Coherent Tunneling in Serially Coupled Double Quantum Dots: A Theoretical Treatment

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Research Article

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Abstract

In this article, through serially (one level) coupled double quantum dots made of different semiconductor materials, a physical model for studying spin-polarized electrons current under coherent tunneling effect, and Rashba spin-orbit effect, RSO, is presented in this article. A magnetic field influences Zeeman splitting of the two dots for a different Zeeman splitting.

The DQD system combines two energy levels with the same spin, resulting in a coherent tunneling effect. As a function of the detuning energy (ϵ), the tunneling spin-polarized current and polarization are calculated.

Using coherent tunneling, Rashba spin-orbit effect, and external magnetic field, I see that the polarization can be controlled exhaustively. Lastly, we have calculated that this system can be used as a spin filter to generate a strong spin-polarized current.

1. Introduction

Recent studies have focused more on the transport properties of double quantum dots [1, 2]. In quantum physics and device applications including Nano-electronics, spintronics, quantum computing, etc..., these structures are perfect systems to study the basic interactions between electrons and their spins [3]. A double quantum dot (DQD) can be considered an artificial molecule. Quantum dots can be coupled as weak tunnel couplings or strong tunnel couplings, depending on the strength of the interdot tunnel coupling, which is easily tuned in the experiment [4].

For fundamental physics and proposed applications, the interaction between charges and spins of electrons in tunnel-coupled quantum dots is crucial As well as being used for the elements of quantum computers, quantum dots can be used as building blocks for single-electron device circuits [5].

Within double quantum dots systems, coherent tunnel coupling is known to be the crucial interaction that manages the exchange of spin and connects the similar energy levels.

In this work, the spin-polarized current which is related to the tunneling through a double quantum-dot system attached to leads in the presence of the coherent tunneling effect t_{12}, the Rashba spin-orbit interaction, and Zeeman splitting is explored. For different Zeeman splitting, we use different dot materials (different g -factors).

The spin-polarized current is obtained by solving the master equation in the Born-Markov approximation.

In the next section and by following the work of Li Zhen-Shan et al. [6], we will present the theoretical treatment to formulate an expression for realizing spin-related currents and polarization in DQD-system.

2. Model And Hamiltonian

Due to the coherent tunneling effect t_{12}, the spin-orbit interaction, and the different Zeeman splitting for each dot ($\Delta_i = g_i \mu B$), I use a system consisting of two tunnel-coupled quantum dots sandwiched between two leads $\alpha = L, R$ with their chemical potentials μ_L, μ_R, connected with DQD by the rates $_L$ and $_R$ (see Fig. (1)), while we neglected the interactions between the left and right leads.

Following our earlier work [7] the Hamiltonian of the quantum junction is described by the sum of the Hamiltonian for isolated system (H^o_{dots}, H_{Leads}) and the dots –leads coupling (H_T).

The total Hamiltonian of the system is described by Anderson Hamiltonian;

$$H = H^o_{dots} + H_{Leads} + H_T$$

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$$H_T = \sum_{\alpha, k\sigma} \gamma_\alpha \left(c^{\dagger}{}_{\alpha k\sigma} d_{\alpha\sigma} + d^{\dagger}{}_{\alpha\sigma} c_{\alpha k\sigma} \right)$$

Assuming that DQD is occupied by one electron, then four basis states are available for this electron; $|\uparrow, 0\rangle$, $|\downarrow, 0\rangle$, $|0, \uparrow\rangle, |0, \downarrow\rangle$ and by including all the interactions that exist between the two dots then, the Hamiltonian matrix elements of H^o_{dots} in Eq. (1) will be,

$H^0_{dots} = H^o + h^o$, where H^o is the diagonal elements and h^o is the off-diagonal elements such that,

$$H^o = -\frac{\Delta_1}{2}\left(\left|\uparrow,0\right\rangle\left\langle\uparrow,0\right| + \left|\downarrow,0\right\rangle\left\langle\downarrow,0\right|\right) - \left(\frac{\Delta_2}{2} + \epsilon\right)\left(\left|0,\uparrow\right\rangle\left\langle 0,\uparrow\right| + \left|0,\downarrow\right\rangle\left\langle 0,\downarrow\right|\right)$$

$$h^o = -t_{12}\left(\left|\uparrow,0\right\rangle\left\langle 0,\uparrow\right| + \left|\downarrow,0\right\rangle\left\langle 0,\downarrow\right| + \left|0,\uparrow\right\rangle\left\langle\uparrow,0\right| + \left|0,\downarrow\right\rangle\left\langle\downarrow,0\right|\right) + t_{so}\left(\left|\uparrow,0\right\rangle\left\langle 0,\downarrow\right| - \left|\downarrow,0\right\rangle\left\langle 0,\uparrow\right| - \left|0,\uparrow\right\rangle\left\langle\downarrow,0\right| + \left|0,\downarrow\right\rangle\left\langle\uparrow,0\right|\right)$$

(2)

Where ϵ is the detuning energy and could be taken to adjust the energies at the two dots.

