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Applications of the Operator ${}_3\phi_2\left(\begin{matrix}a,b,c\\d,e\end{matrix};q,f\theta\right)$

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Abstract

In this paper, we construct the q-exponential operator ${}_3\phi_2\left(\begin{matrix}a,b,c\\d,e\end{matrix};q,f\theta\right)$. We use the operator ${}_3\phi_2$ to obtain an extension of Euler identities, Ramanujan's sum, q-Chu- Vanermonde summation formula and we give some other identities. Also we use the operator ${}_3\phi_2$ to get an extension of the Ramanujan's identity, the Askey beta integral, Ramanujan's beta integral and we give some other integrals formulas.

1- Introduction

In this paper we will use the standard notations for basic hypergeometric series given in [5], we assume that $|q|<1$.

Definition 1.1. [5]. *Let a be a complex variable. The q-shifted factorial is defined by*

$$(a;q)_n=\begin{cases}1, & if\ \ n=0,\\ \prod_{k=0}^{n-1}(1-aq^k), & if\ \ n=1,2,\dots.\end{cases}$$

We define

$$(a;q)_\infty=\prod_{k=0}^{\infty}(1-aq^k).$$

The following notation is used for the multiple q-shifted factorials:

$$(a_1,a_2,\cdots,a_m;q)_n=(a_1;q)_n(a_2;q)_n\cdots(a_m;q)_n,\qquad n=0,1,2,\dots$$

$$a_1,a_2,\cdots,a_m;q)_\infty=\ (a_1;q)_\infty(a_2;q)_\infty\cdots(a_m;q)_\infty.$$

Definition 1.2 [5]. *The generalized basic hypergeometric series is defined by*

$${}_r\phi_s\left(\begin{matrix}a_1,a_2,\cdots,a_r\\b_1,b_2,\cdots,b_s\end{matrix};q,x\right)=\sum_{n=0}^{\infty}\frac{(a_1,\dots,a_r;q)_n}{(q,b_1,\dots,b_s;q)_n}\left[(-1)^nq^{\binom{n}{2}}\right]^{1+s-r}x^n,$$

where $r,s\in\mathbb{N}$; $a_1,\dots,a_r\in C$; $b_1,\dots,b_s\in C\backslash\{q^{-k},k\in N\}$ are assumed to be such that none of the denominator factors evaluate to zero. This series converges absolutely for all x if $r\le s$ and for $|x|<1$ if $r=s+1$.