Mixed Convection Numerical Study Within Lid-Driven Cavity Heat Transferring with Variable Prandtl Number

Zuhal Abdul HadiHamzah [1]
Huda Abdullah Al-Mayahi [2]

Abstract

Heat transfer mixed convection for laminar fluid flow within a vented rectangular cavity is numerically studied. The enclosure's upper wall heated to a consistent temperature while the bottom wall is maintained at a steady speed while it remains cold. An opening in the upper left wall of the cavity allows an external flow to enter, and one in the lower left vertical wall of the cavity allows an external flow to exit. The finite element method is used in conjunction with the Flex PDE software package to solve the conservation governing equations for continuity, energy, and momentum. Results from streamlines, isotherms, and the average Nusselt number demonstrate the impact. of Reynolds number (Re= 20,50,80 ,100) Richardson number (Ri=1,6,10) and the variable Prandtl number(Pr= 0.71, 1.7, 50) for air, water and oil, respectively on heat transfer. The findings indicate that the average Nusselt value rises as Pr and Re grow and falls when Ri number drops. The findings show a good agreement when compared to those of other authors in the literature.

Key Words *Mixed convection ; Free Heating ; Rectangular Enclosure*

Author Details

1 Department of Civil Engineering, University of Basrah, Basrah, Iraq
zuhal.hamza@uobasrah.edu.iq

2 Mechanical Engineering Department, University of Basrah, Basrah, Iraq
huda.abdulkareem@uobasrah.edu.iq

http://dx.doi.org/10.47832/MinarCongress10-8

Introduction

In recent years, there has been a lot of interest in the phenomenon of free convection in cavities with different geometries. This is because it has so many important engineering and scientific uses, like in boilers, energy storage, fire suppression, chemical building insulation, solar collectors, nuclear reactor cooling systems, and lake convection processes [1,2]. When temperature gradients within fluids cause buoyancy-induced flows, natural convection develops as a heat transfer mode. A summary was also provided of the results of internal body processes with various geometries (such as circular, square, and elliptical cylinders) on mixed convection heat transfer. Multiple studies on the phenomenon of natural convection within holes with varying boundary conditions have been conducted to satisfy distinct objectives because of the importance of convection-based heat exchange in engineering systems [3]. The review of the literature revealed that the enclosures under study were either filled with liquid water, oil, or air. As a result, the water-filled enclosures are listed first, followed by the air-filled ones. An investigation by Ali et al. [4] looked at a naturally occurring dimensionless heat transfer efficiency inside two square water-filled chambers. Average Nusselt and modified Rayleigh values were calculated for every enclosure. Using the ratio of aspect (AR = S/H) and the modified Rayleigh number, specific Each enclosure's correlations were obtained, and the two enclosures' combined correlations were also discovered as follows:

$$Nu = 16.676\,(\mathrm{Ra}_{\mathrm{H}}^{*})^{0.0502}\kappa^{-1.018}, 4\times10^{6} < \mathrm{Ra}_{\mathrm{H}}^{*} < 3.5\times10^{8}$$

Shobha et al. [5] presented a mixed convection inside a four-sided lid-driven square porous cavity whose right wall is main tained at a sinusoidal temperature condition, the left wall of the cavity is maintained at a cold temperature, while the top and the bottom walls are adiabatic. They discussed numerically two deferent cases depending upon the direction of the moving walls. They investigated by ranging the various dimensionless numbers such as Grashof number ($10^{3} \le Gra \le 10^{5}$) , Darcy number ($10^{-1} \le Dar \le 10^{-5}$) , Reynolds number ($10 \le Re \le 1000$) while maintaining the Prandtl number ($Pra = 0.7$) fixed.

Olanrewaju et al. looked studied the effects of aspect ratio, Rayleigh, and Nusselt on flow and heat transmission in rectangular holes. [6]. In accordance with the predicted Nusselt numbers with a Prandtl number of (0.71) at air temperature, they provided variations of stream lines and temperatures, contours, and isotherms of the resulting flow fields. They discovered that heat transmission in rectangular cavities rises at larger Raylieh numbers of flow. However, the Nusselt number, which was calculated close to the hot wall's edge, showed that convection was the primary method of heat transmission with a greater aspect ratio. They claimed that when aspect ratio grew, the thickness of the thermal boundary layer also increased continuously, significantly enhancing the amount of fluid heat transfer by convection in the rectangular cavities.

In order to compute the friction of fluid and the spread of entropy produced by the heat move mechanism for Rayleigh numbers ranging from 10^5 to 10^8 and ratios of aspect ranging from 3, 5, 7, and 12, Souda et al. [7] studied the free convection phenomenon using numerical techniques in rectangular holes with different ratios of aspects. It was discovered that the ratios of aspect are significantly influenced by the rate at which Rayleigh numbers grow relative to the generation of all entropy. According to the experimental view, when a cavity is heated at the bottom and cooled at the top, the Rayleigh number increases and the average heat transfer coefficient rises.

Other scholars have examined the natural process of convection of heat transfer for various parameters inside a cavity filled with air [8,9,10]. Not much research has been done on thermally induced flows in triangular cavities with varying aspect ratios in variably warmed vertical walls and adiabatic horizontal walls, notwithstanding the fact that natural convection in cavities has been extensively studied using numerical simulations and experimental methods. This may be partially attributed to the issue's complexity as well as other related problems. It is important to note that research in this field may advance our knowledge and result in better system design and performance, particularly for practical applications like the drying and storage of phenomena. Additionally, it is possible to test for the Navier-Stokes equations in laminar, steady, and incompressible flows using the program validated codes.

The present study's main goals are to show how mixed heating affects mixed convection and how streamlines, isotherms, average temperature, Reynolds number, Prandtl number, and Nusselt number all have an impact.

Theoretic Analysis

A diagram representation of the issue at hand is shown in Figure 1. It is a rectangular, vented enclosure that has a height of W and a base of L.. The side walls are adiabatic, the bottom wall is cold and lid driven with constant velocity while the upper wall is heated at constant temperature (Te_h). The enclosure's inflow opening is on the upper left vertical wall, and its exit opening is on the lower right wall. Radiation and the term for viscous dissipation in the energy equation are not taken into account. In non-dimensional form, the equations that govern steady, laminar, two-dimensional, incompressible flow with constant fluid properties and the Boussinseq approximation look like as this [11]:

Equation of continuity

$$\frac{\partial UU}{\partial X}+\frac{\partial VV}{\partial Y}=0 \tag{1}$$

Equation of momentum in x-direction

$$U\frac{\partial UU}{\partial X}+V\frac{\partial UU}{\partial Y}=-\frac{\partial P^{*}}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^{2}UU}{\partial X^{2}}+\frac{\partial^{2}UU}{\partial Y^{2}}\right) \tag{2}$$

Equation of momentum in y-direction

$$U\frac{\partial VV}{\partial X}+V\frac{\partial VV}{\partial Y}=-\frac{\partial P^{*}}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^{2}VV}{\partial X^{2}}+\frac{\partial^{2}VV}{\partial Y^{2}}\right)+Ric\theta \tag{3}$$

Equation of energy

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}\,a}\left[\frac{\partial^{2}\theta}{\partial X^{2}}+\frac{\partial^{2}\theta}{\partial Y^{2}}\right] \tag{4}$$

The dimensionless variables are defined as:

$$XX=\frac{xx}{L},\quad YY=\frac{y}{L}y,\ U=\frac{u}{u_{in}},\quad V=\frac{v}{u_{in}},\ \theta=\frac{Te-Te_{in}}{T_{h}-Te_{in}}$$

$$P^{**}=\frac{pa}{\rho u_{in}^{2}},\quad \mathrm{Pr}\,a=\frac{\nu}{\alpha},\quad Ric=\frac{Gr}{\mathrm{Re}^{2}},Gr=\frac{g\beta(Te_{h}-Te_{in})L^{3}}{\nu^{2}} \tag{5}$$

The boundary conditions are:

UU=1, VV=0 and $\theta=0$ at the inlet

UU=vv=0 and $\theta=1$ at the upper wall

UU=1,vv=0 and $\theta=0$ at the bottom wall

UU=vv=0 and $\frac{\partial\theta}{\partial N}=0$ at the other walls of enclosure

$P^{**}=0$ at the outlet

The average Nusselt number on the wall that is warm is calculated using the following technique: [12]

$$N_{u}a_{av}=\int_{0}^{1}(\frac{\partial\theta}{\partial y})_{y=w}\ dx \tag{6}$$

$\text{and the bulk average temperature is defined as:}$

$$\theta_{av}=\int_{0}^{1}\frac{\theta d\underline{v}}{\underline{v}} \tag{7}$$

Where $\underline{v}$: $\text{the volume of occupying fluid in rectangular enclousre.}$