3. The Spin-polarized Current Calculation

To study the tunneling current through the double quantum dots, it is very convenient to characterize this dynamic using the equation of motion for the time evolution of density matrix $\rho(t)$ of the quantum system.

Within the Born-Markov approximation, the dynamics equation of motion can be written as follow[s] [8];

$$\dot{\rho}_{mn}(t) = -i\langle m|\left[H^0_{dots},\rho\right]|n\rangle + \sum_{k\neq n}\left(\mathit{\Gamma}_{nk}\rho_{kk} - \mathit{\Gamma}_{nk}\rho_{kk}\right)\delta_{mn} - \Lambda_{mn}\rho_{mn}\left(1-\delta_{mn}\right)$$

(3)

$\mathit{\Gamma}_{nk}$ describes the transition rate from the state $\langle n|$ to the state $|k>$ and the non-adiabatic parameter, Λ_{mn}, whose real part is responsible for the time decay of the off-diagonal matrix elements (coherence) is,

$$_{mn} = \frac{1}{2}\sum_k\left(\mathit{\Gamma}_{km} + \mathit{\Gamma}_{kn}\right)$$

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one can calculate the transition rates using the Fermi Golden rule approximation;

$$\mathit{\Gamma}_{mn} = \sum_{\alpha=L,R}\mathit{\Gamma}_\alpha\left\{f\left(E_m - E_n - \mu_\alpha\right)\delta_{N_m,N_n+1} + \left[1 - f\left(E_n - E_m - \mu_\alpha\right)\delta_{N_m,N_n-1}\right]\right\}$$

(5)

N_m is the number of electrons in the system when it is in state $\langle m|$ and $f\left(E_m - E_n - \mu_\alpha\right)$is the Fermi distribution function at $\alpha = L,R$ lead, and δ_{N_m,N_n} is the delta function.

The available transitions that take place between the stats $\mathrm{n} = 1,2,3,4$ and the states $\mathrm{m} = 1,2,3,4$ reduced the solution of Eq. (3) at the steady states $\dot{\rho}_{mn}(t) = 0$ to;

$$\rho_{11} = \frac{\mathit{\Gamma}_{12}\rho_{22} + \mathit{\Gamma}_{13}\rho_{33} + \mathit{\Gamma}_{14}\rho_{44}}{x}$$

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$$\rho_{22} = \frac{\mathit{\Gamma}_{21}\rho_{11} + \mathit{\Gamma}_{23}\rho_{33} + \mathit{\Gamma}_{24}\rho_{44}}{y}$$

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$\rho_{33} = \frac{\mathit{\Gamma}_{31}\rho_{11} + \mathit{\Gamma}_{32}\rho_{22} + \mathit{\Gamma}_{34}\rho_{44}}{z}$ (8)

$\rho_{44} = \frac{\mathit{\Gamma}_{41}\rho_{11} + \mathit{\Gamma}_{42}\rho_{22} + \mathit{\Gamma}_{43}\rho_{33}}{D}$ (9)

Where,

$$x = \left(\mathit{\Gamma}_{21} + \mathit{\Gamma}_{31} + \mathit{\Gamma}_{41}\right)$$

$$y = \left(\mathit{\Gamma}_{12} + \mathit{\Gamma}_{32} + \mathit{\Gamma}_{42}\right)$$

$$z = \left(\Gamma_{13} + \Gamma_{23} + \Gamma_{43}\right)$$

$$D = (\mathit{\Gamma}_{14} + \mathit{\Gamma}_{24} + \mathit{\Gamma}_{34})$$

The total current that tunnels through the system I_{tot} is considered as the summation of the spin-up current $I_{\uparrow}$ and the spin-down current $I_{\downarrow}$ as [9],

$$I_{tot} = I_{\uparrow} + I_{\downarrow} = \mathit{\Gamma}\left[\rho|\,0,\uparrow\rangle + \rho|0,\downarrow\rangle\right]$$

