RESEARCH ARTICLE | DECEMBER 22 2023

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AIP Conf. Proc. 2977, 030001 (2023)
https://doi.org/10.1063/5.0182112

Composite distribution(Pareto - Amputated Exponential) and estimating its parameters

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Abstract.In this research, the method of synthesis of Pareto distributions of the first type and amputated ace was used to obtain amore flexible distribution of single distributions in the representation of data of complex phenomena and then derived its characteristics (decentralized moments, central moments, variance, torsion coefficient, flattening coefficient, median and mode), and its parameters were estimated in four ways, namely (Maximum Likelihood, moments, fragmentary capabilities and Bayes estimator (Jeffrey method)), and the conclusion was made By using different hypothetical values for the parameters and four different sizes for the samples, namely (100,50,25), the experiment was repeated (1000) times, and using the MSE) criterion to compare the results of the estimate that the torque and bayes methods compete for preference.

Keywords: compound probability distributions, amputation of distributions, exponential distribution, Pareto distribution, simulation.

INTRODUCTION

The researcher faces a lot of difficulties when he analyzes the statistical and from these difficulties, how to find the appropriate distribution of the data of the phenomenon under study and the data for the real phenomena may be represented by a high degree of torsion or that their appearance is limited to the right side without the left side or the axe x, and recently the use of single probability distributions of their continuous and intermittent types does not give the desired result (flexible) when modeling the studied phenomena, so researchers have been interested in studying new ways to generate More flexible distributions in data analysis and one of these methods is the method of composite probability distributions resulting from the composition of standard (single) distributions, and in this research a new compound distribution will be proposed, namely (Pareto - Amputated Exponential).

Search Objective

Proposing a new (Pareto - Amputated Exponential) compound probability distribution, studying some of its properties, and using simulations to generate data suitable for Capricorn distribution and estimating its parameters.

DISCUSSION OF LITERATURE

-In 2010 he introduced **(Wagner Barreto - Souza et al)** the beta generalized exponential distribution, which included the beta exponential distribution and the generalized exponential distribution as special cases, a comprehensive mathematical treatment was introduced for the new distribution, the maximum Likelihood method was used to estimate the parameters, and a set of real data was applied to show that this distribution can be It gives a better fit.

-In 2012, he presented **(Luz M. Zea and others)** the Pareto exponential-beta distribution with an application to susceptibility to bladder cancer, a detailed study was presented on the mathematical properties of the resulting

4th International Scientific Conference of Alkafeel University (ISCKU 2022)
AIP Conf. Proc. 2977, 030001-1–030001-13; https://doi.org/10.1063/5.0182112
Published by AIP Publishing. 978-0-7354-4776-9/$30.00

distribution (moments, quantitative function, mean deviations, etc.), and derivation of the moment function and other functions, use the method of Maximum Likelihood To estimate the model parameters, the information matrix (Fisher) was derived, and the flexibility of the new model was demonstrated by applying a set of real data.

-In 2013, the researcher presented **(Kareama Abed AL_Kadim and another)** the Pareto-exponential resulting from the installation of the exponential distribution with the Pareto distribution, and the researchers contented themselves with providing some mathematical properties (moments, mean, median, variance, and others), and the method of Maximum Likelihood was used to estimate the parameters.

-In 2013 he introduced **(A. Asghar Zadeh et al)** Pareto-Poisson-Lindley distribution with applications, a new distribution based on the Pareto distribution and the composite Poisson-Lindley distribution, many mathematical properties of the distribution were created, and the estimation of parameters was obtained using the maximum Likelihood method, applications of the distribution were presented using three Sets of real data and it appears that the distribution fits better than other relevant distributions in practical uses.

-In 2016, the researcher **(Ayman Alzaatreh)** identified and studied a new distribution, which is the Kama-Pareto distribution, and different properties of the distribution (moments and others) were obtained. He used the method of maximum Likelihood to estimate the parameters of the new model and showed the possibility of the distribution by applying it to three sets of real data.

-In 2016 **(M.H. TAHIR et al)** a new three-parameter Whipple-Pareto distribution was proposed, and the different properties of the new distribution (moments and others) were derived, and it was estimated using the greatest possibility method, and the observed information matrix was determined, and the possibility of the model was demonstrated by applying it to two sets of real data.

