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APPLICATIONS OF THE GENERALIZED HOMOGENEOUS q-SHIFT OPERATOR IN q-POLYNOMIALS

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Abstract. In this paper, we construct the generalized homogeneous q-shift operator ${}_r\Phi_s\left(\begin{array}{c} a_1,\cdots,a_r \\ b_1,\cdots,b_s \end{array};q,cD_{xy}\right)$. Then, we apply this operator to derive some q-identities such as: the generating function and its extension, Rogers formula and its extension, Mehler's formula and its extension, Srivastava-Agarwal type bilinear generating functions for the polynomials $\phi_n^{(\mathbf{a},\mathbf{b})}(x,y,c|q)$. Also, we obtain a transformation formula involving generating functions for $\phi_n^{(\mathbf{a},\mathbf{b})}(x,y,c|q)$. We provide some special values for $\phi_n^{(\mathbf{a},\mathbf{b})}(x,y,c|q)$ in order to establish identities for the polynomials $\phi_n^{(a)}(x)$ and $\phi_n^{(\mathbf{a},\mathbf{b})}(x,y|q)$.

Keywords: the homogeneous q-difference operator, the homogeneous q-shift operator, q-Hahn polynomials, the generalized Al-Salam-Carlitz q-polynomials, generating function, Rogers formula, Mehler's formula.

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1. Introduction

In this paper, we will follow common notations and definitions for the q-series that used in [8]. We assume that $|q| < 1$.

For $a \in \mathbb{C}$, the q-shifted factorial is defined by [8]

$$(a;q)_0 = 1, \quad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), \quad (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k),$$

and the multiple q-shifted factorials by:

$$(a_1,a_2,\ldots,a_r;q)_m = (a_1;q)_m(a_2;q)_m\cdots(a_r;q)_m,$$

where $m \in \mathbb{Z}$ or ∞.

The basic hypergeometric series ${}_r\phi_s$ is presented as follows [8]:

$${}_r\phi_s\left(\begin{array}{c} \alpha_1,\ldots,\alpha_r \\ \beta_1,\ldots,\beta_s \end{array};q,x\right) = \sum_{n=0}^{\infty}\frac{(\alpha_1,\ldots,\alpha_r;q)_n}{(q,\beta_1,\ldots,\beta_s;q)_n}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-r} x^n,$$

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