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SPECIAL CASES AND APPLICATIONS OF THE CAUCHY COMPANION OPERATOR

MOHAMMED A. ABDLHUSEIN[1,*], NADA M. AYAY[2], SAMA ARJIKA[3], HUSAM L. SAAD[4]

[1]College of Education for Women, Shatrah University, Thi-Qar, 64001, Iraq

[2]College of Education for Pure Sciences, University of Thi-Qar, Thi-Qar, Iraq

[3]Department of Mathematics and Informatics, University of Agadez, Post Office Box 199, Agadez, Niger

[4]Department of Mathematics, University of Basrah, Basrah, Iraq

[*]Corresponding author: mmhd@shu.edu.iq

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Abstract. In this paper, proved the famous Cauchy theorem in q-series, also we give some special roles of the Cauchy companion operator $E(a,b;\theta)$ and apply these roles to represent the Cauchy polynomials $P_n(x,y)$ and the finite q-shifted factorial $(a;q)_n$ to derive generating function, Mehler's formula and three Rogers formulas for $P_n(x,y)$ and $(a;q)_n$.

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1. Introduction, Definitions and Preliminaries

In 1998, Chen and Liu [16] have developped a method named "*parameter augmentation*" of deriving hypergeometric identities. Recently, Fang [19] introduced the q-exponential operator ${}_1\Phi_0\left[\begin{matrix} b; \\ & q; -c\theta \\ 0; \end{matrix}\right]$ and give some properties of q-series. This method has more realizations as in [2,3,6,9,10,12–16,24,31,32]. In this paper, we use this method and give easy proofs of results on q-series.

Let us review some common notation and terminology in [20] for basic hypergeometric series. Assume that q is a fixed nonzero real or complex number and $0 < q < 1$. The q-shifted factorial [17,20] is defined for any real or complex parameter a by:

$$(a;q)_0 = 1, \quad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), \quad (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k), \tag{1.1}$$

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