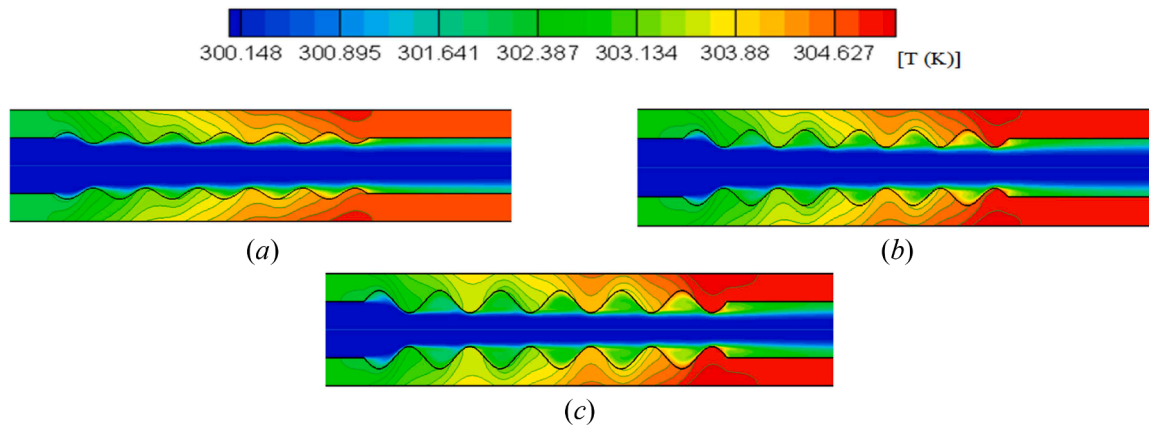


Fig. 14. Temperature contour for wave amplitudes of a) 0.2, b) 0.3 and c) 0.4 m and $Re = 20,000$.

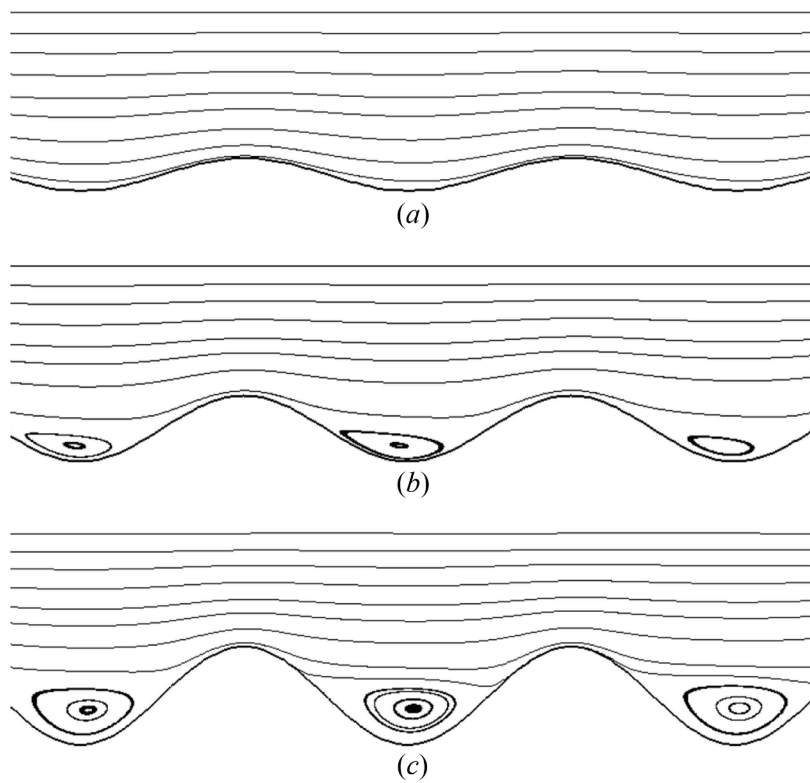


Fig. 15. Flow streamlines for different amplitude values of a) 0.1 m, b) 0.2 m and c) 0.3 m for $Re = 20,000$ and $SVF = 0.03$.

amplitude value, i.e. $a = 0.1$ m, the reverse flow has not been observed and by the wave amplitude increase, put forward the separation point toward the dome; this would increase the separated region length and consequently increase the heat transfer coefficient and friction factor.