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by representing that $\rho\,|0,\uparrow\rangle = \rho_{33}$ and $\rho\,|0,\downarrow\rangle = \rho_{44}$, so the total current in Eq. (10) is taken the form;

$$I_{tot} = \Gamma\left(\rho_{33} + \rho_{44}\right)$$

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To calculate I_{tot} it is convenient to reduce ρ_{33}, and ρ_{44} in terms of ρ_{22} such that,

$$\rho_{33} = \left[\frac{(x1x6 - x3x4)}{x3x5 + x2x6}\right]\rho_{22}$$

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$$\rho_{44} = \left[\frac{(\mathit{\Gamma}_{31}D + \mathit{\Gamma}_{41}\mathit{\Gamma}_{34})\,(z\mathit{\Gamma}_{41} + \mathit{\Gamma}_{31}\mathit{\Gamma}_{43})\,(x1x6 - x3x4)}{(x3x5 + x2x6)} + \frac{(\mathit{\Gamma}_{31}\mathit{\Gamma}_{42} - \mathit{\Gamma}_{41}\mathit{\Gamma}_{32})}{(\mathit{\Gamma}_{31}D + \mathit{\Gamma}_{41}\mathit{\Gamma}_{34})}\right]\rho_{22}$$

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Where;

x1=$(xy - \mathit{\Gamma}_{21}\mathit{\Gamma}_{12})$ x4=$(\mathit{\Gamma}_{31}\mathit{\Gamma}_{42} - \mathit{\Gamma}_{41}\mathit{\Gamma}_{32})$
x2=$(x\mathit{\Gamma}_{23} + \mathit{\Gamma}_{21}\mathit{\Gamma}_{13})$ x5=$(z\mathit{\Gamma}_{41} + \mathit{\Gamma}_{31}\mathit{\Gamma}_{43})$ (14)

x3=$(\mathit{\Gamma}_{21}\mathit{\Gamma}_{14} + x\mathit{\Gamma}_{24})$ x6=$(\mathit{\Gamma}_{31}D + \mathit{\Gamma}_{41}\mathit{\Gamma}_{34})$

By subtitling the values of ρ_{33} and ρ_{44} defined in Eq. (12), (13) and get used to the definitions in Eq. (14), then an expression for I_{tot} will be,

$$I_{tot} = \Gamma\left\{\left[\frac{(x1x6 - x3x4)\,\rho_{22}}{x3x5 + x2x6}\right] + \left[\frac{(\mathit{\Gamma}_{31}D + \mathit{\Gamma}_{41}\mathit{\Gamma}_{34})\,(z\mathit{\Gamma}_{41} + \mathit{\Gamma}_{31}\mathit{\Gamma}_{43})\,(x1x6 - x3x4)}{(x3x5 + x2x6)} + \frac{(\mathit{\Gamma}_{31}\mathit{\Gamma}_{42} - \mathit{\Gamma}_{41}\mathit{\Gamma}_{32})}{(\mathit{\Gamma}_{31}D + \mathit{\Gamma}_{41}\mathit{\Gamma}_{34})}\right]\rho_{22}\right\}$$

(15)

4. Results And Discussions

To study the effect of the coherent tunneling t_{12} on the spin current in the DQD system, $I_{\uparrow}, (I_{\downarrow})$ is calculated numerically from Eq. (13) and Eq. (14) as a function of the detuning energy (ϵ) for different t_{12} and at fixed values for other system parameters $[t_{so}, \mathit{\Delta}_i and B]$ and the parameter Γ is related to the density of state of the leads, these results are presented in Fig. (2).

From Fig. (2), we see that the spin-polarized current increases as t_{12} increases and the current maximum are shifted towards the $\epsilon = 0$ the physical meaning of this behavior could be taken from the Hamiltonian matrix H^0_{dots} Eq. (2), where t_{12} couples the energy levels of the two dots have the same spin when increasing t_{12} the energy levels of the two dots are easier to become close together, so it is easier to determine spin-polarized currents $I_{\uparrow}, (I_{\downarrow})$.

the spin-up polarized currents Fig. (2a) for all values of t_{12} were shown between $\epsilon = (-0.1 - 0)$ and spin-down polarized currents, Fig. (2b) have the same behavior at $\epsilon = (0 - 0.1)$ and the polarization stopped at $\epsilon = 0$ where no spin-up or spin –down polarized currents exist.