-In 2021, the researcher **(Al-Amri)** proposed a new probability distribution, which is the left truncated Campbell distribution And his family, which included two distributions, the left-Whipple truncated Campbell distribution and the left-exponential Campbell truncated distribution, and the properties were derived for the proposed distributions (central moment, eccentric moment, oblateness, skewness, mode, and median), the parameters were estimated using four methods of estimation, namely ((Maximum Likelihood, moments, partial potentials, and estimator Cran), The comparison between the estimation methods was carried out using the criterion(MSE), Mathematical models have been proposed to estimate the optimal period of preventive replacement under the criterion of least cost and least downtime by employing the proposed probability distributions, as well as an expert system was built to estimate the optimal period of preventive replacement. The experiment was repeated (1000) times, and after comparison the results showed that the method of estimator (Cran) is the best among the methods used.

Our study differs from previous studies in terms of building a new composite probabilistic model, (Pareto-Amputated Exponential), studying its properties and estimating its parameters using four methods of estimation: (greatest possibility, moments, fractional abilities, and Bayes (Jeffrey).

STUDY METHODOLOGY

Preface

This chapter includes some basic concepts and the method of synthesis of the inevitable distributions of mechanism, the derivation of the new composite distribution and some of its properties.

Reliability Function [5] [6]

It represents the non-failure probability of the machine (the machine), which means that it will not stop working during the time (t), It is in the following mathematical form:

$$S(t) = P(T > t) = \int_{t}^{\infty} f(x)dx$$

$$S(t) = 1 - F(t) \qquad (1)$$

F(t) is the cumulative function of failure

Risk Function [10]

It is defined as the very probability of the failure of the machine (machine) in the period

$(t, t + \Delta t)$ provided that it approaches Δt zero $(\Delta t \to 0)$, and is defined mathematically as follows:

$$h(t) = \lim_{\Delta t \to 0}[\frac{P(t < T < t + \Delta t \ \vdots T > t)}{\Delta t}]$$
$$= \lim_{\Delta t \to 0}\left[\frac{P(t<T<t+\Delta t)}{\Delta t}\right] * (P(T > t))^{-1}$$
$$= [S(t)]^{-1} * \lim_{\Delta t \to 0}[\frac{F(t+\Delta t)-F(t)}{\Delta t}]$$
$$h(t) = \frac{f(t)}{S(t)} \qquad (2)$$

Exponential Distribution [9]

Suppose the random variable (x) has an exponential distribution with the parameter (λ) and has a probability density function, which is

$$f(x, \lambda) = \lambda e^{-\lambda x} \ ; x > 0 \ ; \lambda > 0 \qquad (3)$$

And it has a cumulative function which is

$$F(X, \lambda) = 1 - e^{-\lambda x} \ ; x > 0 \qquad (4)$$

Expectation

$$E(X) = \frac{1}{\lambda} \qquad (5)$$

Variance

$$Var(X) = \frac{1}{\lambda^2} \qquad (6)$$

Pareto Distribution [9]

Pareto's distribution is attributed to the Italian economist Vilfredo Pareto (1923-1848), which laid the foundations for this distribution, Pareto's distribution found a great deal of scope for application despite its lack of use, especially in economics by studying the distribution of income when it is beyond a known extent such as (a).

Let's say the random variable x is distributed Pareto with the parameters (θ, α) and has a probability density function which is

$$f(x; \theta, \alpha) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}} \ ; x > \theta \quad , \alpha > 0, \theta > 0 \qquad (7)$$

And it has a cumulative function which is

$$F(X; \theta, \alpha) = 1 - \left(\frac{\theta}{x}\right)^\alpha \ ; x > 0 \qquad (8)$$

Expectation

$$E(X) = \frac{\alpha\theta}{\alpha - 1} \qquad (9)$$

Variance

$$Var(X) = \frac{\alpha\theta^2}{(\alpha - 2)(\alpha - 1)^2} \qquad (10)$$

Composition of Probability Distributions [5][15]

In recent years, many researchers have been interested in the study and formation of composite probability distributions in order to obtain more flexible distributions than standard distributions in modeling and describing real data and phenomena. There are many methods used to synthesize probability distributions, including the cumulative distributive function method.