The variation of the Nu_{avg} versus the wave amplitude for different values of Re numbers and SVF are shown in Fig. 16(a) and (b), respectively. As could be seen, the overall variation of Nu_{avg} vs the wave amplitude in different Re numbers and $SVFs$ are increasing; in a specific SVF and Re number, and by the wave amplitude increment from 0 – 0.4 m, Nu_{avg} increases by nearly 50 %. By increasing the wave amplitude and by the emergence of rotational flows behind the wave and the reverse flow process, the Nu_{avg} decreased; on the other hand, Nu_{avg} is grown by the increased velocity in the convergence part of the channel. As the flow passes through the curves of the channel, the symmetrical flow and turbulence form which eventually leads to excessive mixing of fluids and the increased heat transfer rate. Also, it could be seen that with the Re

number increment, Nu_{avg} increases too; as the Re number increases from 8000 – 40,000, the Nu_{avg} increases approximately by 75 %.

As an important factor from the viewpoint of fluid flow, the pressure drop, versus the wavelength at different SVF and Re numbers are shown in Fig. 17(a) and (b), respectively. The pressure drop is determined by subtracting the average outlet pressure from the inlet one. As expected, the pressure drop increased with increasing wave amplitude, SVF , and Re number; from a wavelength of 0.1 m – 0.4 m, the pressure drop in the case of $Re = 8000$ and $SVF = 1\%$ increased by 73 % (from 0.12 Pa– 0.45 Pa). The pressure drop in corrugated ducts is higher than in flat-walled ducts due to the collision of the fluid with the ridges of the duct wall and the subsequent formation of vortices. It could be also revealed from Fig. 17(a) and (b) that the pressure drop inclination rate vs. the amplitude number is the highest in cases of the highest SVF and Re number; by increasing the wave amplitude number from 0.1 – 0.4 the pressure drop increment is 2.67 and 2.7 times for SVF value of 1 % and 4 % and 1.6