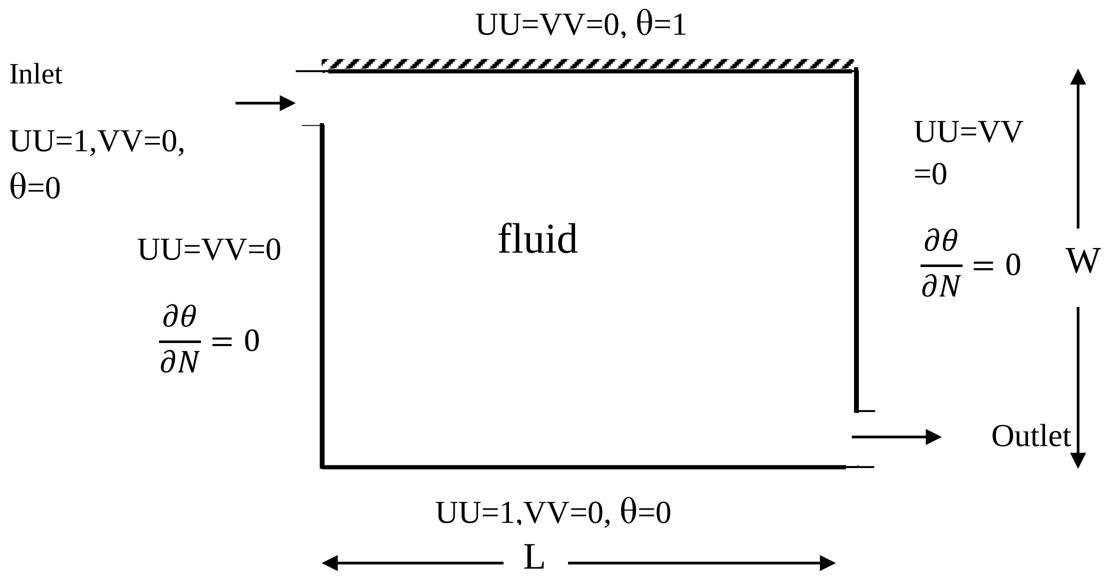


Figure 1. An illustration of the physical model's boundary conditions

Numerical Solution

Using the software program Flex PDE, the finite element method is used to numerically solve the boundary conditions and governing equations (1) to (4) [13]. Due to mass conservation, the equation of continuity (1) must be adopted as a constraint, so this restraint can be employed to determine the pressure distribution [6]. Using a penalty parameter and the equation-based compressibility criterion (1), we can solve equations (3) to (5), yielding:

$$\nabla^2 P = \gamma\left(\frac{\partial uu}{\partial x} + \frac{\partial vv}{\partial y}\right) \tag{8}$$

γ is a penalty parameter that ought to be determined either through other methods or by using physical knowledge [14]. In this study, a most practical value for was found to be (10^{11} L^{-2}). The numerical solutions are derived in terms of isotherms, streamlines. The heated surface's average Nusselt number (upper wall) is used to calculate the heat transfer coefficient.

$Nu_{a_{av}} = \int_0^1 (\frac{\partial\theta}{\partial y})_{y=0}\, dx$. Also, the bulk temperature is calculated from the relation $\theta_{av} = \int_0^1 \frac{\theta dv}{v}$

Comparison and Validation of the Study:

The geometry under study in this paper is a mixed ventilated cavity, so a number of grid size sensitivity tests, along with the continuity equation $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$, were conducted. The results showed that the distribution of velocity for the grid size determined by enforcing a precision of 10^{-3} was validated precisely. This precision represents a compromise between run-by-run time and result accuracy. Fig. (2-a)depicts the grid domain for Ri = 1, Re = 20 and Figure (2-b)shows the distribution of $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$ over the domain. The grid independency test is presented in table-1 . this table shows that the Nu_{av} becomes stable beyond grid 5 therefore, the grid 6 (nodes=4445) is adopted in this paper.

Abood et al. [15] and Rahman et al. [16] validate a computational model for mixed convection heat transfer through examining the correlation of mixed convection with uniform circulation of heat in the left side . Fig. (3) illustrates how the current investigation's flow and thermal fields compare, and Abood et al [15] and Rahman et al [16]. The findings indicate a good agreement.

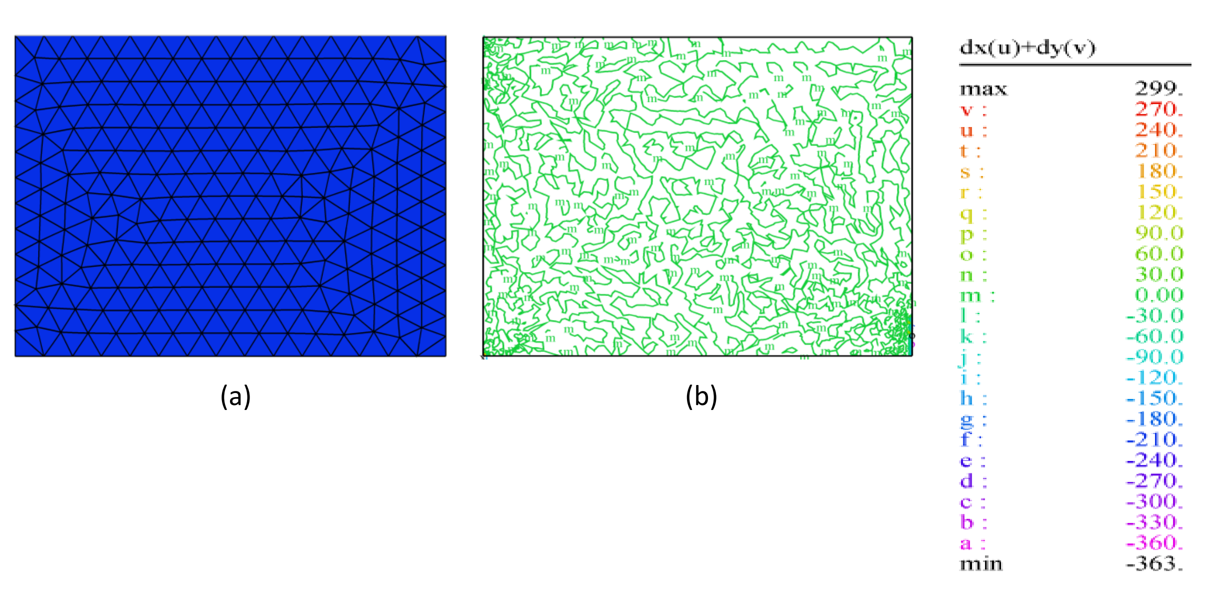


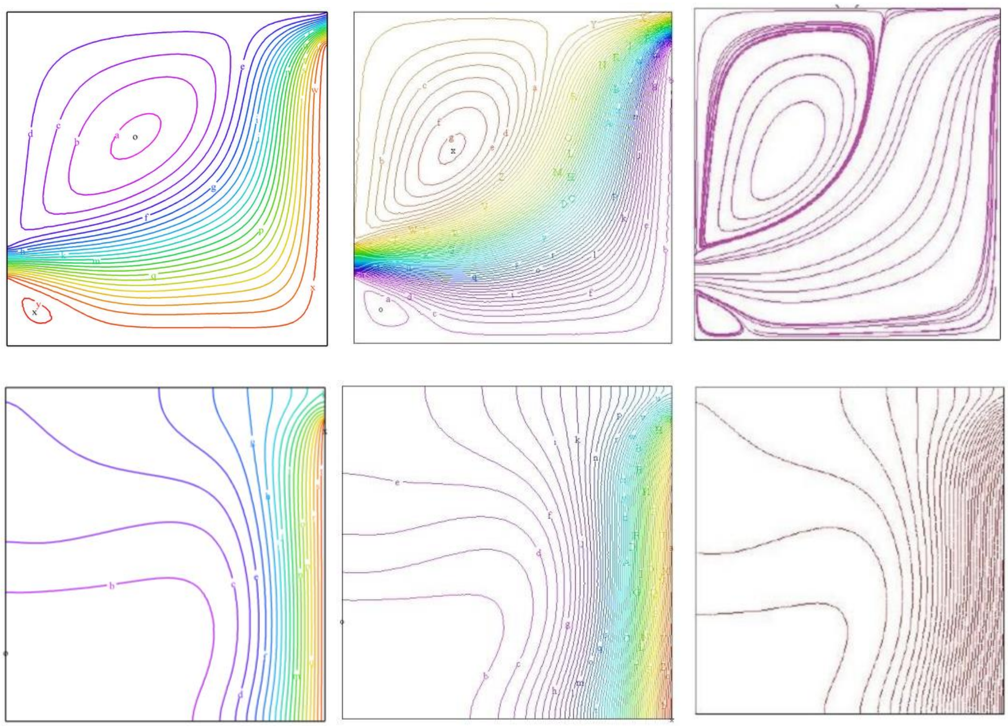
Figure 2. (a) domain-wide grid distribution (b) Verification of continuity equation