And I_{tot} define in Eq. (15) is shown in Fig. (3) for differentt_{12}

To confine the effect of coherent tunneling t_{12} on the spin-polarized currents, we drew I_{tot}as a function of t_{so} and for different values of t_{12}. Our results are given in Fig. 4.

The electron polarization in the DQD system is always defined as [10];

$$P = \frac{I_{\uparrow} - I_{\downarrow}}{I_{\uparrow} + I_{\downarrow}}$$

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Our numerical results for P are shown in Fig. (5), we see here a beak at $\epsilon = -0.005$where the spin-up current is polarized ($P = 1$) and the spin-down current equals zero. But at $\epsilon = +0.005$ the spin-up current equal zero and the spin-down current is polarized ($P = -1$). Also, the two polarization peaks are symmetric around $\epsilon = 0$. Also, this fig. gives the effects of t_{12} on the polarization, we notice here that the polarization peak shifted towards $\epsilon = 0$ as t_{12} increased.

5. Conclusion

we study spin-polarized electrons current in the presence of coherent tunneling effect, t_{12}, and Rashba spin-orbit effect, RSO, through a serially (one level) coupled double quantum dots made of different semiconductor materials to give different Zeeman splitting for the two dots under the influence of an external magnetic field. Besides the Rashba spin-orbit effect we focused our attention on the coherent tunneling effect that couples two energy levels that have the same spin in the DQD system. The tunneling spin-polarized current and polarization are all calculated as a function of the detuning energy (ϵ). Here we see that the current takes a Lorentzian shape, the spin-up and spin-down polarized currents always have the same behavior, where we noticed that the peak position of the spin-up polarized current is at the value of $\epsilon = -0.05$ and the peak position of the spin-down polarized current is at the value of $\epsilon = +0.05$. The polarization could be controlled by dominating coherent tunneling, Rashba spin-orbit effect, and the external magnetic field, which may give an exhaustively spin-polarized current. Finally, our calculations related to this system make sure that one can use it as a spin filter to get a strong spin-polarized current.

Declarations

Ethical Approval

(Not applicable)

Competing interests

(Not applicable)

Authors' contributions

(Not applicable)

Funding

(Not applicable)