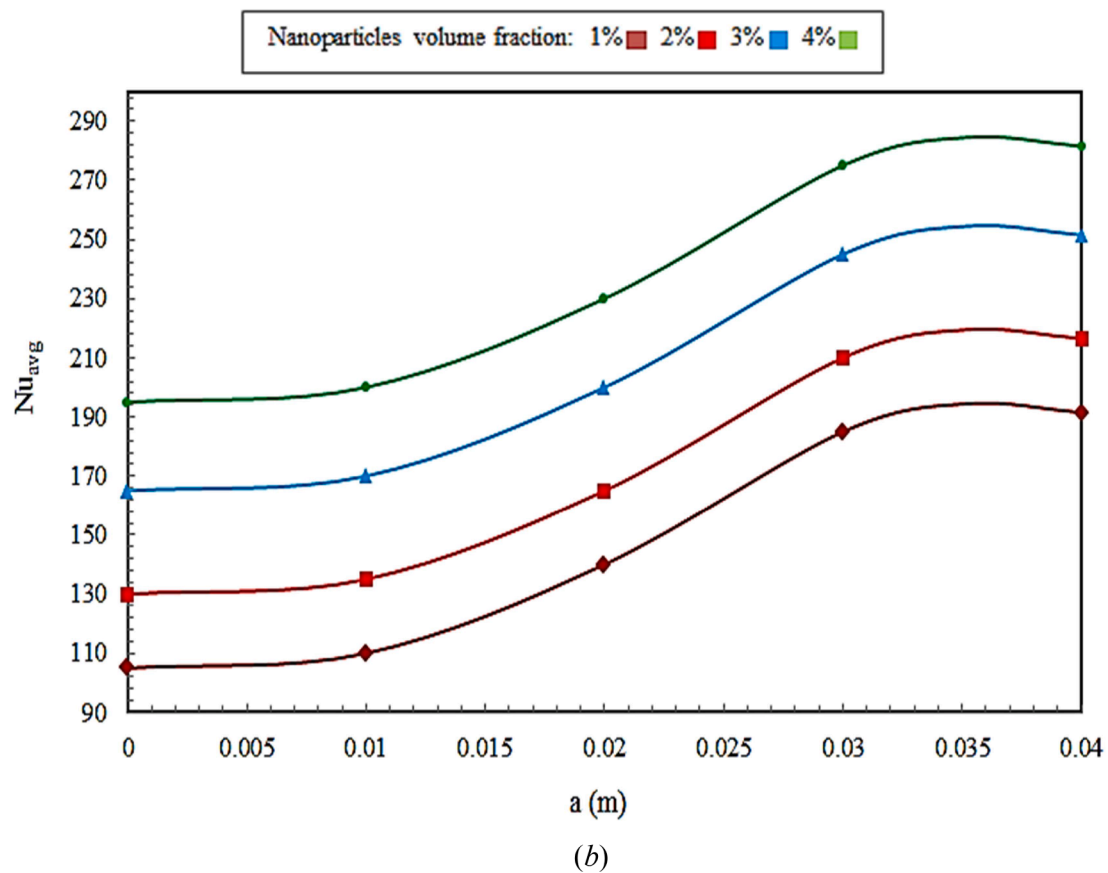
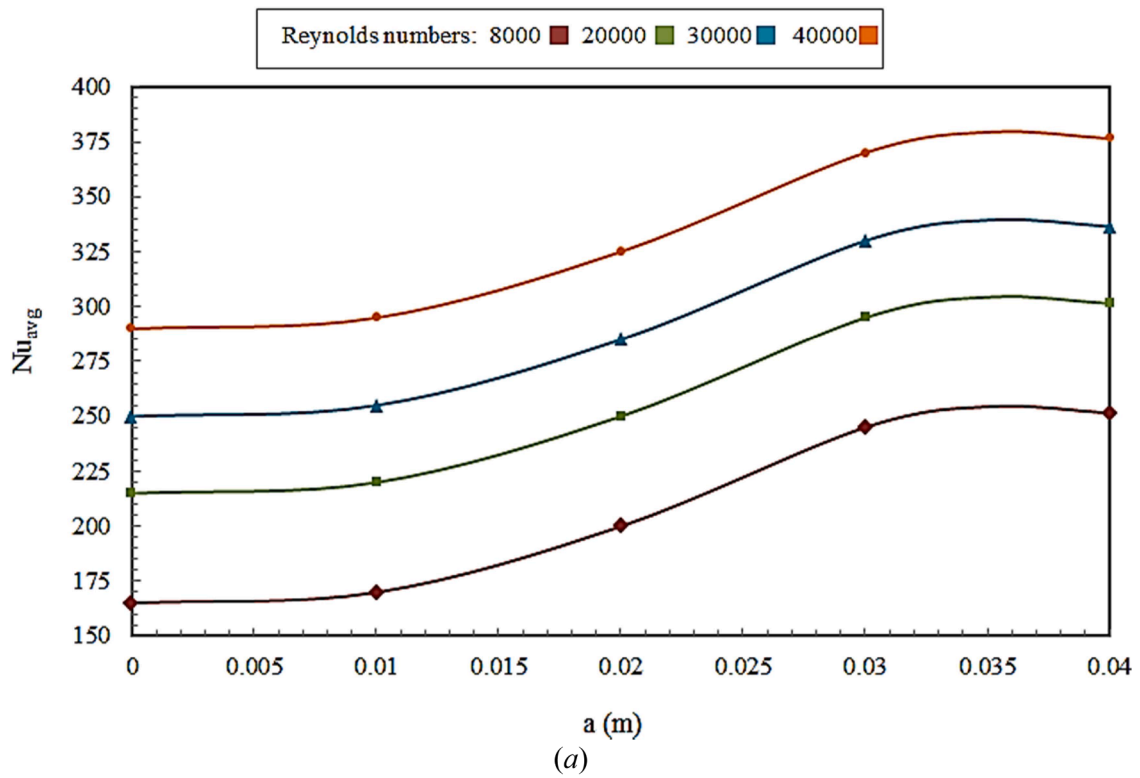


Fig. 16. The Nu_{ave} vs. the wave amplitude in different a) Re numbers and $SVF=3\%$ and b) $SVFs$ and $Re=8000$.