Table 1. Independency grid test

Grids	Nodes	$Nu_{av.}$		
		Air	Water	Oil
1	930	2.9212	3.3298	7.6952
2	2074	3.7942	3.7595	8.9601
3	2699	4.3006	4.1887	10.3718
4	3421	4.7522	4.6251	11.0615
5	4152	5.6539	5.69	11.948
6	4445	5.654	5.70	11.95

Streamlines

Isotherms



Present Ref. [15] Ref. [16]

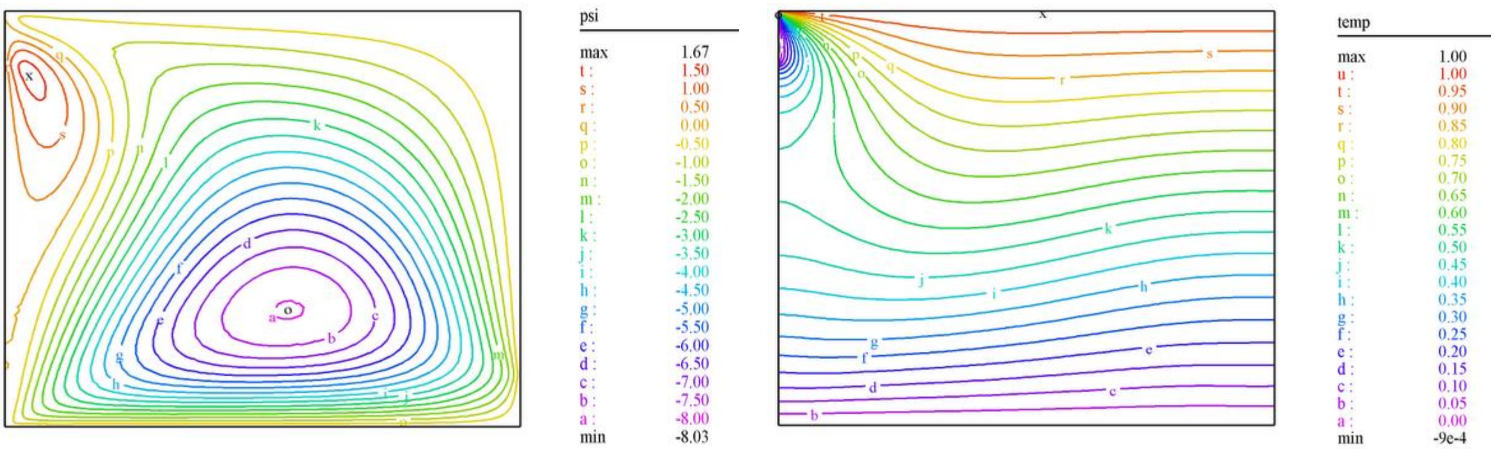
Figure 3. Validation through comparison of streamlines and isotherms at Pra=0.71, Ric=1 and Re=100 with Ref. [15] and Ref. [16]

Results and Discussion:

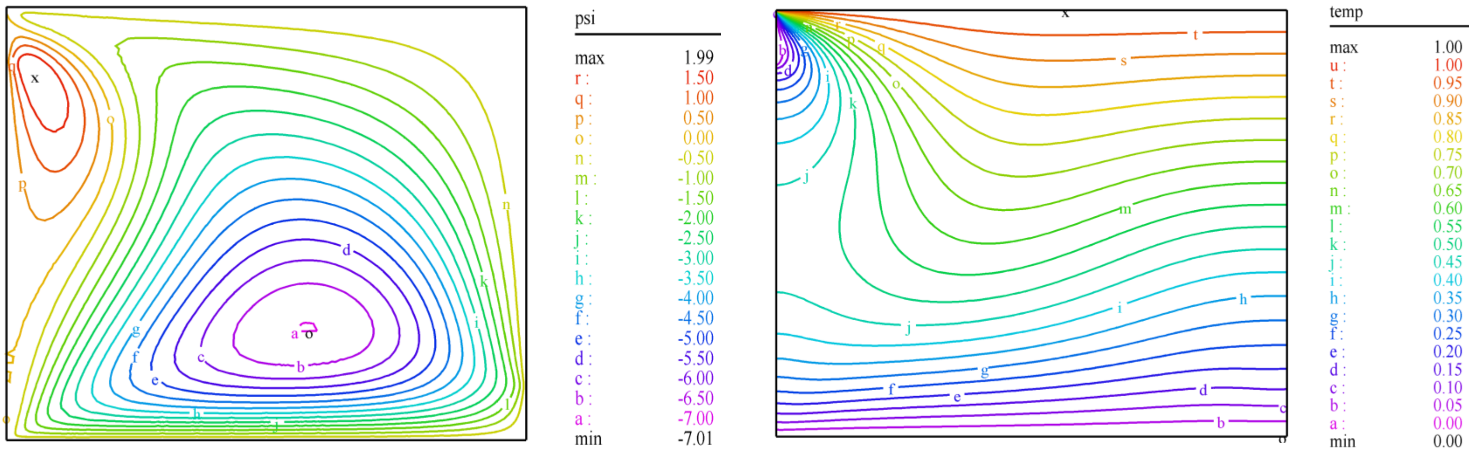
1. **Effect of Re**

Numerical simulations are run for various Pra (varying from 0.71, 1.7 and 50) to cover various working fluids (water, air, and oil) under the influence of Re, and Ric, varying from (20, 80, and 100) for Re, (1,6,10) for Ric, respectively. In the case of mixed convection, Pra affects the temperature distribution and the rate of heat transfer regine are mainly investigated. For a better layout, notice that the streamlines' and isotherms' contours are arranged horizontally.

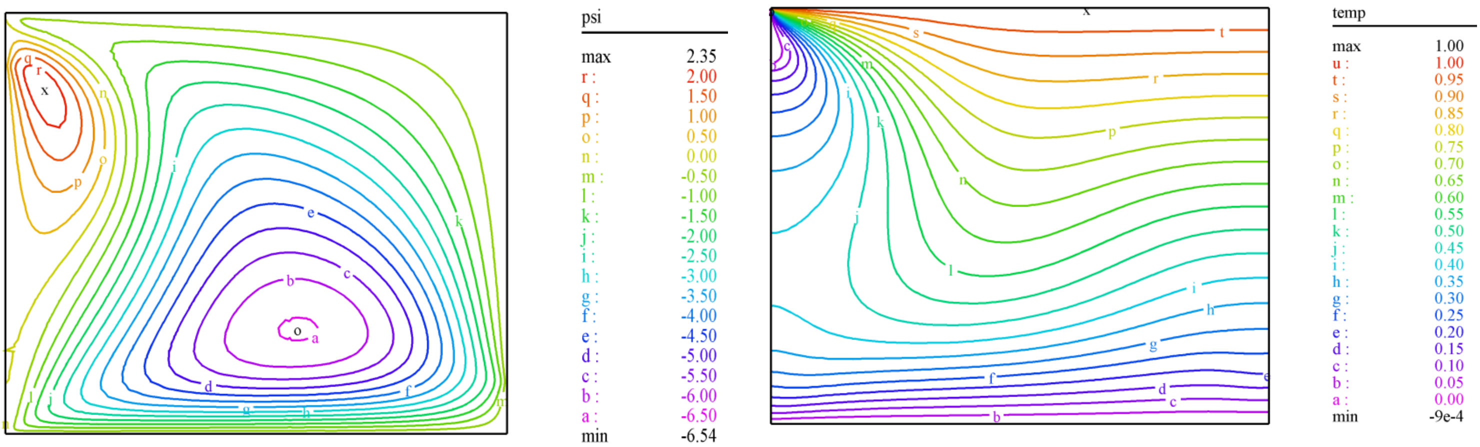
The velocity direction always aligns with the y direction and is the opposite of the gravity direction. The streamlines, isotherms lines and Nu_{av} as representations of the numerical results. The studied parameters have the following ranges: Ric= (1,6,10), Pra = (0.71,1.7,50), Re = 20–80, and 100, respectively. For air, Pra = 1.7, Ric = 1and Re = (20,50, 80). Figure 4(a, b, and c) illustrate the effect of variations in Reynolds number on streamelines (left) and isotherms (right). As evidenced by this figure that The formation of two circulating cells, one located the bottom and the other in the left upper corner. The streamelines increased with increasing Re number due to the flow circulation's impact. Additionally, this figure demonstrates a growth in the expansion of the thermal boundary layer along the top wall of the space.



