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New Cubic Transmuted Burr XII Distribution, Properties and Application

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Abstract. In this paper, We propose a new lifetime distribution with more flexibility called the New Cubic Transmuted Burr XII (NCTBXII) distribution. develop the proposed distribution on the basis of the cubic ranking transmutation map proposed by Granzotto et al. [1] . The distributional properties as (r^{th}) moment, moment generating function, Characteristic function, reliability function and hazard rate are discussed for the proposed distribution. We derive representation for the order statistics.The maximum likelihood estimation of the model parameters is described. Applications of the proposed model and their fit are illustrated with data set and the corresponding diagnostic analyses are also provided.

INTRODUCTION

When the experiment is, a number of uncertainty and models occur. Different distributions are used for. When they were previously implemented, the standard and the only known distributions are not enough to eliminate these doubts It is unable to a good model for different data collections. However, in recent years are different get one more convenient distributions for mixed data groups using methods enabled. For example, with mixed distributions, converted components and distributions. New distributions are more flexible than other distributions. Thus, they are one of the important practices of statistical theory to develop new distributions. Many statisticians in the literature strive to generalize some existing distributions by introducing new parameters to the base distributions. The idea behind this is to make these distributions more flexible and more practicable, and thus to find the true nature of the data and how it behaves.

The statistical literature contains many of continuous univariate distributions which have several applied sciences such as engineering, lifetime analysis, reliability, economics, medical, actuarial , demography, finance and insurance studies. These applications have shown that data sets following the well-known models are more often the exception rather than the reality.

Shaw and Buckley (2009) proposed ranking quadratic transmutation map to solve financial problems

Quadratic Ranking Transmutation Map

Theorem 1.1: Let X_1 and X_2 be non negative random variables denoting lifetimes of two individuals in some population with are independent and identically distributed with distribution $G(x)$. Then, according to Shaw and Buckley [2], a ranking quadratic transmutation map has the following simple form

$$F(x) = (1+\lambda)G(x) - \lambda[G(x)]^2, \lambda \in [-1,1] \tag{1}$$

Proof

Let X_1 and X_2 be *i.i.d* random variables with common cumulative distribution function $G(x)$. Then,consider $X_{1:2} = min(X_1,X_2)$ and $X_{2:2} = \max(X_1,X_2)$

Let$Y =^d X_{1:2}$ with probability π , and $Y = X_{2:2}$ with probability 1-π, where $0 \leq \pi \leq 1$.

The cumulative distribution function of Y is

$$F_Y(x) = \pi P(X_{1:2} \leq x) + (1-\pi)P(X_{2:2} \leq x)$$

$$F_Y(x) = \pi[1-[1-G(x)]^2] + (1-\pi)[G(x)]^2$$

$$F_Y(x) = 2\pi G(x) + (1-2\pi)[G(x)]^2 \tag{2}$$

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if we let $\lambda = 2\pi - 1$, The distribution in equation (2) are called transmuted distribution or ranking quadratic transformation map as proposed in equation (1).

Cubic Ranking Transmutation Map

Theorem 1.2: Let X_1 , X_2 and X_3 be *i.i.d* random variables with common cumulative distribution function $G(x)$. Then,the cubic rank transformation map is

$$F(x) = \lambda_1 G(x) + (\lambda_2 - \lambda_1)[G(x)]^2 + (1-\lambda_2)[G(x)]^3, \lambda_1 \in [0,1], \lambda_2 \in [-1,1] \tag{3}$$

Proof

Consider the following order statistics

$X_{1:3}$ = min(X_1,X_2,X_3) ,$X_{2:3}$ and $X_{3:3}$ = max(X_1,X_2,X_3)

Let $Y =^d X_{1:3}$ with probability π_1 , $Y = X_{2:3}$ with probability π_2,and $Y = X_{3:3}$ with probability π_3,

where $0 \le \pi_i \le 1$, π_3=1-π_1-$\pi_2$and$\sum_{i=1}^{3} \pi_i$.

The cumulative distribution function of$Y$is

$$F_Y(x) = \pi_1 P(X_{1:3} \le x) + \pi_2 P(X_{2:3} \le x) + 1 - \pi_1 - \pi_2 P(X_{3:3} \le x),$$

where

$$P(X_{1:3} \le x) = 1 - [1-G(x)]^3, P(X_{2:3} \le x) = 3[G(x)]^2 - 2[G(x)^3, P(X_{3:3} \le x) = [G(x)]^3$$

Now,the cumulative distribution function of Y is write as follow:

$$F_Y(x) = 3\pi_1 G(x) + 3(\pi_2 - \pi_1)[G(x)]^2 + (1-2\pi_2)[G(x)]^3 \tag{4}$$

if we let $\lambda_1 = 3\pi_1, \lambda_2 = 3\pi_2$ he distribution in equation (4) are called transmuted distribution of order 2 or cubic transformation map as proposed in equation(3).