Suppose c(t) is the probability density function of the random variable T$\{T \in [a, b]; -\infty < a < b < \infty\}$

The cumulative distribution function of the $H(x)$ random variable x, and the cumulative function of the family (T-X) is as follows:

$$G(X) = \int_{a}^{w(H(x))} c(t)\, dt \qquad (11)$$

$$g(x) = \frac{\partial w(H(x))}{\partial x}\, c\big[w\big(H(x)\big)\big] \qquad (12)$$

Where w(H(x)) represents a function of the cumulative distribution function H(x) and has conditions:

Must fall within the period of random variable (x) $w(H(x)) \in [a, b]$

W(H(x)) must be calculus and non-diminishing

$$w(H(x)) \to a \;\; AS\; (x \to -\infty) \quad and \quad w\big(H(x)\big) \to b \;\; AS\; (x \to \infty)$$

Derivation of the (Pareto - Amputated Exponential) Distribution

Amputation of Exponential Distribution [11][9]

Amputation from the left: It is a process by which a probability density function is deduced from the random variable (X) which is within the period, and$[a, \infty]$a constant defined within the sample space Ω. It is known mathematically

$$f^{*}(x) = \frac{f(x)}{F(\infty) - F(a)} \quad ; x > a$$

Whereas $F(\infty) = 1$

$$f^{*}(x) = \frac{f(x)}{1 - F(a)} \quad ; a \le x < \infty \qquad (13)$$

Using equations (3), (4) and (13) we get:

$$f^{*}(x, \lambda, \theta) = \lambda e^{-\lambda(x-\theta)}\, ; x \ge \theta \qquad (14)$$

We conclude that $f^{*}(x)$ in equation (14) the following conditions are met if a probability density function represents the amputated exponential distribution.

1-$f^{*}(x) \ge 0$

2-$\int_{\theta}^{\infty} f^{*}(x) dx \;\; = 1$

By deriving the probability density function of amputated exponential distribution, we get the following:

$$F^{*}(t) = \int_{\theta}^{x} f^{*}(t) dt$$

$$F^{*}(t) = \int_{\theta}^{x} \lambda e^{-\lambda(t-\theta)}\, dt$$

$$= 1 - e^{-\lambda(x-\theta)}\, ; x \ge \theta \qquad (15)$$

Distribution Composition (Pareto - Amputated Exponential)

Using equations (7), (8), (1 1), (1 2) and (1 4) we get the following:

$$W\big(H(x)\big) = W(1 - (\frac{\theta}{x})^{\alpha})$$

$$W\big(H(x)\big) = \frac{1 - F(x)}{f(x)}$$

$$W\big(H(x)\big) = \frac{x}{\alpha}$$

$$F(x) = \int_{\theta}^{\frac{x}{\alpha}} \lambda e^{-\lambda(t-\theta)} dt \qquad (16)$$

$$f(x) = \frac{\lambda}{\alpha}\, e^{-\frac{\lambda x}{\alpha} + \lambda\theta}\, ; x \ge \theta \qquad (17)$$

We note that the probability density function does not meet the following conditions:

1-$f(x) \ge 0$

2-$\int_{\theta}^{\infty} f(x) dx \;\; = e^{\lambda\theta - \frac{\lambda\theta}{\alpha}}$

We multiply the equation (17) by inverting the product of the integration $[e^{\lambda\theta - \frac{\lambda\theta}{\alpha}}]^{-1}$ to achieve the second condition, and thus the function of probability density is as follows:

$$f(x) = \frac{\lambda}{\alpha}\, e^{-\frac{\lambda}{\alpha}(x-\theta)}\; ; x \geq \theta\;\; , \lambda, \alpha > 0 \, , \theta > 0 \qquad (18)$$

The cumulative distribution function is as follows

$$F(x) = \int_{\theta}^{x} f(t)\; dt$$

$$F(x) = 1 - e^{-\frac{\lambda}{\alpha}(x-\theta)}\; ; x \geq \theta \qquad (19)$$

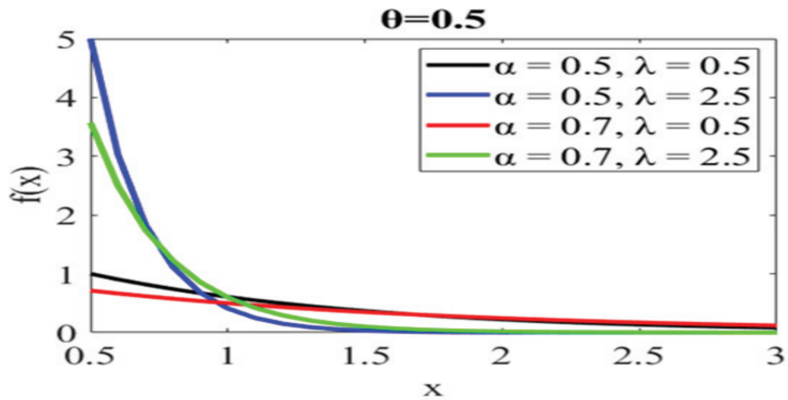

FIGURE 1: Drawing the probability density function of the amputated (Pareto- Amputated Exponential) distribution of different values of the shape (λ, α) parameters and the measurement parameter$(\theta = 0.5)$