(a) Re =20



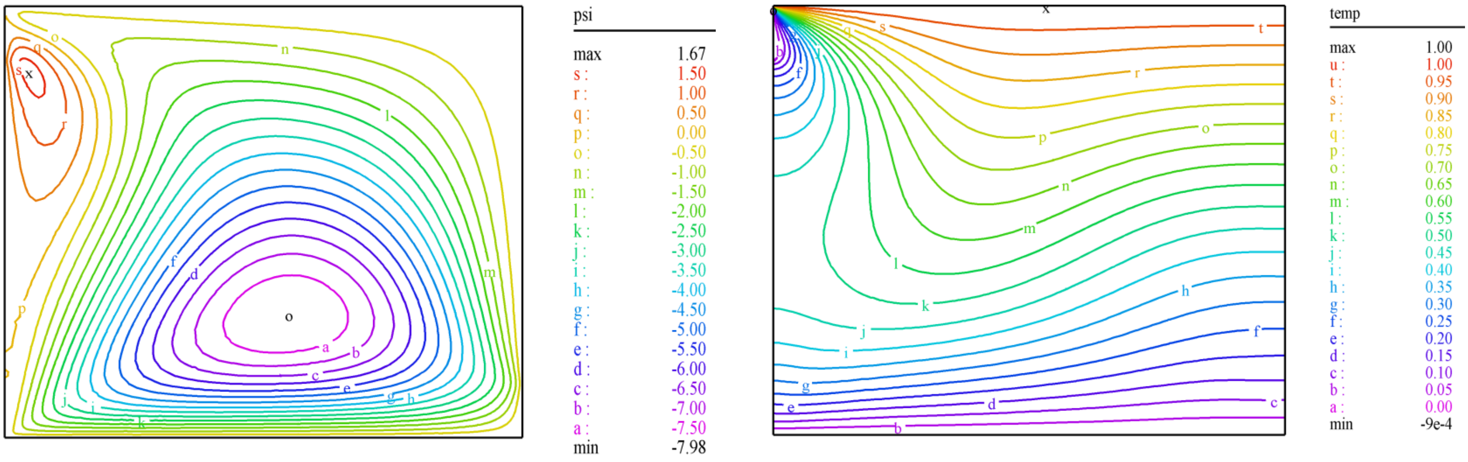
(b) Re=50



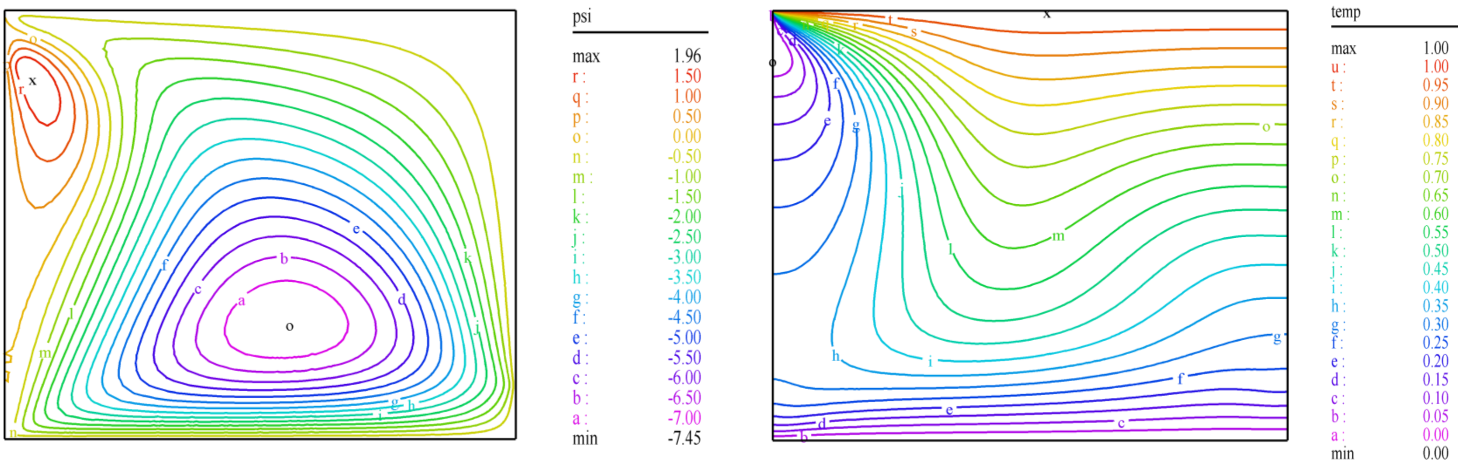
(c) Re =80

Figure 4. Streamlines (left) and isotherms (right) at Ric =1 and Pra =0.71, (a) Re =20, (b)Re=50 and (c) Re=80.

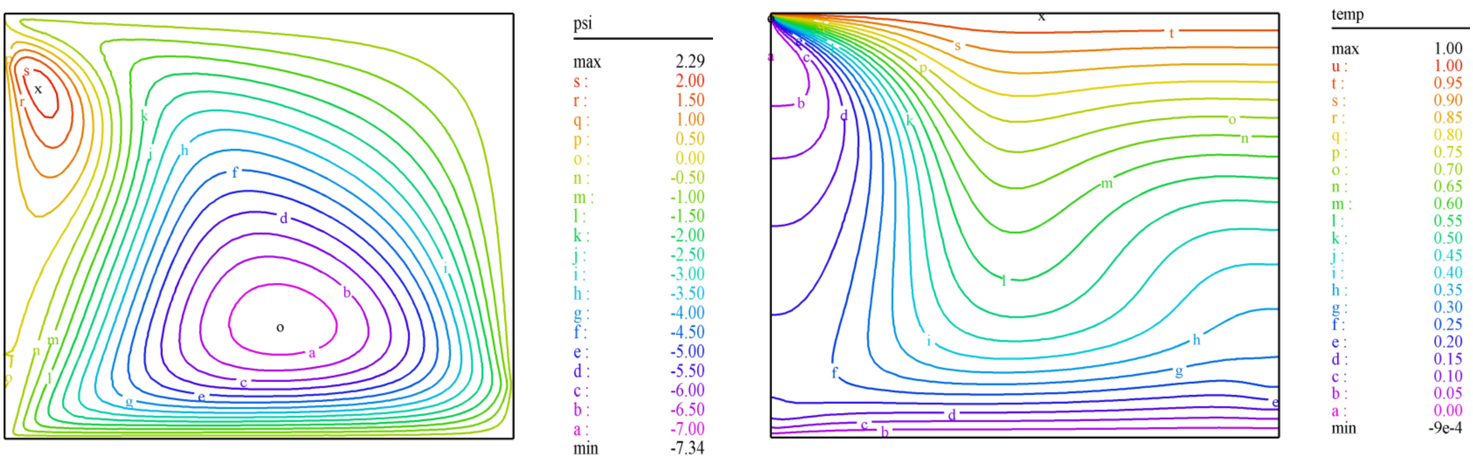
For water, Pra = 1.7, Re = 20,50, 80 and Ric =1, Figure 5(a, b, and c) illustrate the effect of variations in Reynolds number on streamelines (left) and isotherms (right). The streame lines increased with increasing Re number due to the flow circulation's impact,it can be seen the thickness of votex lines neer the bottom moved wall. Furthermore, this image shows an increase in the temperature boundary layer's expansion across the space's upper heated wall.



