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GENERALIZED q-DIFFERENCE EQUATION FOR THE GENERALIZED q-OPERATOR ${}_t\Phi_s(D_q)$ AND ITS APPLICATIONS IN q-POLYNOMIALS

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Abstract The solution to a generalized q-difference equation is described in q-operator form in this paper, which is a generalization of Fang's work [9]. We solve a general q-difference equation to get a general q-operator identity. The generalized q-polynomial $\phi_n^{(A,B)}(b,c|q)$ is defined. We present two types generating function and a generalization of the transformational identity for the polynomials $\phi_n^{(A,B)}(b,c|q)$ using the q-difference equation. By assigning specific values to the parameters in the findings for the polynomials $\phi_n^{(A,B)}(b,c|q)$, we get two types generating functions and the transformational identity for polynomials $H_n(a_0,\dots,a_s;b_1,\dots,b_s;b,c)$, $P_n(x,y,a)$, $h_n(x,y,a,b|q)$ and $h_n(a_1,\cdots,a_r;b_1,\cdots,b_s;x,y|q)$.

1 Introduction

In this paper, the notations that was used in [10] is followed and we assume that $|q|<1$. We're going to mention to a few notations that we depend on during this paper.

The q-shifted factorial is defined by [10]:

$$(a;q)_0=1,\qquad (a;q)_n=\prod_{k=0}^{n-1}(1-aq^k),\qquad (a;q)_\infty=\prod_{k=0}^{\infty}(1-aq^k).$$

Also the multiple q-shifted factorials:

$$(a_1,a_2,\dots,a_m;q)_n=(a_1;q)_n(a_2;q)_n\dots(a_m;q)_n.$$

The basic hypergeometric series ${}_t\phi_s$ is given by [10]:

$${}_t\phi_s\left(\begin{matrix}a_0,a_1,\dots,a_{t-1}\\ b_1,b_2,\dots,b_s\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(a_0,a_1\dots,a_{t-1};q)_n}{(q,b_1,b_2\dots,b_s;q)_n}\left[(-1)^nq^{\binom{n}{2}}\right]^{1+s-t}x^n,$$

where $q\neq 0$ when $t>s+1$. Note that

$${}_{s+1}\phi_s\left(\begin{matrix}a_1,a_2,\dots,a_{s+1}\\ b_1,b_2,\dots,b_s\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(a_1,a_2,\dots,a_{s+1};q)_n}{(q,b_1,b_2,\dots,b_s;q)_n}x^n,\quad |x|<1.$$

The q-binomial coefficients is given by [10]:

$$\begin{bmatrix}n\\k\end{bmatrix}=\frac{(q;q)_n}{(q;q)_k(q;q)_{n-k}},\quad 0\leqslant k\leqslant n.$$