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GENERALIZED q-DIFFERENCE EQUATION FOR THE GENERALIZED q-OPERATOR $_r\Phi_s(D_q)$ AND ITS APPLICATIONS IN q-INTEGRALS

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Abstract. In 2014, Fang [12] discovered a general q-exponential operator identity by solving a q-difference equation. Fang [12] developed some generalizations of q-integrals using this q-difference equation. Reshem and Saad [20] presented the solution to a generalized q-difference equation in q-operator form, which is a generalization of Fang's work [12]. Using the q-difference equation technique, Reshem and Saad [20] discussed some properties of q-polynomials. In this paper, the generalized q-difference equation technique is used to generalize some well-known integrals such as fractional q-integrals, the q-Barnes contour integral, and Ramanujan q-integrals.

Keywords: q-difference equation, q-operator, q-integral, fractional q-integrals, q-Barnes contour integral, Ramanujan q-integrals

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1. Introduction

In this paper, the notations that was used in [13] is followed, and we assume that $|q| < 1$. We mention to some notations that we depend on during this paper.

The q-shifted factorial is defined by [13]:

$$(a;q)_0 = 1, \qquad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k) \qquad and \qquad (a,q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

Also the multiple q-shifted factorials:

$$(a_1, a_2, \ldots, a_m; q)_n = (a_1;q)_n(a_2;q)_n \ldots (a_m;q)_n.$$

The basic hypergeometric series $_t\phi_s$ is given by [13]:

$$_t\phi_s\left(\begin{array}{c} a_0, a_1, \ldots, a_{t-1} \\ b_1, b_2, \ldots, b_s \end{array}; q, x\right) = \sum_{n=0}^{\infty} \frac{(a_0, a_1 \ldots, a_{t-1}; q)_n}{(q, b_1, b_2 \ldots, b_s; q)_n} \left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-t} x^n,$$

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