Source: prepared by the researcher

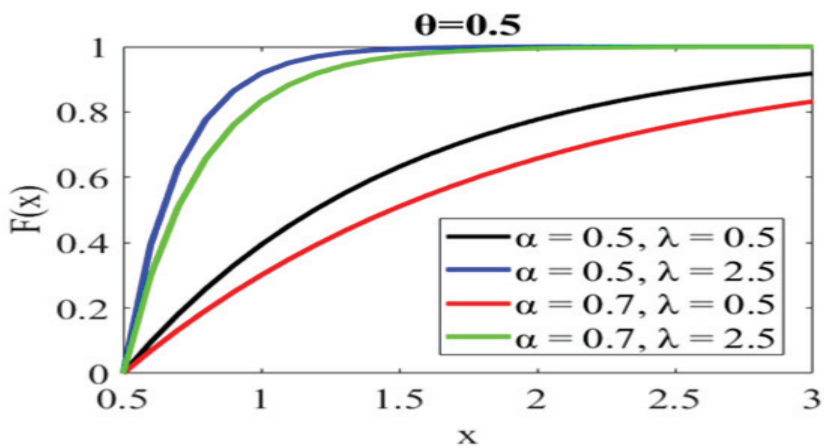

FIGURE 2: Drawing the cumulative distributional function of the composite distribution of different values of the shape (λ, α) parameters and the measurement parameter $(\theta = 0.5)$

Source: Prepared by the researcher

We extract the function of reliability using equation (19) and the following agencies:

$$S(x) = P(X > x) = \int_{x}^{\infty} f(u)\, du = 1 - F(x)$$

$$S(x) = e^{-\frac{\lambda}{\alpha}(x-\theta)} \qquad (20)$$

We extract the risk function using equations (18) and (20) agencies:

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\lambda}{\alpha} \qquad (21)$$

Decentralized Resolve

$$E(x^r) = \int_{\theta}^{\infty} x^r f(x)dx$$

$$E(x^r) = \frac{\lambda}{\alpha} e^{\frac{\lambda\theta}{\alpha}} \int_{\theta}^{\infty} x^r\, e^{-\frac{\lambda x}{\alpha}} dx \quad \text{... (21)}$$

Let's suppose that

$$y = \frac{\lambda x}{\alpha}$$

$$x = \frac{\alpha}{\lambda} y$$

$$dx = \frac{\alpha}{\lambda} dy$$

After compensating in equation (2 2) we get:

$$E(x^r) = \frac{\lambda}{\alpha} (\frac{\alpha}{\lambda})^{r+1} e^{\frac{\lambda\theta}{\alpha}} \int_{\frac{\lambda\theta}{\alpha}}^{\infty} y^r e^{-y} dy \quad (23)$$

And using the incomplete upper gamma law [18] to solve the integral in the equation (23)

$$\Gamma r + 1, x = \int_{x}^{\infty} y^r\, e^{-y} dy$$

Thus, we obtain the eccentric moment of degree (r)

$$E(x^r) = (\frac{\alpha}{\lambda})^r e^{\frac{\lambda\theta}{\alpha}} \left(\Gamma r + 1, \frac{\lambda\theta}{\alpha}\right) \quad (24)$$

Substituting (r=1) gives us the first moment, which is

$$E(x) = \frac{\alpha}{\lambda} e^{\frac{\lambda\theta}{\alpha}} \left(\Gamma 2, \frac{\lambda\theta}{\alpha}\right) \quad (25)$$

Substituting (r=2) we get the second moment, which is

$$E(x^2) = \left(\frac{\alpha}{\lambda}\right)^2 e^{\frac{\lambda\theta}{\alpha}} \left(\Gamma 3, \frac{\lambda\theta}{\alpha}\right) \quad (26)$$

Central Moments

$$E(x-\mu)^r = \int_{\theta}^{\infty} (x-\mu)^r f(x)dx$$

And using the binomial theory we get

$$= \frac{\lambda}{\alpha} e^{\frac{\lambda\theta}{\alpha}} \left[\sum_{j=0}^{r} C_j^r (-\mu)^{r-j}\right] \int_{\theta}^{\infty} x^j e^{-\frac{\lambda x}{\alpha}} dx \quad (27)$$

and I am imposing

$$y = \frac{\lambda}{\alpha}$$

$$x = \frac{\alpha}{\lambda} y$$

$$dx = \frac{\alpha}{\lambda} dy$$

And after compensation and simplification we get the central determination

$$E(x-\mu)^r = e^{\frac{\lambda\theta}{\alpha}} \left[\sum_{j=0}^{r} \left(\frac{\alpha}{\lambda}\right)^j C_j^r (-\mu)^{r-j} \left(\Gamma j + 1, \frac{\lambda\theta}{\alpha}\right)\right] \quad \text{... (28)}$$