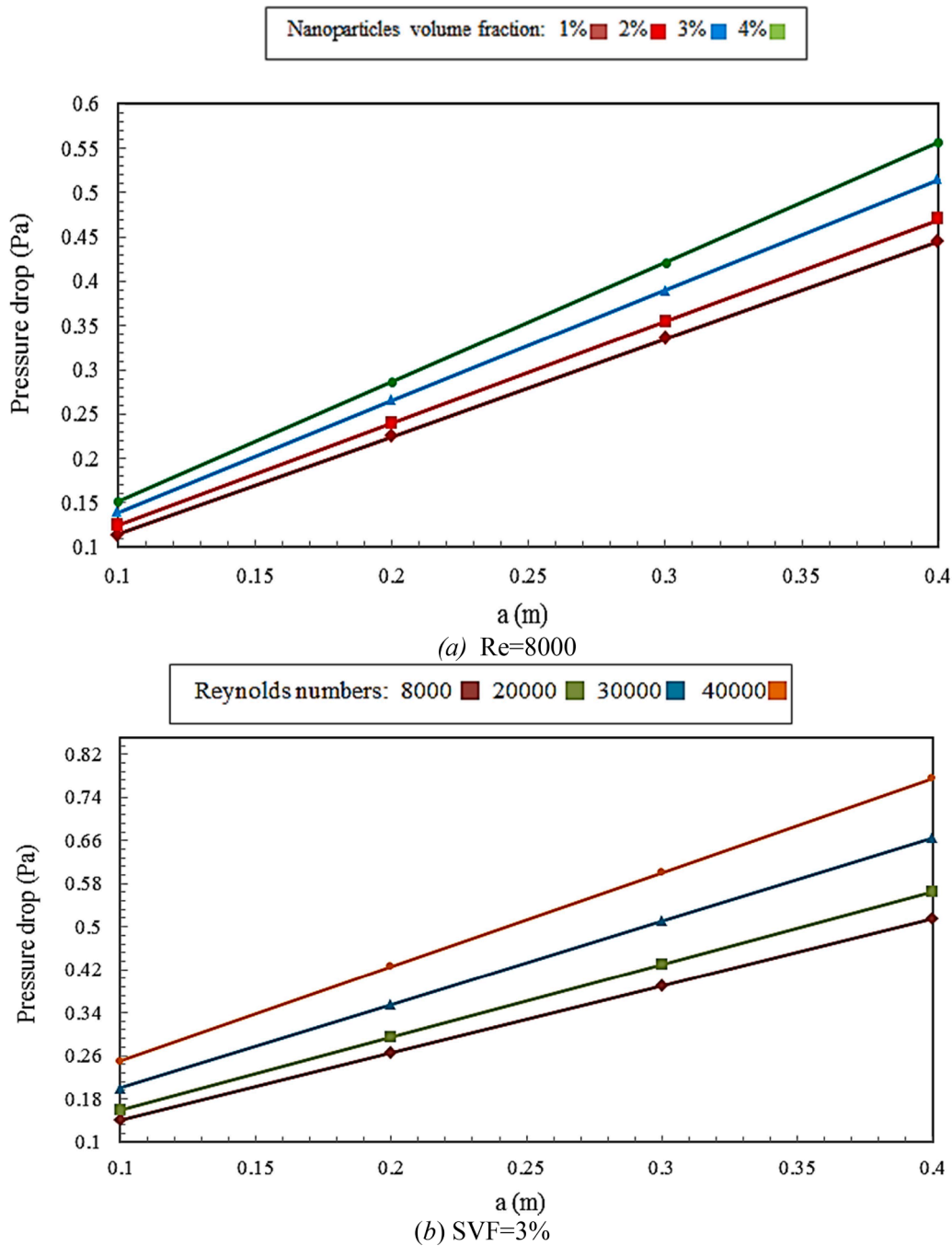


Fig. 17. The flow pressure drop vs the wave amplitude number in different a) SVFs, b) Re numbers.

times and 2.7 times for $Re=8000$ and $40,000$, respectively. This demonstrates the more beneficial effect of increasing the SVF than increasing the Re numbers on system performance under the present operating conditions.

4.4. The effect of SVF

The addition of NPs to the fluid increases both the values of heat transfer and pressure drop; to reveal the effect of using NP on the overall thermo-flow characteristics of the system, the J/f factor is used, as the quotient Eq. (27). The variation of J/f factor vs. the SVF for different wave amplitudes and Re numbers are shown in Fig. 18(a) and (b), respectively. The figure shows the improving effect of increasing SVF on the value of J/f at each Re and wave amplitude numbers; at wave

amplitude of $a = 0.3$ m and by increasing the SVF from the $SVF = 0.01$ to $SVF = 0.04$, the J/f increases by 30 %. This approves the heat transfer improving effect of SVF increment is greater than pressure drop increment at all Re numbers and wall amplitudes. Also, the Re number increment harms J/f value, and by being increased from 4000 – 80,000, J/f deteriorates by approximately 75 %.