Definition 1.1: The (cdf) of cubic rank transmuted distribution are givens in equation (3) by derivative for x we can find the (pdf) of the distribution as:

$$f(x) = g(x)[\lambda_1 + (\lambda_2 - \lambda_1)[G(x)] + (1-\lambda_2)[G(x)]^2, x > 0 \tag{5}$$

Granzotto et al. (2017) [1] proposed a cubic ranking transmutation map and its studied different properties. They studied properties of cubic rank transmuted Weibull distribution and cubic rank transmuted log logistic distribution.Afify et al. (2017) proposed the beta transmuted-H family of distributions [3]. Al-Kadim and Mohammed (2017) [4] presented the cubic transmuted Weibull distribution in terms of basic mathematical properties . Rahman et al. (2018) [5] developed general family of transmuted distributions.sara (2018) [6]studied properties of cubic rank transmuted kumaraswamy distribution. Riffi (2019) [7] presented higher rank transmuted families of distributions. Rahman et al. (2019) [8] studied properties of cubic rank transmuted Weibull distribution.

The Burr XII (BXII) distribution originally proposed by Burr (1942) (see [9]) has many applications in different areas including reliability, failure time modeling and acceptance sampling plans. [10] Introduced transmuted Burr XII (TBXII) distribution. [11] proposed cubic transmuted Burr XII (CTBXII) distribution. [12] studied the BXII model and its related models, namely: Pareto II (Lomax), log-logistic, compound Weibull gamma and Weibull exponential distributions

The main target of this article is to increase the applicability of the Burr XII distribution specially in the area of household income, environmental, biology, engineering, reliability, insurance and other areas of life. For doing so, a second-order Burr XII distribution is introduced that can capture the complex behavior in the real-life datasets. The layout plan of the article: In Section 2, the new cubic transmuted Burr XII distribution is introduced. Some of the distributional properties are described in Section 3 along with the distributions of the different order statistics in Section 4. The maximum likelihood estimation of the model parameters is described in Section 5. Section 6 describes two real-life applications of the proposed distribution. Some concluding remarks are listed at the end.

NCTBXII DISTRIBUTION

The Burr XII distribution was first introduced in the literature by Burr [9], which has the distribution function as

$$G(x) = 1-(1+x^{\beta})^{-\alpha-1}, x>0 \tag{6}$$

Where $\alpha > 0 and \beta > 0$ are both shape parameters.Further detail studying the distribution ,see [13]

Maurya et al. [10], use (6) in (1) and developed transmuted Burr XII distribution (TBXII) which has the following cumulative distribution function

$$F(x) = 1+[(\lambda-1)(1+x^{\beta})^{-\alpha}-\lambda(1+x^{\beta})^{-2\alpha}], x>0 \tag{7}$$

Where $\alpha > 0, \beta > 0$ and $\lambda \in [-1,1]$

the distribution function of a cubic transmuted family proposed by Rahman et al. [14], which is written as

$$F(x) = (1-\lambda)G(x)+3\lambda[G(x)]^2-2\lambda[G(x)]^3, x>0 \tag{8}$$

Akhtar et .al [11], use (6) in (8) and developed cubic transmuted Burr XII distribution (CTBXII) which has the following cumulative distribution function

$$F(x) = (1+x^{\beta})^{-3\alpha}((1+x^{\beta})^{\alpha}-1)[\lambda((1+x^{\beta})^{\alpha}-2)+(1+x^{\beta})^{2\alpha}], x>0 \tag{9}$$

Where $\alpha > 0, \beta > 0$ and $\lambda \in [-1,1]$

Here, the NCTMBXII distribution is introduced with use of (6) and (3). The (cdf) and (pdf) of the NCTBXII distribution are given, respectively by

$$F(x) = \lambda_1[1-(1+x^{\beta})^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^{\beta})^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^{\beta})^{-\alpha-1}]^3, x>0 \tag{10}$$

and

$$f(x) = \alpha\beta x^{\beta-1}(1+x^{\beta})^{-3\alpha-1}[3-3\lambda_2-(1+x^{\beta})^{2\alpha}(3-\lambda_1-\lambda_2)+2(1+x^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)], x>0 \tag{11}$$