Availability of data and materials

I got the data through my own work using Matlab arithmetic program

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Figures

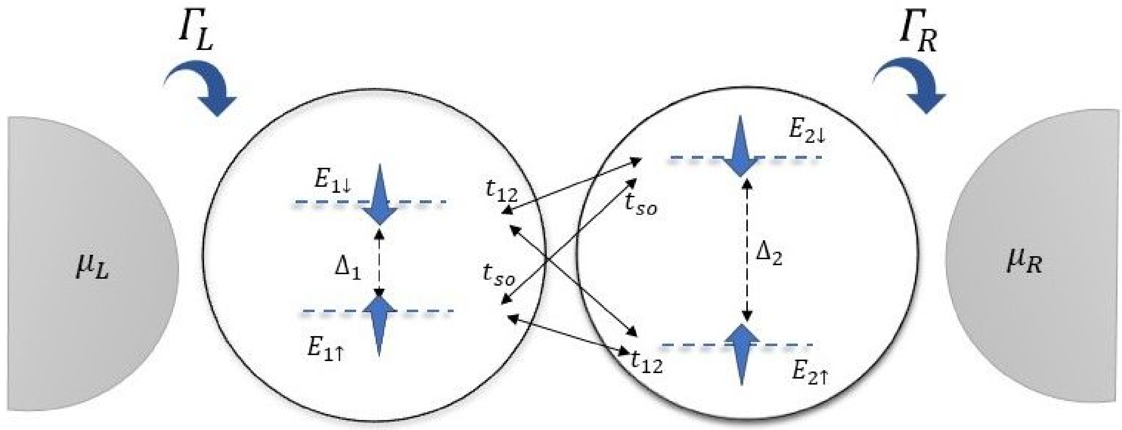

Figure 1

The system under consideration was described in a schematic diagram

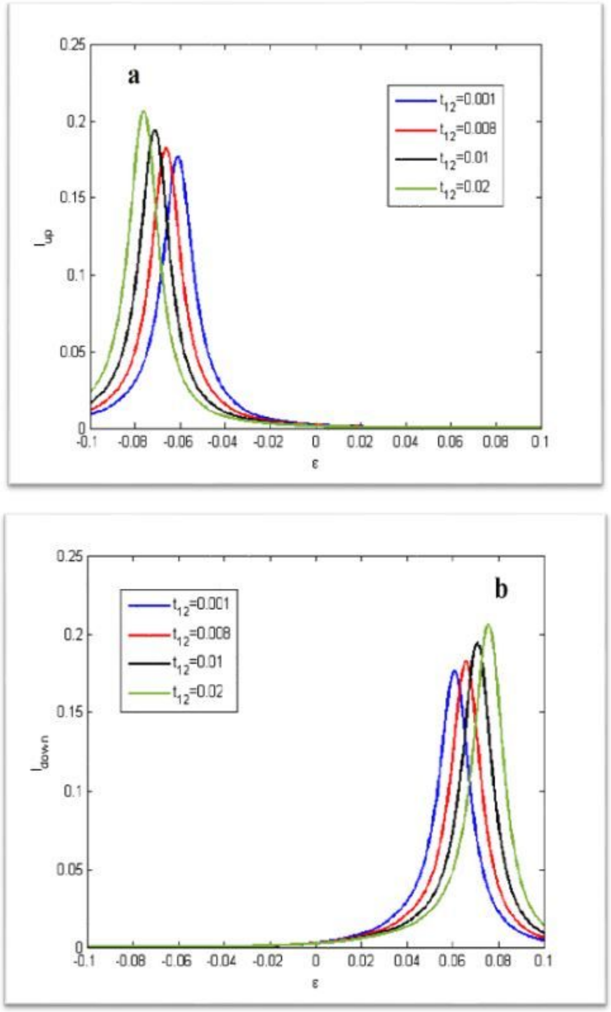

Fig.2: Spin-polarized currents, $I_{\uparrow}, (I_{\downarrow})$ for different t_{12} and at fixed other parameters $\Delta_2 = 0.1$, $\Delta_1 = 0.02$, $\Gamma = 0.001, t_{so} = 0.005$.

Figure 2

See image above for figure legend.