To obtain the variance we compensate (r=2) and$\mu = E(x)$ *in the equation (28)*

$$\sigma^2 = e^{\frac{\lambda\theta}{\alpha}} \left[\mu^2 \left(\Gamma(1, \frac{\lambda\theta}{\alpha})\right) - 2\mu \left(\frac{\alpha}{\lambda}\right) \left(\Gamma\left(2, \frac{\lambda\theta}{\alpha}\right)\right) + \left(\frac{\alpha}{\lambda}\right)^2 \left(\Gamma(3, \frac{\lambda\theta}{\alpha})\right)\right] \quad \text{... (29)}$$

Torsion Coefficient

$$C.S = \frac{E(x-\mu)^3}{\sigma^3}$$

$$C.S=\frac{\left[e^{\frac{\lambda\theta}{\alpha}}\left[\begin{array}{c}(-\mu)^3\left(\Gamma\left(1,\frac{\lambda\theta}{\alpha}\right)\right)+3\mu^2\left(\frac{\alpha}{\lambda}\right)^1\left(\Gamma\left(2,\frac{\lambda\theta}{\alpha}\right)\right)\\ -3\mu\left(\frac{\alpha}{\lambda}\right)^2\left(\Gamma\left(3,\frac{\lambda\theta}{\alpha}\right)\right)+\left(\frac{\alpha}{\lambda}\right)^3(\Gamma(4,\frac{\lambda\theta}{\alpha})\end{array}\right]\right]}{\left[e^{\frac{\lambda\theta}{\alpha}}\left[\begin{array}{c}\mu^2\left(\Gamma\left(1,\frac{\lambda\theta}{\alpha}\right)\right)-2\mu\left(\frac{\alpha}{\lambda}\right)^1\left(\Gamma(2,\frac{\lambda\theta}{\alpha})\right)\\ +\left(\frac{\alpha}{\lambda}\right)^2\left(\Gamma(3,\frac{\lambda\theta}{\alpha})\right)\end{array}\right]\right]^{3/2}} \quad \ldots(30)$$

Flattening Coefficient

$$C.K=\frac{E(x-\mu)^4}{\sigma^4}-3$$

$$C.K=\frac{\left[e^{\frac{\lambda\theta}{\alpha}}\left[\begin{array}{c}\mu^4\left(\Gamma 1,\frac{\lambda\theta}{\alpha}\right)-4\mu^3\left(\frac{\alpha}{\lambda}\right)^1\left(\Gamma 2,\frac{\lambda\theta}{\alpha}\right)\\ +6\mu^2\left(\frac{\alpha}{\lambda}\right)^2\left(\Gamma 3,\frac{\lambda\theta}{\alpha}\right)-4\mu\left(\frac{\alpha}{\lambda}\right)^3\left(\Gamma 4,\frac{\lambda\theta}{\alpha}\right)\\ +\left(\frac{\alpha}{\lambda}\right)^4\left(\Gamma 5,\frac{\lambda\theta}{\alpha}\right)\end{array}\right]\right]}{\left[e^{\frac{\lambda\theta}{\alpha}}\left[\begin{array}{c}\mu^2\left(\Gamma\left(1,\frac{\lambda\theta}{\alpha}\right)\right)-2\mu\left(\frac{\alpha}{\lambda}\right)^1\left(\Gamma\left(2,\frac{\lambda\theta}{\alpha}\right)\right)\\ +\left(\frac{\alpha}{\lambda}\right)^2\left(\Gamma\left(3,\frac{\lambda\theta}{\alpha}\right)\right)\end{array}\right]\right]^{2}} \quad (31)$$

Mode

And we derive the probability density function of the compound distribution.

$$\frac{\partial f(x)}{\partial x}=0$$

$$(-\frac{\lambda}{\alpha})^2 e^{-\frac{\lambda}{\alpha}(x-\theta)}=0$$

And by simplification we get

$$x=\theta \quad (32)$$

Mediator

$$\frac{1}{2}=\int_{\theta}^{m}\frac{\lambda}{\alpha}e^{-\frac{\lambda}{\alpha}(x-\theta)}\,dx$$

$$m=\frac{\lambda\theta+0.7\alpha}{\lambda} \quad (33)$$

SIMULATION (RESULTS)

Preface

The experimental aspect was reviewed, which includes the use of the simulation method to generate data that follows the composite distribution, the use of three different sample sizes and different hypothetical values of the parameters, and the comparison of the estimation of the parameters of the (Pareto- Amputated Exponential) distribution obtained.