Where $\alpha > 0, \beta > 0, \lambda_{\in}[0,1]$ and $\lambda_2 \in [-1,1]$

In future, the pdf in (11) is denoted by X$\sim NCTBXII(\alpha,\beta,\lambda_1,\lambda_2)$

Structural Properties

For X$\sim NCTBXII(\alpha,\beta,\lambda_1,\lambda_2)$ the survival, hazard,and cumulative hazard are given respectively by

$$S(x) = 1-[\lambda_1[1-(1+x^{\beta})^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^{\beta})^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^{\beta})^{-\alpha-1}]^3], x\geq 0 \tag{12}$$

$$h(x) = \frac{\alpha\beta x^{\beta-1}(1+x^{\beta})^{-3\alpha-1}[3-3\lambda_2-(1+x^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)]}{1-[\lambda_1[1-(1+x^{\beta})^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^{\beta})^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^{\beta})^{-\alpha-1}]^3]} \tag{13}$$

$$H(x) = -log[1-[\lambda_1[1-(1+x^{\beta})^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^{\beta})^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^{\beta})^{-\alpha-1}]^3]], x>0 \tag{14}$$

Shapes of the NCTBXII Density and Cumulative Functions

The following graphs show that shapes of NCTBXII density are arc, exponential, positively skewed, and symmetrical (Fig.1).And the NCTXIII cumulative distribution (cdf) are shown in (Fig. 2).