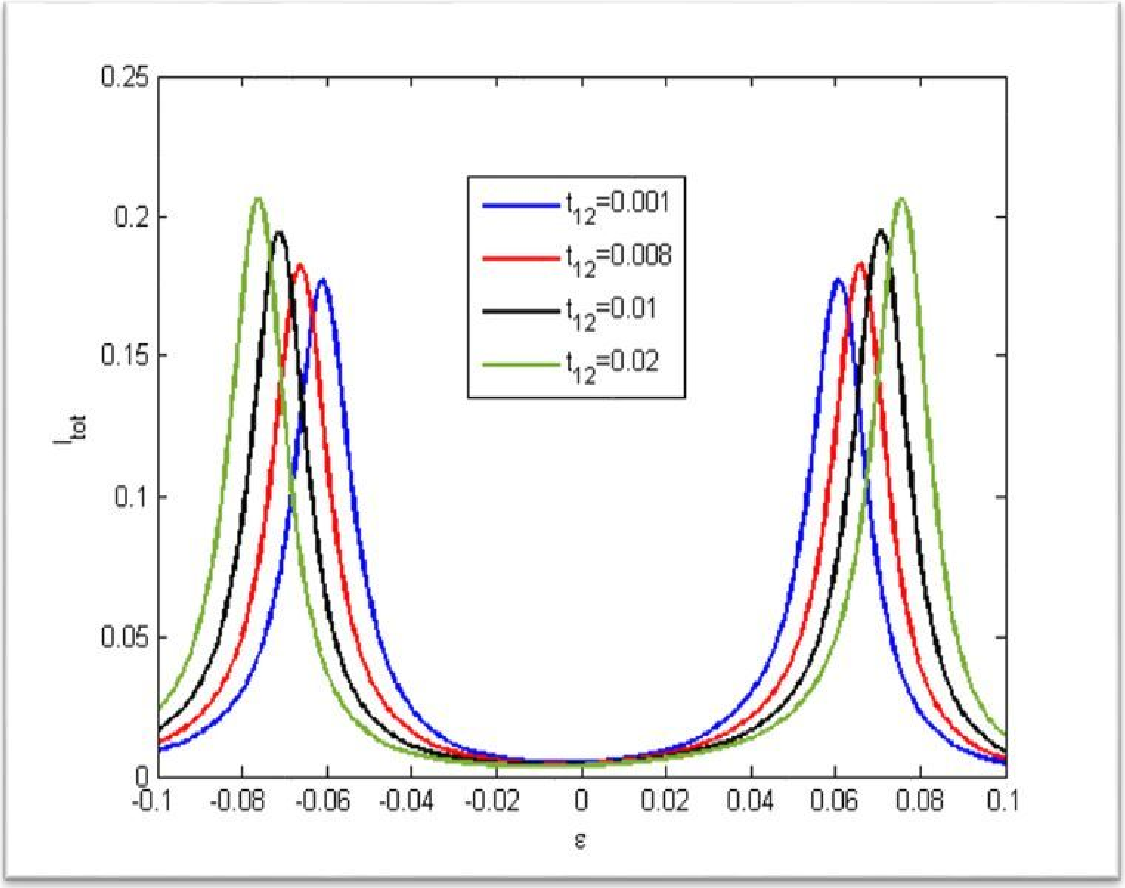

Fig.3: Spin-polarized currents for different t_{12} and at fixed other parameters $\Delta_2 = 0.1$, $\Delta_1 = 0.02$, $t_{so} = 0.005$

Figure 3

See image above for figure legend.

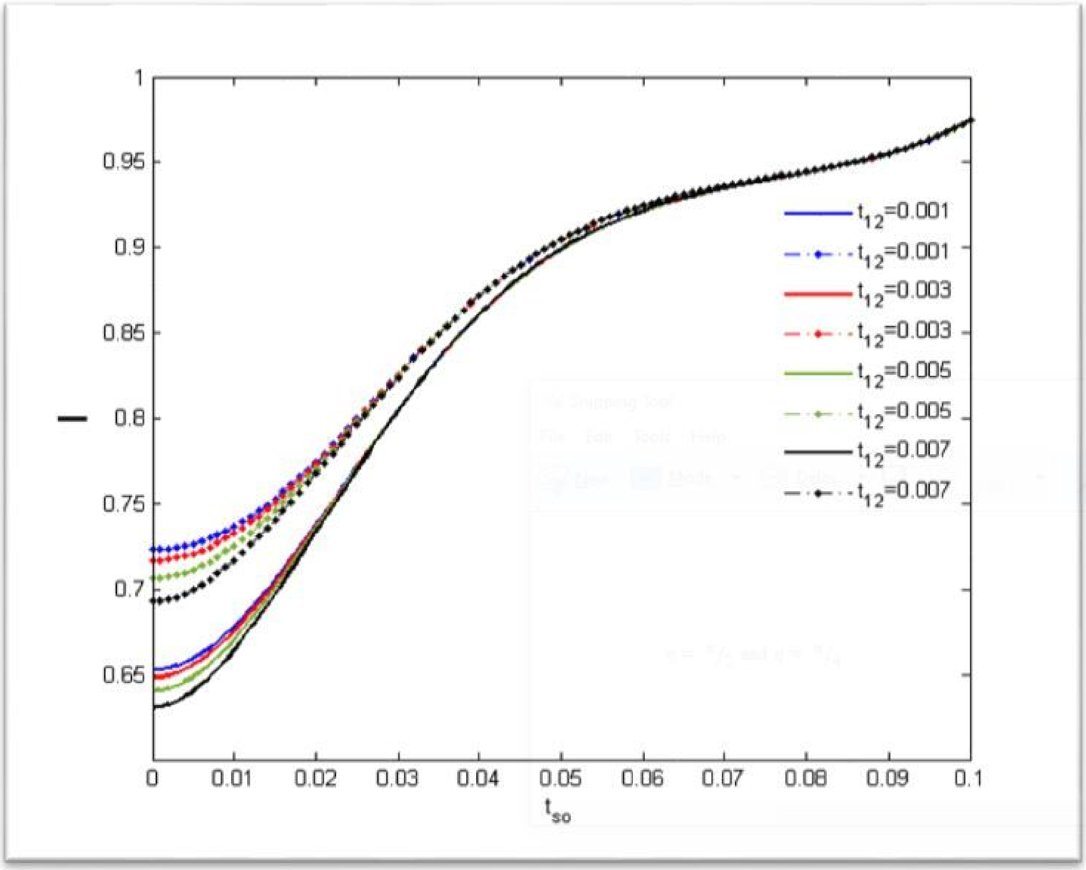

Fig.4: Spin-polarized currents as a function of t_{so} for different t_{12}

Figure 4

See image above for figure legend.