Simulation Concept [5]

Simulation is defined as a method similar to real reality using certain models, for example, in real reality there are theories that are difficult to analyze, so simulation is used to describe these theories in a similar way to reality with certain models.

ESTIMATION OF THE PARAMETERS OF THE (PARETO- AMPUTATED EXPONENTIAL)

At this stage, the estimation of the parameters of the (Pareto- Amputated Exponential) distribution was performed using the estimation methods used in the research.

TABLE 1. Studied Simulation Models

L	A	I	Cases
0.5	0.5	0.5	1
2.5	0.5	0.5	2
0.5	1.0	0.5	3
2.5	1.0	0.5	4
0.5	0.5	2.5	5
2.5	0.5	2.5	6
0.5	1.0	2.5	7
2.5	1.0	2.5	8

Source: Prepared by the researcher

TABLE 2. Estimation values and values (MSE) accompanying different estimation methods when (θ=0.5, α=0.5, λ=0. 5)

Best	Bayes	For	Moments	MLE	Methods	
	0.5433	0.4418	0.556	0.5433	$\hat{\theta}$	
Bayes	0.4696	0.5552	0.3051	0.4996	$\hat{\alpha}$	n=25
	0.4954	0.4949	0.6359	0.527	$\hat{\lambda}$	
	0.0163	0.0565	0.0693	0.0177	MSE	
	0.521	0.4624	0.5543	0.521	$\hat{\theta}$	
Bayes	0.4899	0.538	0.3141	0.505	$\hat{\alpha}$	n=50
	0.5035	0.5018	0.6329	0.5191	$\hat{\lambda}$	
	0.0035	0.0286	0.0606	0.0039	MSE	
	0.5101	0.472	0.5526	0.5101	$\hat{\theta}$	
Bayes	0.5011	0.532	0.3214	0.5087	$\hat{\alpha}$	n=100
	0.5078	0.5073	0.6298	0.5156	$\hat{\lambda}$	
	0.0016	0.0158	0.0542	0.0019	MSE	

Source: Prepared by the researcher

We note from the previous table that

- The Bayes method is considered the best of all estimation methods because it has the lowest MSE value.
- We find that the rest of the estimation methods approach the Biz method at larger sample sizes, especially the MLE method.
- And that all capabilities possess special consistency because its MSE decreases by increasing the sample size.

TABLE 3. Estimation values and values (MSE) accompanying different estimation methods when (θ=0.5,α=0.5, λ=2.5)

Best	Bayes	For	Moments	MLE	Methods	
	0.5078	0.4887	0.47	0.5078	$\hat{\theta}$	
Bayes	0.4753	0.5744	0.5556	0.5057	$\hat{\alpha}$	n=25
	2.5082	2.59	2.6193	2.6683	$\hat{\lambda}$	
	0.0222	0.0457	0.0287	0.0527	MSE	
	0.5042	0.493	0.4772	0.5042	$\hat{\theta}$	
Bayes	0.5025	0.5555	0.5509	0.518	$\hat{\alpha}$	n=50
	2.5689	2.5891	2.6146	2.6483	$\hat{\lambda}$	
	0.0192	0.0276	0.0244	0.0377	MSE	
	0.5019	0.4933	0.4837	0.5019	$\hat{\theta}$	
For	0.5128	0.5446	0.5436	0.5206	$\hat{\alpha}$	n=100
	2.5998	2.5888	2.6113	2.6394	$\hat{\lambda}$	
	0.0209	0.0196	0.0200	0.0309	MSE	

Source: Prepared by the researcher

It is clear from the previous table that

- The Bayes method is considered the best of all estimation methods because it has the lowest MSE value at a sample size of 25 and50.
- ThePartiality Method (Per) is best at a sample size of 100.

