Newton-Kantorovich Method for Solving One of the Non-Linear Sturm-Liouville Problems

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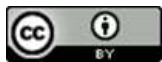
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Abstract:

Due to its importance in physics and applied mathematics, the non-linear Sturm-Liouville problems witnessed massive attention since 1960. A powerful Mathematical technique called the Newton-Kantorovich method is applied in this work to one of the non-linear Sturm-Liouville problems. To the best of the authors' knowledge, this technique of Newton-Kantorovich has never been applied before to solve the non-linear Sturm-Liouville problems under consideration. Accordingly, the purpose of this work is to show that this important specific kind of non-linear Sturm-Liouville differential equations problems can be solved by applying the well-known Newton-Kantorovich method. Also, to show the efficiency of applying this method to solve these problems, a comparison is made in this paper between the Newton-Kantorovich method and the Adomian decomposition method applied to the same non-linear Sturm-Liouville problems under consideration in this work. As a result of this comparison, the results of the Newton-Kantorovich method agreed with the results obtained by applying Adomian's decomposition method.

Keywords: Adomian's decomposition method, Central finite-difference approximation, Newton-Kantorovich method, Non-linear Sturm-Liouville problems, Non-Linear differential equations.

Introduction:

Many researchers have shown an interest in the non-linear Sturm-Liouville problems along the time since 1960, and it has applications in physics and applied mathematics. To illustrate this interest, some of the recent research work that has been done regarding these problems will be mentioned here as follows: One of the most recent research works about the Sturm-Liouville problems is the work presented in 2022 by Aal-Rkhais, Kamil, and Oweidi[1] in which they discussed solving the second order singular Sturm-Liouville equation. Also, many interesting theories regarding the non-linear eigenvalue problems of the Sturm-Liouville Type are presented by Kurseeva, Moskaleva, and Valovik[2] in 2019 including deriving solvability results, asymptotics of positive and negative eigenvalues, and also applications were given. Moreover, among other recent works that have to be mentioned here is that presented by He and Yang[3] in 2019 and by Al-Khaled and Hazaimeh[4] in 2020, wherein the work of He and Yang, the existence of positive solutions for systems of non-linear Sturm-Liouville differential equations with weight functions was studied, while in the work of Al-Khaled and Hazaimeh a comparative study between a modified version of the variational iteration method and the Sinc-Galerkin method was presented to solve non-linear Sturm-Liouville eigenvalue problem. In this paper, the Newton-Kantorovich method is applied to approximate the solution for one of the non-linear Sturm-Liouville problems that are the problem: $-y''(x) + y^2(x) = \lambda\, y(x); y(x) > 0, x \in I = (0,1)$ subject to the boundary conditions $y(0) = y(1) = 0$ where $\lambda > 0$ is an eigenvalue parameter. Newton - Kantorovich method is an important numerical method that is used to solve some non-linear ordinary and partial differential equations. This mathematical technique is very well-known and there is a massive number of research about it. One of the most recent studies in this regard is the work of Regmi, Argyros, George and Argyros[5], published in 2022, in which they introduced a technique that