5. Conclusion

Considering the widespread application of heat exchangers in industry, heat transfer improvement in these devices became an inevitable task. The NFs by their superior thermal properties are a good choice for this purpose. In this work, the application of CuO/water NF in a corrugated channel heat exchanger was studied. The heat exchanger surface is

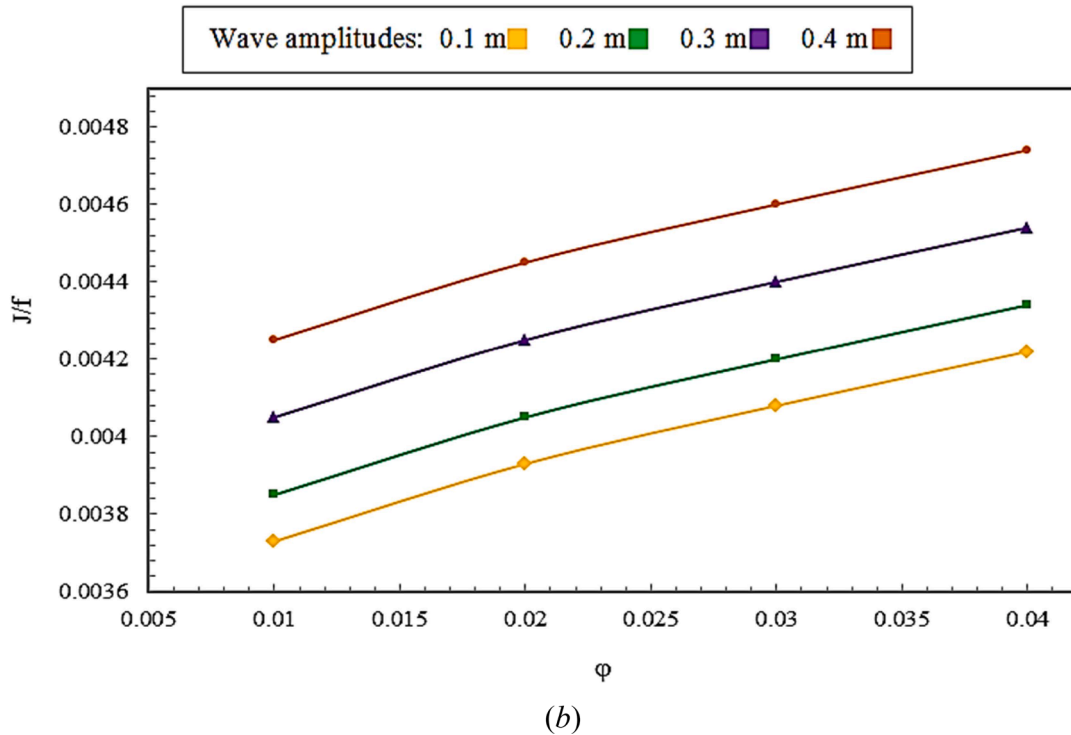
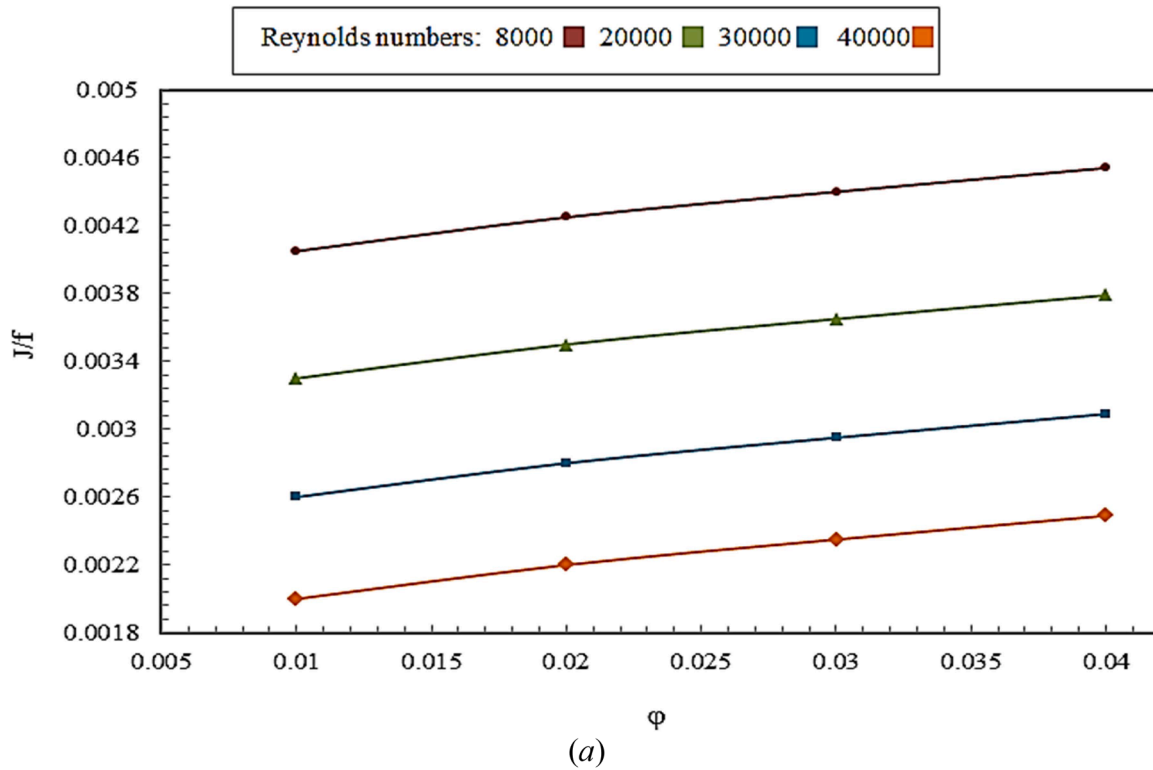