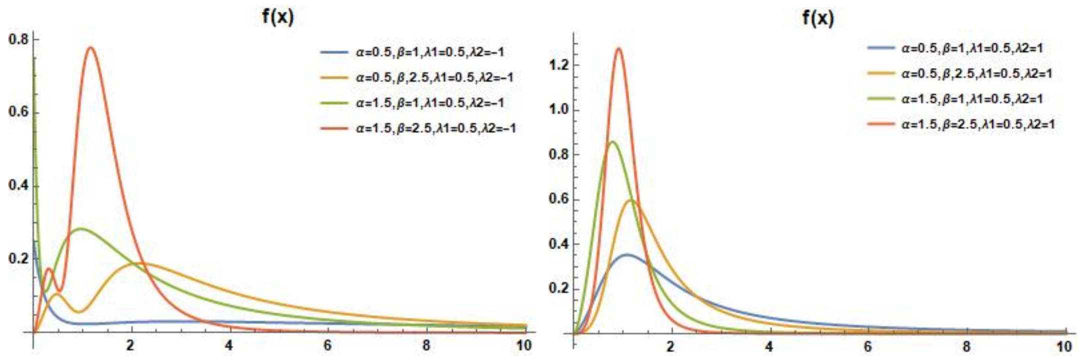

FIGURE 1. The density function of NCTBXII

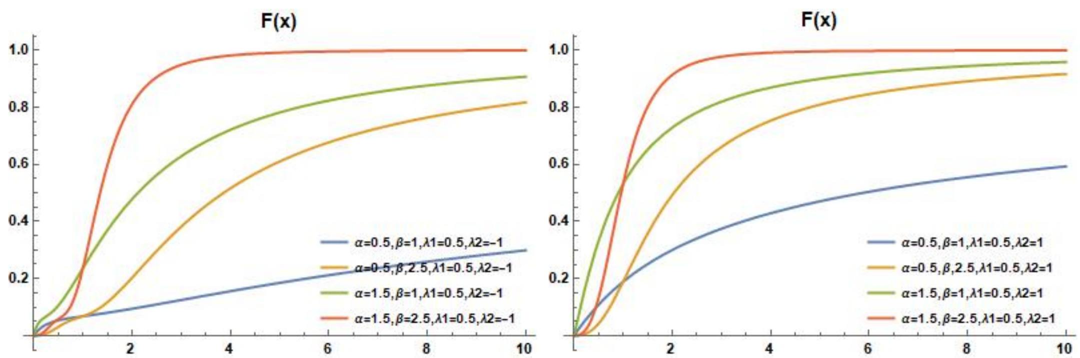

FIGURE 2. The cumulative function of NCTBXII

MATHEMATICAL PROPERTIES

Moments

The rth raw moments of the proposed NCTBXII distribution is obtained as

$$\mu_r' = E(x^r) = \int_0^\infty x^r f(x)dx$$

$E(x^r) = \int_0^\infty \alpha\beta x^{\beta+r-1}(1+x^\beta)^{-3\alpha-1}[3-3\lambda_2+(1+x^\beta)^{2\alpha}(3-\lambda_1-\lambda_2)+2(1+x^\beta)^\alpha(\lambda_1+2\lambda_2-3)]dx$
letting $x^\beta = y, x = y^{\frac{1}{\beta}}, dx = \frac{1}{\beta}y^{\frac{1}{\beta}-1}dy, then$
$\mu_r' = (3-\lambda_1-\lambda_2)\alpha B(1+\frac{r}{\beta},\alpha-\frac{r}{\beta})+(\lambda_1+2\lambda_2-3)(2\alpha)B(1+\frac{r}{\beta},2\alpha-\frac{r}{\beta})+(1-\lambda_2)(3\alpha)B(1+\frac{r}{\beta},3\alpha-\frac{r}{\beta})$

$$\mu_r' = \Gamma(1+\frac{r}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{r}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{r}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{r}{\beta})}{\Gamma(2\alpha)}], r = 1,2,.... \quad (15)$$

where $\Gamma(.)$ is gamma function.

Mean and Variance of the NCTBXII distribution are

$$E(x) = \Gamma(1+\frac{1}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{1}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{1}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{1}{\beta})}{\Gamma(2\alpha)}]$$

$$var(x) = \Gamma(1+\frac{2}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{2}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{2}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{2}{\beta})}{\Gamma(2\alpha)}]$$

$$-(\Gamma(1+\frac{1}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{1}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{1}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{1}{\beta})}{\Gamma(2\alpha)}])^2$$

Note that can obtain all the higher moments by using r > 2 in equation (15)

Moment Generating Function

The moment generating function for the proposed NCTBXII distribution is stated by the following theorem.

Theorem 1. Let a continuous random variable X follows the NCTBXII distribution, then the moment generating function $M_x(t)$ of X is

$$M_x(t)=\sum_{i=1}^{n}\frac{t^r}{r!}.\Gamma(1+\frac{r}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{r}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{r}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{r}{\beta})}{\Gamma(2\alpha)}]$$

Proof

The moment generating function is defined as

$$E[e^{tx}]=\int_0^\infty e^{tx}f(x)dx$$

where $f(x)$ is given in (11). Using the series representation of e^{tx} given by Gradshteyn and Ryzhik [15], we have

$$M_x(t)=\int_0^\infty\sum_{i=0}^{n}\frac{t^r}{r!}.x^r f(x)dx=\sum_{i=1}^{n}\frac{t^r}{r!}E(x^r) \tag{16}$$

Using $E(X^r)$ from (15) in (16), we have the moment generating function $M_x(t)$.

Characteristic Function

The characteristic function for the proposed NCTBXII distribution is stated by the following theorem.

Theorem 2. Let a continuous random X follows the NCTBXII distribution, then the characteristic function $\phi_x(t)$ is

$$\phi_x(t)=\sum_{i=0}^{n}\frac{(ti)^r}{r!}.\Gamma(1+\frac{r}{\beta})[(3-\lambda_1-\lambda_2)\frac{\Gamma(\alpha-\frac{r}{\beta})}{\Gamma(\alpha)}+2(\lambda_1+2\lambda_2-3)\frac{\Gamma(3\alpha-\frac{r}{\beta})}{\Gamma(3\alpha)}+3(1-\lambda_2)\frac{\Gamma(2\alpha-\frac{r}{\beta})}{\Gamma(2\alpha)}]$$

where,$i=\sqrt{-1}$ is the imaginary unit and $t\in R$.

Proof. The proof is simple as moment generating function.

Order Statistics

Order statistics (OS) have wide applications in climatology, life testing and reliability. Moments of OS are also designed for replacement policy with the prediction of failure of future items determined from few early failures. Let $X_1,...,X_n$ be a random sample from the NCTBXII model and let $X_{1:n},,...,X_{n:n}$ be the corresponding order statistics. The pdf of the ith order statistic, say $X_{i:n}$, is given by