(a) Re =20



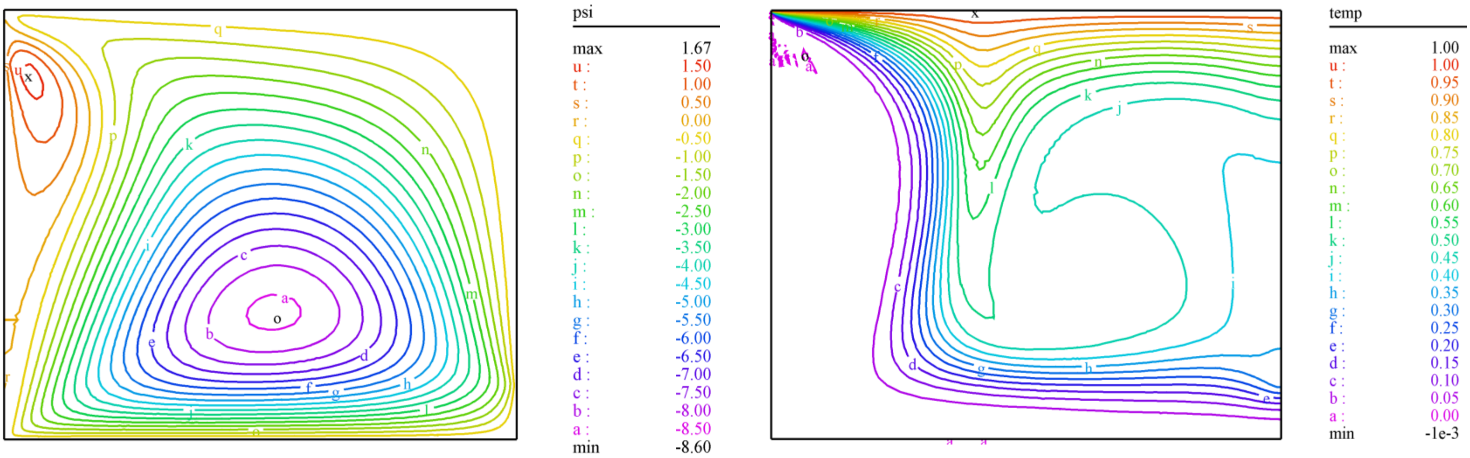
(b) Re=50



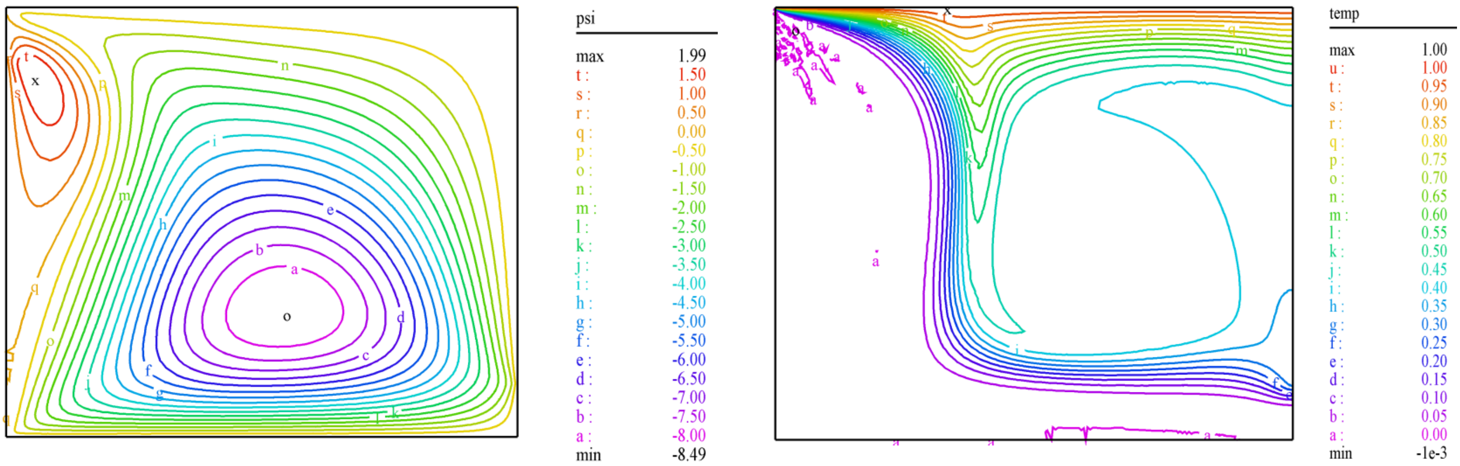
(c) Re =80

Figure 5. Streamlines (left) and isotherms (right) at Ric =1 and Pra =1.7, (a) Re =20, (b)Re=50 and (c) Re=80

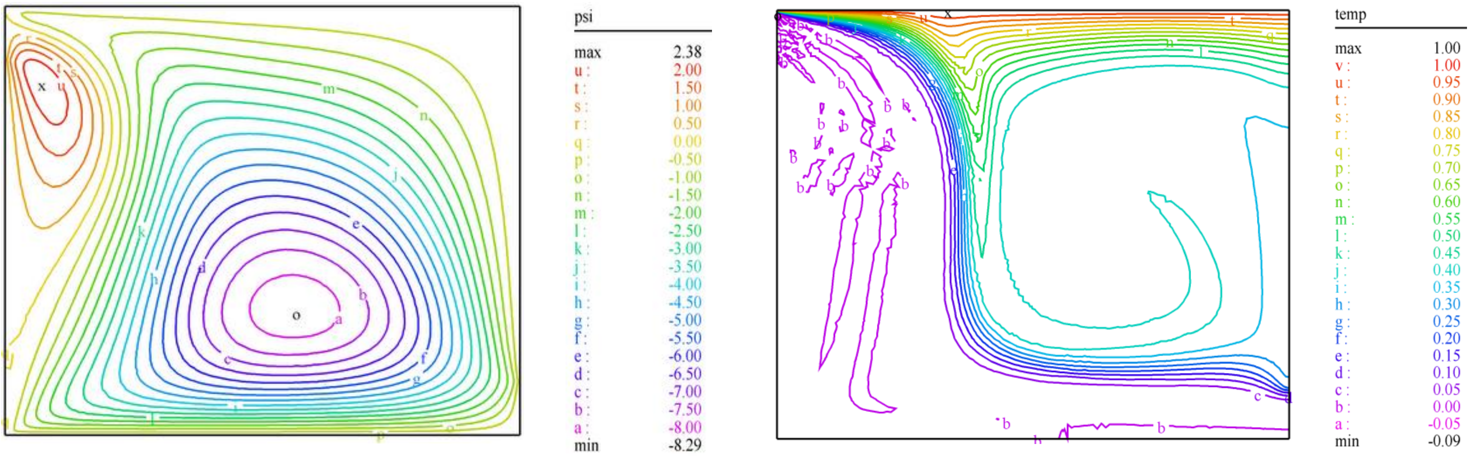
For oil, Pra =50, Re = 20,50, 80 and Ric =1, Figure 6(a, b, and c) illustrate the effect of variations in Reynolds number on streame lines (left) and isotherms (right). In steamlines contours, the secondary vortex developed near the left top corner will be grown. The streame lines increased with increasing Re number due to the flow circulation's impact. The same graph also shows an increase in the thermal boundary layer's extension along the space's upper wall. This illustrated how buoyancy increases when Re is low, resulting in a drop in air density and a flow toward the exit.