Fig. 18. The J/f factor vs the SVF in different a) Re numbers and $a = 0.3$ and b) wave amplitudes and $Re=8000$.

under a constant heat flux and the thermal and fluid flow performance of NF is simulated using the two-phase mixture model. The obtained results could be summarized as follows.

- Compared to single-phase modeling, the mixture model predicts a higher thermal conductivity for the fluid which results in an increased higher heat removal rate and lower wall temperature.

- Between the study cases, the lowest wall temperature is for the highest SVF , highest Re numbers and highest wave amplitudes which is due to the highest Nu numbers in these cases and the result of more intensified turbulent flow.
- The temperature difference between different study cases increased through the channel length. This shows that the heat transfer enhancing effect of increasing Re number, SVF , and wave amplitude becomes more prominent by distancing from the entrance region; in

other words by departing from the entrance region, the effect of heat transfer improving factors augments which increases the bulk fluid temperature more.

- Increasing the *SVF*, decreases the centerline velocity, particularly in wavy regions of the channel; by increasing the *SVF* from 1% to 4% the dimensionless velocity is reduced by 0.16 in the corrugated area and remains nearly unchanged in the smooth area. The higher effect of *NP* increment on decreasing the axial velocity in the corrugated zone is discussed as the effect of vortices and therefore, the higher effect of fluid viscosity on velocity reduction
- The improving effect of using *NF* is more significant where the effect of other enhancing factors (such as the *Re* number or *SVF* increment) is weak. For wall amplitude of 0.1 m, by increasing the *SVF* from 1 to 4 % the Nu_{avg} increased by 35 % and 22 % in $Re = 8000$ and 40,000, respectively.

CRedit authorship contribution statement

Mohammad N. Fares: Formal analysis, Data curation, Conceptualization, Supervision. **Mohammad AL-Saad:** Formal analysis, Writing – original draft. **Heider H.J. Almutter:** Writing – original draft, Writing – review & editing. **Dheyaa J. Jasim:** Data curation, Formal analysis, Validation. **Mohammad Ali Fazilati:** Conceptualization, Data curation, Formal analysis, Supervision, Writing – original draft, Writing – review & editing. **Soheil Salahshour:** Writing – original draft. **Sh. Baghaei:** Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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