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}[\lambda_1[1-(1+x^\beta)^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^\beta)^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^\beta)^{-\alpha-1}]^3]^{i-1}$$
$$[1\text{-}(\lambda_1[1-(1+x^\beta)^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^\beta)^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^\beta)^{-\alpha-1}]^3)]^{n-i}$$
$$[\alpha\beta x^{\beta-1}(1+x^\beta)^{-3\alpha-1}[3-3\lambda_2-(1+x^\beta)^{2\alpha}(3-\lambda_1-\lambda_2)+2(1+x^\beta)^\alpha(\lambda_1+2\lambda_2-3)]]$$

where $r=1,2,,n$. Using $i=1$, obtain the density function of lowest order statistic$X_{1:n}$, and is given as

$$f_{1:n}(x)=n[1-(\lambda_1[1-(1+x^\beta)^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^\beta)^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^\beta)^{-\alpha-1}]^3)]^{n-1}$$
$$[\alpha\beta x^{\beta-1}(1+x^\beta)^{-3\alpha-1}[3-3\lambda_2-(1+x^\beta)^{2\alpha}(3-\lambda_1-\lambda_2)+2(1+x^\beta)^\alpha(\lambda_1+2\lambda_2-3)]],$$

also for using $i=n$, the density function of highest order statistic$X_{n:n}$, is obtain by

$$f_{n:n}(x)=n[\lambda_1[1-(1+x^\beta)^{-\alpha-1}]+(\lambda_2-\lambda_1)[1-(1+x^\beta)^{-\alpha-1}]^2+(1-\lambda_2)[1-(1+x^\beta)^{-\alpha-1}]^3]^{n-1}$$
$$[\alpha\beta x^{\beta-1}(1+x^\beta)^{-3\alpha-1}[3-3\lambda_2-(1+x^\beta)^{2\alpha}(3-\lambda_1-\lambda_2)+2(1+x^\beta)^\alpha(\lambda_1+2\lambda_2-3)]]$$

MAXIMUM LIKELIHOOD ESTIMATION

In this section, the estimation of the model parameters for the proposed NCTBXII distribution has been conducted. This was down by using maximum likelihood estimation technique. For doing this, consider a random sample$x_1, x_2, , x_n$of size n from the proposed NCTBXII distribution, which has the likelihood function as

$$L = \prod_{i=1}^{n}[\alpha\beta x_i^{\beta-1}(1+x_i^{\beta})^{-3\alpha-1}[3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)]]$$

and the equivalent log-likelihood function $l = log(L)$ is

$$\begin{aligned} Logl &= nlog[\alpha\beta]+(\beta-1)\sum_{i=1}^{n} log[x_i]-(3\alpha+1)\sum_{i=1}^{n} log[1+x_i^{\beta}] \\ &+\sum_{i=1}^{n} log[3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)] \end{aligned} \tag{17}$$

The maximum likelihood estimates of α, β, λ_1 and λ_2 are obtained by maximizing the log-likelihood function given in (12). For doing so, taking the derivatives with respect to unknown parameters and further proceed as

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - 3\sum_{i=1}^{n} log[1+x_i^{\beta}] - \sum_{i=1}^{n} \frac{2(log[1+x_i^{\beta}](1+x_i^{\beta})^{2\alpha}(\lambda_1+\lambda_2-3)-log[1+x_i^{\beta}](1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3))}{3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)} \tag{18}$$

$$\begin{aligned} \frac{\partial l}{\partial \beta} &= \frac{n}{\beta} - \sum_{i=1}^{n} log[x_i] - (3\alpha+1)\sum_{i=1}^{n} \frac{log[x_i]x_i^{\beta}}{1+x_i^{\beta}} \\ &- \sum_{i=1}^{n} \frac{2\alpha(log[x_i]x_i^{\beta}(1+x_i^{\beta})^{2\alpha-1}(\lambda_1+\lambda_2-3)-log[x_i]x_i\beta(1+x_i^{\beta})^{\alpha-1}(\lambda_1+2\lambda_2-3))}{3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)} \end{aligned} \tag{19}$$

$$\frac{\partial l}{\partial \lambda_1} = \sum_{i=1}^{n} \frac{2(1+x_i^{\beta})^{\alpha}-(1+x_i^{\beta})^{2\alpha}}{3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)} \tag{20}$$

$$\frac{\partial l}{\partial \lambda_2} = \sum_{i=1}^{n} \frac{4(1+x_i^{\beta})^{\alpha}-(1+x_i^{\beta})^{2\alpha}-3}{3-3\lambda_2-(1+x_i^{\beta})^{3\alpha}(\lambda_1+\lambda_2-3)+2(1+x_i^{\beta})^{\alpha}(\lambda_1+2\lambda_2-3)} \tag{21}$$

Now setting $\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \beta} = 0, \frac{\partial l}{\partial \lambda_1} = 0, \frac{\partial l}{\partial \lambda_2} = 0$ and solving the resulting non-linear system of equations gives the maximum likelihood estimate $(\hat{\alpha}, \hat{\beta}, \hat{\lambda_1}, \hat{\lambda_2})$,Hence, theoretical solution is very much complex even sometimes impossible for this nonlinear set of equations. In order to get the numerical solution, apply Mathematica.