(a) Re =20



(b) Re=50

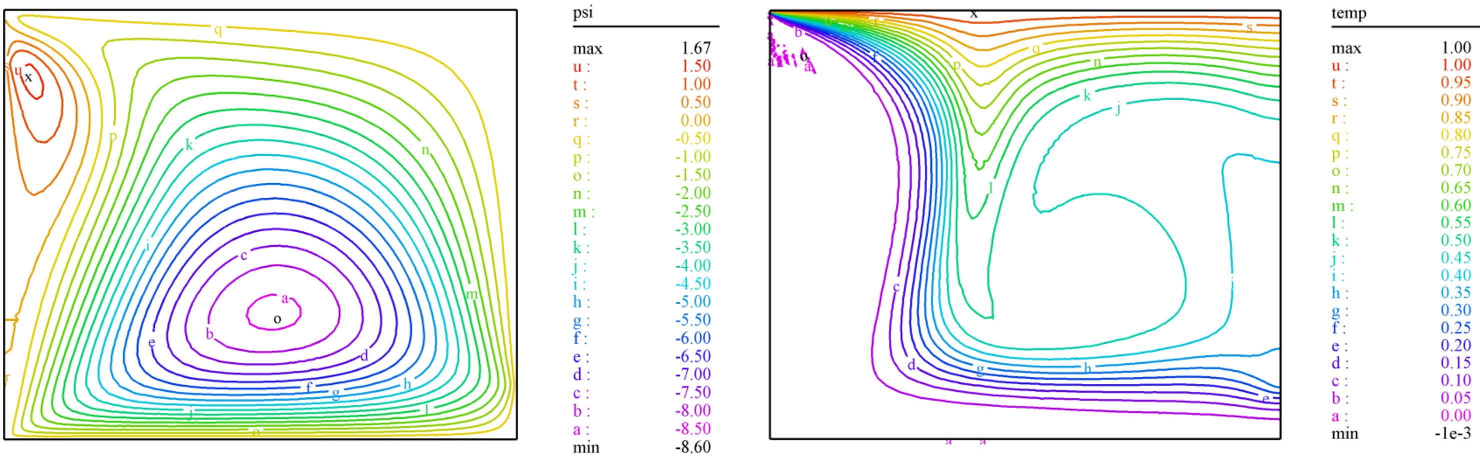


(c) Re =80

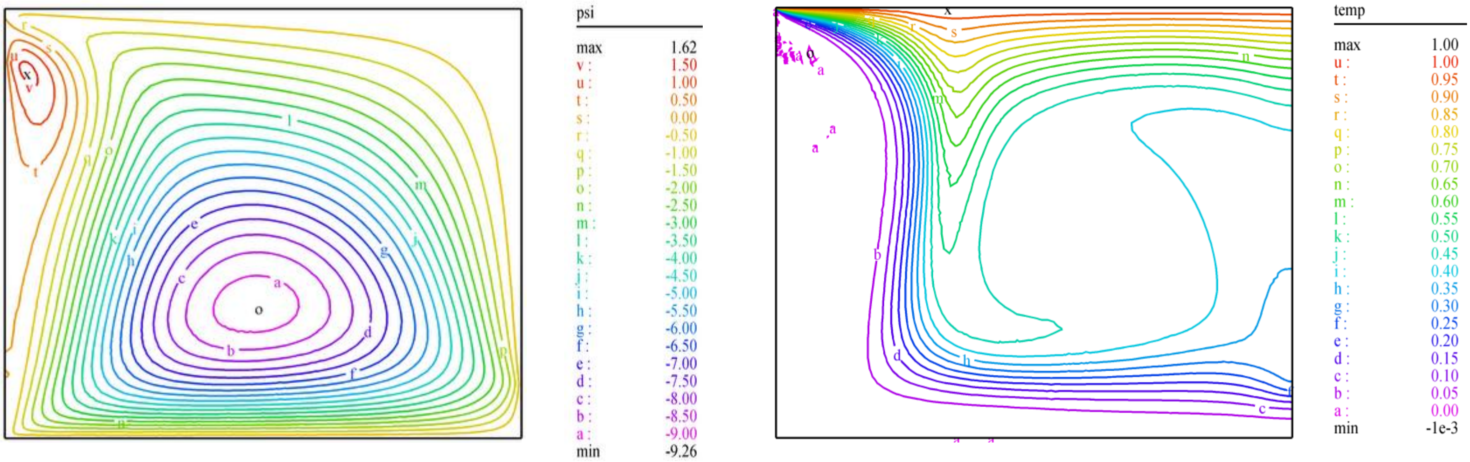
Figure 6. Streamlines (left) and isotherms (right) at Ric =1 and Pra =50, (a) Re =20, (b)Re=50 and (c) Re=80.

2. Effect of Ri

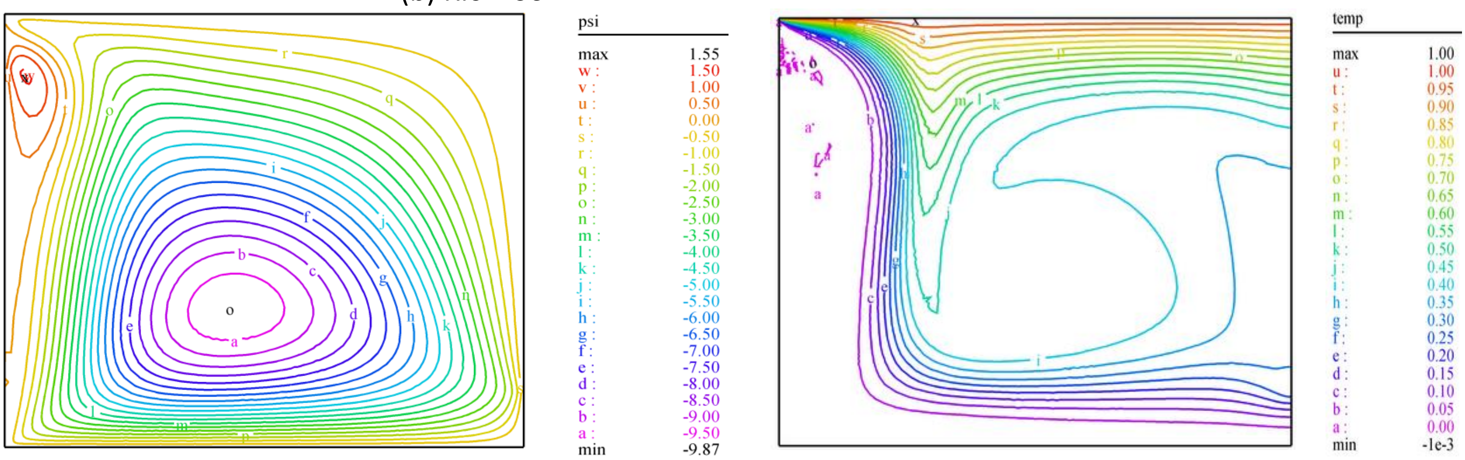
Richardson's number Ric is in order to accurately predict the weather and investigate density and turbidity currents in lakes, oceans, and reservoirs, RIC is crucial. It represents the proportion of the buoyancy term to the flow shear term as a dimensionless number. Figure 7. (a, b, and c) show the effect of different values of Ric number variation on Nua_{av} for Re=20, and Pra=50. It is evident that the Nuaav increases progressively as the Ric number increases. Due to the buoyancy effect, this increases roughly linearly.



(a) Ric = 01



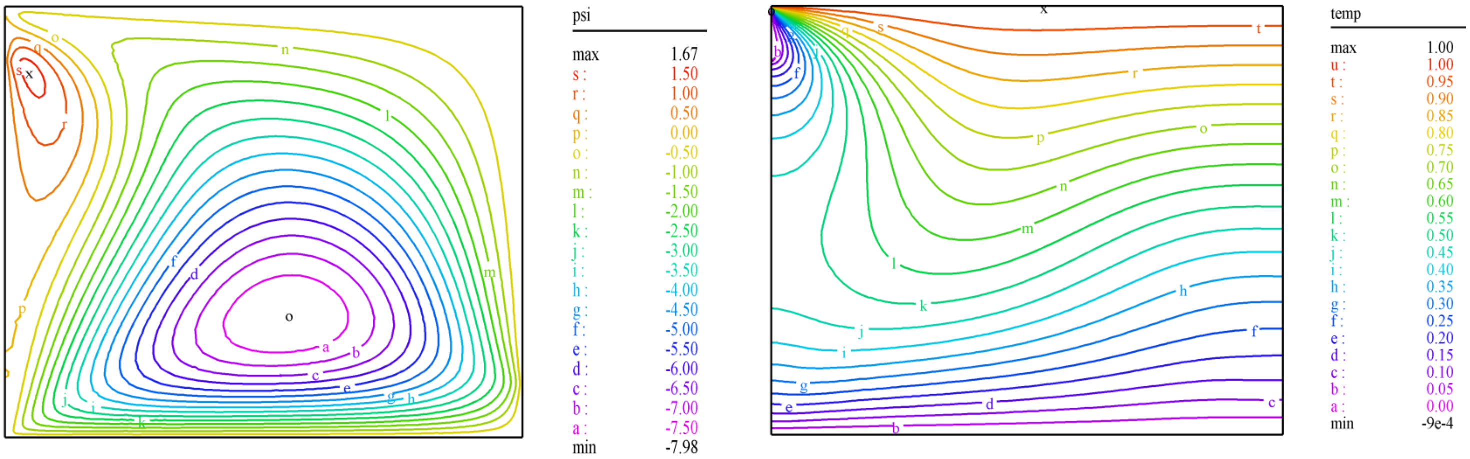
(b) Ric = 06



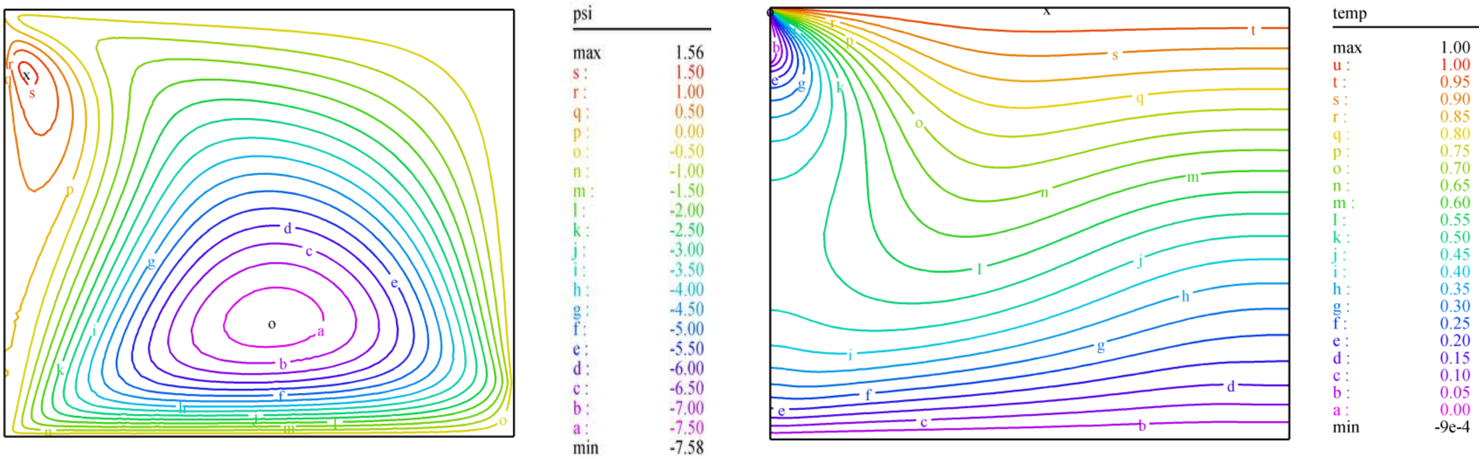
(c) Ric = 10

Figure 7. Streamlines (left) and isotherms (right) at Re =20 and Pra =50, (a) Ric =1, (b)Ric=6 and (c) Ric=10.