TABLE 4 . Estimation values and values (MSE) accompanying different estimation methods when (θ=0.5, α=1.0, λ=0.5)

Best	Bayes	For	Moments	MLE	Methods	
	0.5814	0.4029	0.5288	0.5814	$\hat{\theta}$	
Moments	0.954	1.078	0.9753	1.0149	$\hat{\alpha}$	n=25
	0.5169	0.5006	0.5426	0.5499	$\hat{\lambda}$	
	0.0253	0.1796	0.0171	0.0269	MSE	
	0.538	0.4097	0.5329	0.538	$\hat{\theta}$	
Bayes	1.0055	1.0794	1.002	1.0366	$\hat{\alpha}$	n=50
	0.5166	0.5007	0.5234	0.5325	$\hat{\lambda}$	
	0.0087	0.113	0.0102	0.0111	MSE	
	0.5193	0.4346	0.5327	0.5193	$\hat{\theta}$	
Bayes	1.0228	1.0666	1.0068	1.0384	$\hat{\alpha}$	n=100
	0.5204	0.5088	0.5185	0.5283	$\hat{\lambda}$	
	0.0047	0.0617	0.0064	0.0062	MSE	

Source: Prepared by the researcher

nwill result from the previous table that

- The Bayes method is considered the best of all estimation methods because it has the lowest MSE value at a sample size of 50 and100.
- The moment method is best at a sample size of 25.

TABLE 5. Estimation values and values (MSE) accompanying different estimation methods when (θ=0.5, α=1.0, λ=2.5)

Best	Bayes	For	Moments	MLE	Methods	
	0.5166	0.4793	0.4393	0.5166	$\hat{\theta}$	
Bayes	0.9411	1.1324	1.0817	1.0011	$\hat{\alpha}$	n=25
	2.4749	2.5558	2.6107	2.6329	$\hat{\lambda}$	
	0.0502	0.1066	0.0624	0.0697	MSE	
	0.5079	0.4822	0.4553	0.5079	$\hat{\theta}$	
Bayes	0.9863	1.1046	1.0759	1.0168	$\hat{\alpha}$	n=50
	2.5366	2.5684	2.6052	2.615	$\hat{\lambda}$	
	0.0282	0.0588	0.041	0.0419	MSE	
	0.504	0.4882	0.4608	0.504	$\hat{\theta}$	
Bayes	1.012	1.0831	1.0741	1.0274	$\hat{\alpha}$	n=100
	2.5623	2.5787	2.5938	2.6013	$\hat{\lambda}$	
	0.0206	0.0359	0.0301	0.0281	MSE	

Source: Prepared by the researcher

We can see from the table above that the Bayes method is considered the best of all estimation methods because it has the lowest MSE value.

TABLE 6 . Estimation values and values (MSE) accompanying different estimation methods when (θ=2.5, α=0.5, λ=0.5)

Best	Bayes	For	Moments	MLE	Methods	
	2.5349	2.434	2.6051	2.5349	$\hat{\theta}$	
Bayes	0.4629	0.6073	0.4985	0.4924	$\hat{\alpha}$	n=25
	0.4983	0.5554	0.563	0.5301	$\hat{\lambda}$	
	0.0086	0.0878	0.0251	0.0088	MSE	
	2.5224	2.4603	2.616	2.5224	$\hat{\theta}$	
Bayes	0.4875	0.5892	0.4922	0.5026	$\hat{\alpha}$	n=50
	0.5061	0.5522	0.5451	0.5218	$\hat{\lambda}$	
	0.0037	0.0408	0.0222	0.0041	MSE	
	2.5126	2.4738	2.6162	2.5126	$\hat{\theta}$	
Bayes	0.5017	0.5724	0.4934	0.5093	$\hat{\alpha}$	n=100
	0.5082	0.5449	0.5358	0.516	$\hat{\lambda}$	
	0.0017	0.0232	0.0192	0.0020	MSE	

Source: Prepared by the researcher

It will be concluded from the table above that the Bayes method is considered the best of all estimation methods because it has the lowest MSE value.

TABLE 7 . Estimation values and values (MSE) accompanying different estimation methods when (θ=2.5, α=0.5, λ=2.5)

Best	Bayes	For	Moments	MLE	Methods	
Moments	2.5082	2.4911	2.4906	2.5082	$\hat{\theta}$	
	0.4787	0.5741	0.5428	0.5093	$\hat{\alpha}$	n=25
	2.4973	2.5827	2.6063	2.6567	$\hat{\lambda}$	
	0.0225	0.0422	0.0189	0.0496	MSE	
Moments	2.5042	2.4916	2.4948	2.5042	$\hat{\theta}$	
	0.5033	0.5598	0.5337	0.5188	$\hat{\alpha}$	n=50
	2.5672	2.5863	2.6035	2.6466	$\hat{\lambda}$	
	0.0192	0.028	0.0157	0.0374	MSE	
Moments	2.5021	2.4942	2.4957	2.5021	$\hat{\theta}$	
	0.5139	0.544	0.5287	0.5217	$\hat{\alpha}$	n=100
	2.6036	2.593	2.6027	2.6433	$\hat{\lambda}$	
	0.0177	0.0202	0.0148	0.0331	MSE	

Source: Prepared by the researcher

From the previous table we conclude that

- The moment method is considered the best of all estimation methods because it has the lowest MSE value.
- While the rest of the methods approach this method at larger sample sizes.