APPLICATION

In order to check the applicability, real-life application have been conducted for the proposed NCTBXII distribution, which are described by the following dataset. This dataset represents the life of fatigue fracture of Kevlar 373/ epoxy subjected to constant pressure at 90% until all had failed. The data was extracted from [16] and it has previously been used by Barlow et al. [17]. The summary statistics of the dataset are given in Table 1, and observed that it has positively skewed distribution.

TABLE 1. The summary of the fatigue fracture dataset

Data	Min	Median	Mean	Max
Fatigue Fracture	0.0251	1.7362	1.9592	9.096

TABLE 2. MLE of the parameters and respective SE for selected distribution along with proposed NCTBXII distribution

Distribution	Parameters	Etimate
NCTBXXII	α	1.5999
	β	1.4945
	λ_1	0.4990
	λ_2	-0.9044
CTBXII	α	0.6609
	β	1.7377
	λ	0.7227
TBXII	α	0.3328
	β	2.2306
	λ	0.9999
BXII	α	0.6656
	β	2.2306

In order to asses the practicality of the proposed NCTBXII distribution, several other distributions like CTBXII, TBXII and BXII are selected. As first step, estimate the model parameters of the proposed model along with selected models, and estimated values are presented in Table 2.

The estimated plots for the selected models along with the proposed NCTBXII distribution are plotted over the empirical pdf and cdf plots, and presented in the Fig. 3.

Hence, from the figures, observed that the dataset fitted well with the proposed distribution as compared with other selected models. Critically observed that four parameters proposed NCTBXII distribution is suitable than,three parameters CTBXII distribution ,three parameters TBXII distribution for modeling this dataset. Again, some model selection criteria like Log-likelihood, Akaike's information criterion (AIC), corrected Akaike information criterion (AICc), Bayesian information criterion (BIC) are selected to asses the practicality of the proposed model. The calculated model selection criteria values are presented in Table 3. According to the obtained model selection criteria values, it has been seen that the proposed NCTBXII model fitted well as comparedwith other competingmodel.

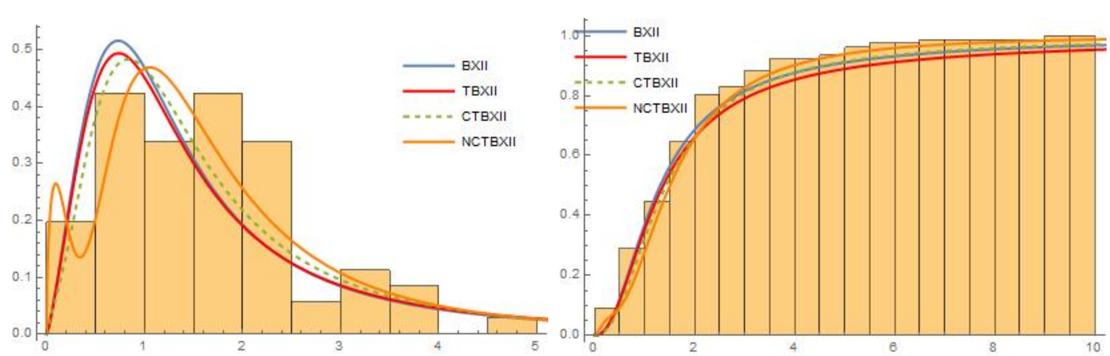

FIGURE 3. The estimated cumulative and density functions of selected models along with the proposed NCTBXII distribution

TABLE 3. Selection criteria values obtained for selected models

Distribution	Log-likelihood	AIC	AICc	BIC
NCTBXII	-120.364	248.729	249.292	258.052
CTBXII	-125.951	257.902	258.235	264.894
TBXII	-128.553	263.107	263.44	270.099
BXII	-128.553	261.107	261.271	265.768

CONCLUSION

This article introduced a NCTBXII distribution. The important distributional properties along with the distributions of different order statistics are discussed. The proposed distribution is applied on the life-time datasets and observed thequite better fit than any other selected models used in this study. Hopefully, this distribution will be flexible enoughto handle more complex real-life datasets arising in different areas of life.

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