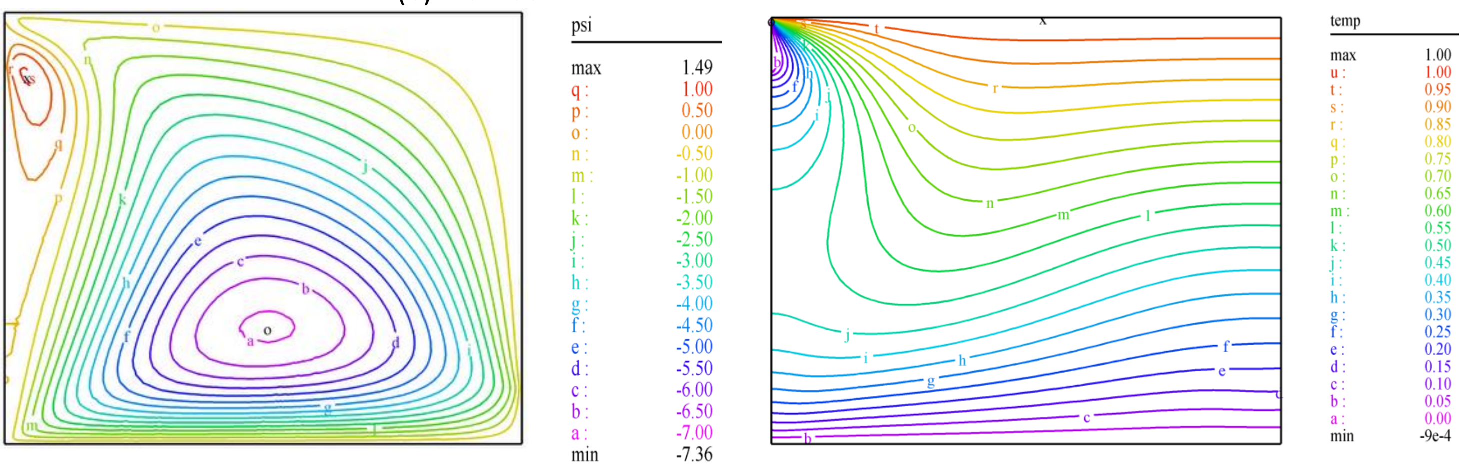
Figure 8. (a, b, and c) show the effect of different values of Ric number variation on Nu_{av} for Re=20, and Pra=1.7. It is evident that when the Ric number increases, the Nua_{av} increases progressively. This increases almost linearly as a result of the buoyancy impact.



(a) Ric = 01



(b) Ric = 06



(c) Ric = 10

Figure 8. Streamlines (left) and isotherms (right) at Re =20 and Pra =1.7, (a) Ric =1, (b)Ric=6 and (c) Ric=10.

Figure 9. (a, b, and c) show the effect of different values of Ri number variation on Nua_{av} for Re=20 , and Pra=0.71. As can be seen, the Nuav gradually increases as the number of Rics increases. This increases roughly linearly as a result of the buoyancy effect.

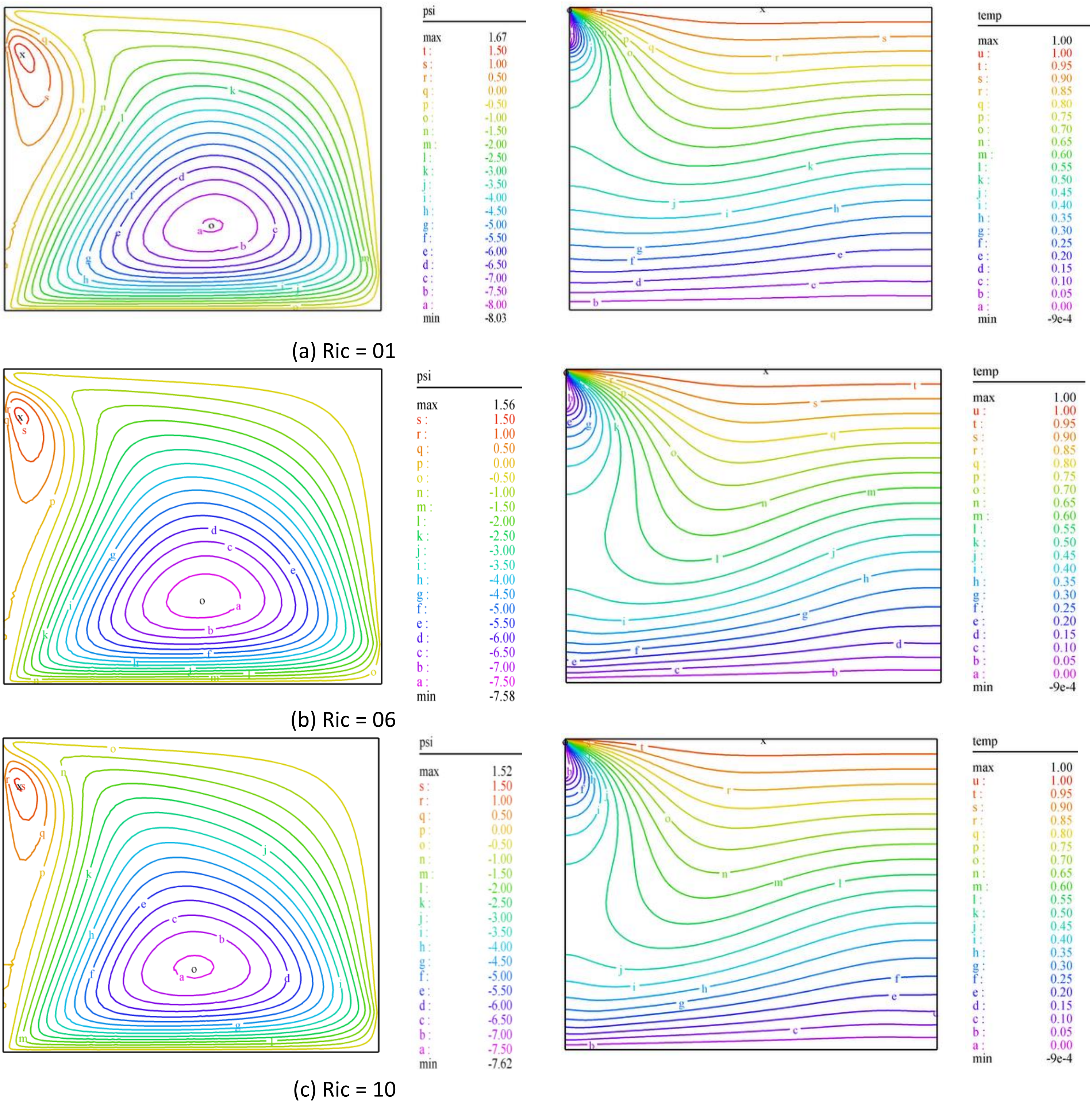


Figure 9. Streamlines (left) and isotherms (right) at Re =20 and Pra =0.7, (a) Ric =1, (b)Ric=6 and (c) Ric=10.

3. Nusselt number and average temperature

Figure 10 illustrates how the average Nusselt value is affected by the Re number. It is evident that when R_e and Prandtl numbers rise, Nuav's value also rises.The impact of Ri number on the Nu_{av}, which falls with increasing Ri number, is depicted in Fig. 11. Table 2 shows how θ_{av} varies with Re at Ri=1 and for water, oil, and air. It has been seen that as Re increases, the average temperature falls. Table 3 shows how θ_{av} varies with Ri for air, water, and oil at Re=20. Because forced convection predominates, it has been noticed that the average temperature lowers as Re increases.