TABLE 8 . Estimation values and values (MSE) accompanying different estimation methods when (θ=2.5, α=1.0, λ=0.5)

Best	Bayes	For	Moments	MLE	Methods	
	2.579	2.3928	2.5921	2.579	$\hat{\theta}$	
Bayes	0.9609	1.1643	1.0154	1.0223	$\hat{\alpha}$	n=25
	0.5116	0.5281	0.5515	0.5443	$\hat{\lambda}$	
	0.0245	0.234	0.0273	0.0266	MSE	
	2.5394	2.4082	2.5983	2.5394	$\hat{\theta}$	
Bayes	1.0058	1.1457	1.0218	1.0369	$\hat{\alpha}$	n=50
	0.5167	0.5291	0.5365	0.5326	$\hat{\lambda}$	
	0.0096	0.1381	0.0215	0.0121	MSE	
	2.52	2.4408	2.5974	2.52	$\hat{\theta}$	
Bayes	1.0252	1.1101	1.0183	1.0408	$\hat{\alpha}$	n=100
	0.521	0.5288	0.5302	0.5289	$\hat{\lambda}$	
	0.0049	0.0698	0.0167	0.0064	MSE	

Source: Prepared by the researcher

From the previous table we see that the Bayes method is considered the best of all estimation methods because it has the lowest MSE value.

TABLE 9. Estimation values and values (MSE) accompanying different estimation methods when (θ=2.5, α=1.0, λ=2.5)

Best	Bayes	For	Moments	MLE	Methods	
	2.5156	2.4795	2.48	2.5156	$\hat{\theta}$	
Moments	0.9384	1.1265	1.0639	0.9983	$\hat{\alpha}$	n=25
	2.4815	2.5617	2.5971	2.6399	$\hat{\lambda}$	
	0.0497	0.1042	0.0297	0.0711	MSE	
	2.5076	2.4854	2.4834	2.5076	$\hat{\theta}$	
Moments	0.983	1.0937	1.058	1.0134	$\hat{\alpha}$	n=50
	2.5456	2.5798	2.6026	2.6243	$\hat{\lambda}$	
	0.0322	0.0576	0.024	0.0473	MSE	
	2.5039	2.487	2.492	2.5039	$\hat{\theta}$	
Moments	1.0128	1.0867	1.053	1.0282	$\hat{\alpha}$	n=100
	2.5597	2.5767	2.592	2.5987	$\hat{\lambda}$	
	0.0206	0.038	0.0185	0.0279	MSE	

Source: Prepared by the researcher

One of the results of the table above is that the moment method is considered the best of all estimation methods because it has the lowest MSE value.

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

Through simulation experiments to estimate the parameters, the best of my method of estimation was obtained among the methods used by the message, which is the Biz method, and competed with the preference of the moment method because they have less value than the average error squares (MSE).

All estimators have consistency because the average error squares (MSE) decrease as the sample size increases.

When (θ=0.5,α=0.5, λ=0.5) we find that the method of greatest possibility when estimating the parameters is close to the Bayes method, especially when using samples larger than (100,50,25).

When (θ=0.5,α=0.5, λ=2.5) the Biz method is better at estimating the parameters for small and medium sample sizes, while the fractional method is better for large sample sizes.

When (θ=0.5,α=1.0, λ=0.5) we find that the Bayes method is the best for estimating the parameters for medium and large sample sizes, while the moment method is the best for small sample sizes.

When (θ=2.5,α=0.5, λ=2.5) we find that the moment method is the best for estimating the parameters between the estimation methods because it has less value than (MSE), and that the rest of the methods approach it when the sizes of samples greater than (100,50,25).

RECOMMENDATIONS

The researcher recommends using probabilistic distribution (Pareto- Amputated Exponential) with other distributions using different synthesis methods for the purpose of obtaining new distributions that are important in the study and analysis of complex data.

Application of the proposed model (Pareto- Amputated Exponential) in other aspects such as scientific aspects (e.g. medical, engineering, etc.).

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