FLT-HPM for Two-dimensional Transient Natural Convection in a Horizontal Cylindrical Concentric Annulus

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Abstract: A hybrid procedure FLT-HPM was proposed in this study, by combining the homotopy perturbation method (HPM) with Fourier transform and Laplace transform which aimed to find an approximate analytical solution to the problem of two-dimensional transient natural convection in a horizontal cylindrical concentric annulus bounded by two isothermal surfaces. The effect of the Grashof number, Prandtl number, and the radius ratio on fluid flow (air) and heat transfer with different values awreas discussed. Moreover, the velocity distributions and the mean Nusselt numbers were studied, and the Nusselt numbers were used to represent local and general heat transfer rates. Finally, the convergence of FLT-HPM was tested theoretically through the proof of some theorems. In addition, these theorems were applied to the results of the new solutions obtained using FLT-HPM.

Keywords: FLT-HPM; Homotopy perturbation method; Fourier transform; Laplace transform; Natural convection; Cylindrical annulus

1 Introduction

Researchers and scientists have been interested in the heat transfer theory of natural convection recently because of its wide applications in various science and technology fields, including aircraft cabin insulation, nuclear reactors, solar collector sensors, heat storage systems, power transmission cables, cooling of electronic components, etc [1–5]. Two-dimensional transient natural convection between two concentric circular horizontal cylinders is one of the most famous problems of natural convection, and has attracted the attention of quite a few researchers. Among the first researchers to provide experimental solutions to this problem, Crawford and Lemlich [6] carried out a numerical study and succeeded in approximating the steady-state differential equations with appropriate difference equations. Moreover, the effect of diameter ratios at 2, 8 and 57 was discussed with 0.7 as the Prandtl number. This study paved the way for many authors and researchers who succeeded in finding numerical and analytical solutions to the above problem. For example, Mack and Bishop [7] used the Rayleigh number power series to solve the problem of natural convection between two concentric horizontal cylinders with slight temperature difference. In their analytical study, the effect of Prandtl number, Rayleigh number and radius ratio on streamline formation, local heat transfer rates, velocity and temperature distributions were discussed. Kuehn and Goldstein [2] introduced experimental and theoretical-numerical studies. In the experimental study, the Mach-Zehnder interferometer was used to locate temperature distributions and coefficients of local heat-transfer. In addition, the governing invariant property equations were solved numerically using the finite difference method. The comparisons between experimental and numerical results under similar conditions illustrated good consistence. Tsui and Tremblay [4] conducted a theoretical-numerical study, and discussed the effect of both the Grashof number from 7×10^2 to 9×10^4 and diameter ratio variations of 1.2, 1.5 and 2 with 0.7 as the Prandtl number. Pop et al. [8] obtained an approximate analytical solution to the presented problem. The method of matched asymptotic expansions was used, which obtained the solutions for three regions (core, inner and outer boundary layers) in a short time. It was found that the solution was distinct from the steady state solution. Hassan and Al-lateef [9] conducted a numerical study in which the energy and vorticity equation was solved using the alternating direction implicit (ADI) method, and the stream function equation was solved using the successive over relaxation (SOR) method. The results of the numerical solutions were discussed, based on the difference in diameter ratios of 1.2, 1.5 and 2, the effect of the Grashof number ranging from