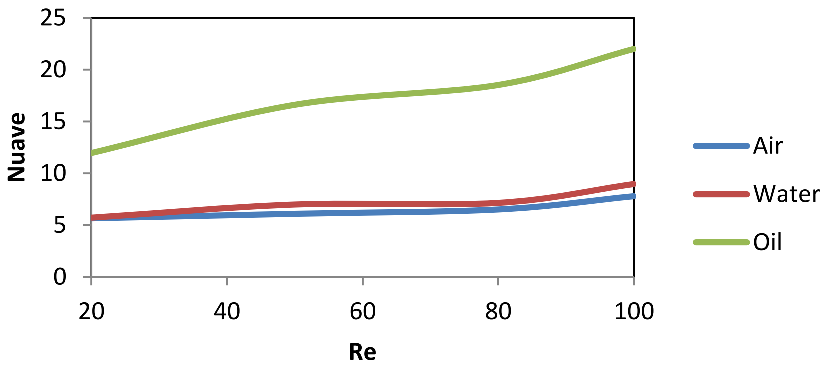


Figure 10. effect of Re number on the average of Nua for different fluids

Table 2. Variation of average fluid temperature with Reynolds number at Ric=1for different Prandtl number

Air Pra 0.71				
Re	20	50	80	100
θ_{av}	0.4217	0.4200	0.4179	0.401
Water Pra 1.7				
Re	20	50	80	100
θ_{av}	0.4154	0.3948	0.371	0.358
Oil Pra 50				
Re	20	50	80	100
θ_{av}	0.2679	0.2283	0.2057	0.192

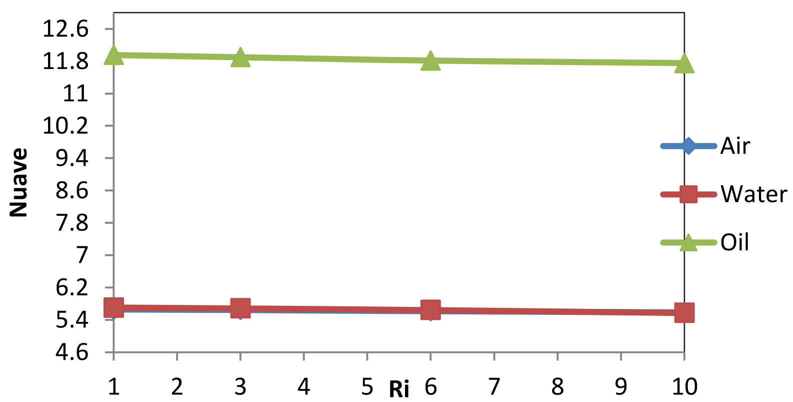


Figure 11. effect of Ric number on the average of Nua for different fluids

Table 3. Variation of average fluid temperature with Richardson number at Re=20 for different Prandtl number

Air Pra 0.71				
Ri	1	3	6	10
θ_{av}	0.4217	0.40128	0.40076	0.382
Water Pra 1.7				
Ri	1	3	6	10
θ_{av}	0.4154	0.4142	0.403	0.392
Oil Pra 50				
Ri	1	3	6	10
θ_{av}	0.2679	0.266	0.253	0.19

Conclusions

Flex PDF software has been used in this study to investigate the effects of variable Prandtl number on mixed convection of a laminar flow and heat transfer in a rectangular enclosure. By varying the Reynolds Number (20 Re 100) and the Richardson Number (1 Ric 10) fluid flows, the numerical results are examined. The following finding have been summarized:

A) The average Nusselt number is maximum at the heated surface with the highest Reynolds value. b. Nusselt numbers decline as Ric rises.

B) the value of average temperature θ_{av} decreasing with increasing Re and Ric.

C) For oil, the average Nusselt number is higher than water, and water is higher than air (due to the difference in density).

D) The average temperature θ_{av} for oil is less than that for water and air for the same value of Re.

Nomenclature

g Gravitational acceleration, meter/sec^2

W The square enclosure's height, meter

k Conductivity of thermal, Watt/meter^2.K

L The heated wall's length, meter

N Dimensionless normal distance

Nus Nusselt Number

Nus_{av} Average Nusselt Number

P^{**} A pressure without dimensions, pascal /ρ u_{in}^2

Pra Prandtl Number, υ/ α

Ric Richardson Number, Gr/Re^2

Re Reynolds Number, u_{in} L/υ

Te Fluid in the enclosure's temperature, K

UU,VV Components of non-dimensional velocity, uu/u_{in}, vv/ u_{in}

w Inlet opening height, (1/10) L

W The inlet's non-dimensional height, w/L

XX, YY coordinates that are not dimensional, XX=xx/L, YY=yy/L

Greek Symbols

α Fluid thermal diffusivity, meter^2/sec

β coefficient of thermal expansion, 1/K

θ Temperature dimensionless, $\theta=(Te-Te_{in})/(T_h-Te_{in})$

υ Kinematics viscosity, meter^2/s

ρ Density , kg/meter^3

Subscripts

h Hot

in Inlet

Reference

Esfahani, J.A.; Alinejad, J. Entropy generation of conjugate natural convection in enclusures: The lattice Boltzmann method. J. Thermophys. Heat Transf. 2013, 27, 498–505. [CrossRef]

Raisi, A., Arvin, A., 2018, "A numerical study of the effect of fluid - structure interaction on transient natural convection in an air -filled square cavity ", International Journal of Thermal Sciences, 128, pp. 1-14.

Mohamed Loukili, KamilaKotrasova, Denys Dutykh.,2020, "A Computational simulation of steady natural convection in an H -form cavity", 4th computational methods in systems and software 2020, Szczecin, Poland. https://hal.archives -ouvertes.fr/hal -02615655.

Ali, M.E.-S.; Nuhait, A.O.; Alabdulkarem, A.; Almuzaiqer, R. Free convection heat transfer inside square water-filled shallow enclosures. PLoS ONE 2018, 13, e0204251. [CrossRef]

Shobha B., Manoj K., Arvind P. "Mixed convection in four-sided lid-driven sinusoidally heated porous cavity using stream function-vorticity formulation " SN Applied Sciences (2020) 2:2066 | https://doi.org/10.1007/s42452-020-03815-7.

Olanrewaju M. Oyewola, Samuel Afolabi, Olawale S. Ismail." NUMERICAL SIMULATION OF NATURAL CONVECTION IN RECTANGULAR CAVITIES WITH DIFFERENT ASPECT RATIOS" Frontiers in Heat and Mass Transfer (FHMT), 17, 11 (2021) DOI: 10.5098/hmt.17.11.

SoudaMorsli, AminaSabeur, Mohammed El Ganaoui., 2017, "Influence of aspect ratio on the natural convection and entropy generation in rectangular cavities with wavy -wall", Energy Procedia. Vol 139, December, pp 29 -36. https://doi.org/10.1016/j.egypro.2017.11.168.

Ambethkar V, Kumar M (2017) Numerical solutions of 2-d unsteady incompressible ow with heat transfer in a driven square cavity using stream function-vorticity formulation. Int J Heat Technol 35(3):459–473

Ambethkar V, Kushawaha D (2017) Numerical simulations of uidow and heat transfer in a four-sided lid-driven rectangular domain. arXiv :1705.00707 v1 [physics.u-dyn]

Saglam, M.; Sarper, B.; Aydin, O. Natural Convection in an Enclosure with a Pair of Discrete Heat Sources. J. Thermophys. Heat Transf. 2019, 33, 234–245. [CrossRef].

Alshara A. K., Abood F. A. and Al-mayahi H. S., 2013, "Mixed Convection Heat Transfer Inside A vented Square Enclosure with Concentric Rotation Inner Cylinder "Thi-Qar university Journal for engineering Sciences ,Vol. 4,No.3, PP. 60-76.

Alshara A. K.M. , 2012, " Effect of single or multi rotating horizntal cylinder on the mixed convection heat transfer inside a triangular enclosure" Al-Qadisiya Journal for Engineering Sciences, Vol. 5, No. 1, PP. 95-110.

GunorBackstrom, 2005, "Field of physics by finite element analysis using Flex PDE " GB Publishing and Gunner Backstrom Malmo, Sweedn.

Roy S. and Basak T., 2005, " Finite element analysis of natural convection flows in square cavity with non uniformly heated wall" Int. J. Eng. Sci., 43, 668-680.

FalahAbood , Sana M. Shrama , Zainab K. Radhi , 2011,"Study of mixed convection heat transfer inside a vented square cavity with inner heated cylinder", Thi-Qar university Journal for engineering Sciences ,Vol. 2,No.3, PP. 113-130.

Rahman, M. M., Alim, M. A., Mamun, M. A. H., Chowdhury M. K. and Islam A. K. M. S., 2007, "Numerical study of opposing mixed convection in a vented enclosure", ARBN Journal of engineering and Applied Sciences, Vol. 2, No.2, PP. 25-36.