American Journal of Mathematics and Statistics 2017, 7(1): 38-43
DOI: 10.5923/j.ajms.20170701.06

Conharmonically Flat Vaisman-Gray Manifold

Habeeb M. Abood*, Yasir A. Abdulameer

Department of Mathematics, College of Education for Pure Sciences, Basra University, Basra, Iraq

Abstract This paper is devoted to study some geometrical properties of conharmonic curvature tensor of Vaisman-Gray manifold. In particular, we have found the necessary and sufficient condition that flat conharmonic Vaisman-Gray manifold is an Einstein manifold.

Keywords Almost Hermitian Manifold, Vaisman-Gray manifold, Conharmonic tensor

1. Introduction

One of the representative work of differential geometry is an almost Hermitian structure. Gray and Hervalla [1] found that the action of the unitary group $U(n)$ on the space of all tensors of type (3,0) decomposed this space into sixteen classes. The conditions that determined each one of these classes belongs to the type of almost Hermitian structure have been identified. These conditions were formulated by using the method of Kozel's operator [2].

The Russian researcher Kirichenko found an interesting method to study the different classes of almost Hermitian manifold. This method depending on the space of the principal fiber bundle of all complex frames of manifold M with structure group is the unitary group $U(n)$. This space is called an adjoined G-structure space, more details about this space can be found in [3-6].

One of the most important classes of almost Hermitian structures is denoted by $W_1 \oplus W_4$, where W_1 and W_4 respectively denoted to the nearly Kähler manifold and local conformal Kähler manifold.

A harmonic function is a function whose Laplacian vanishes. Related to this fact, Y. Ishi [7] has studied conharmonic transformation which is a conformal transformation that preserves the harmonicity of a certain function. Agaoka, et al. [8] studied the twisted product manifold with vanishing conharmonic curvature tensor. Agaoka, et al. [9] studied the fibred Riemannian space with flat conharmonic curvature tensor, in particular, they proved that a conharmonically flat manifold is locally the product manifold of two spaces of constant curvature tensor with constant scalar curvatures. Siddiqui and Ahsan [10] gave an interesting application when they studied the conharmonic curvature tensor on the four dimensional space-time that satisfy the Einstein field equations. Abood and Lafta [11] studied the conharmonic curvature tensor of nearly Kähler and almost Kähler manifolds. The present work devoted to study the flatness of conharmonic curvature tensor of Vaisman-Gray manifold by using the methodology of an adjoined G-structure space.

2. Preliminaries

Suppose that M is $2n$-dimensional smooth manifold, $C^\infty(M)$ is a set of all smooth functions on M, $X(M)$ is the module of smooth vector fields on M. An almost Hermitian manifold (AH-manifold) is the set $\{M, J, g = <.,.>\}$, where M is a smooth manifold, and J is an almost complex structure, and $g = <.,.>$ is a Riemannian metric, such that $< JX, JY > = < X, Y >$; $X, Y \in X(M)$.

Suppose that $T_p^c(M)$ is the complexification of tangent space $T_p(M)$ at the point $p \in M$ and $\{e_1, \dots, e_n, Je_1, \dots, Je_n\}$ is a real adapted basis of AH-manifold. Then in the module $T_p^c(M)$ there exists a basis given by $\{\varepsilon_1, \dots, \varepsilon_n, \hat{\varepsilon}_1, \dots, \hat{\varepsilon}_n\}$ which is called adapted basis, where, $\varepsilon_a = \sigma(e_a)$ and $\hat{\varepsilon}_a = \bar{\sigma}(e_a)$ and $\sigma, \bar{\sigma}$ are two endomorphisms in the module $X^c(M)$ which are defined by $\sigma = \frac{1}{2}\left(id - \sqrt{-1}\ J^c\right)$ and $\bar{\sigma} = \frac{1}{2}\left(id + \sqrt{-1}\ J^c\right)$, such that, $X^c(M)$ and J^c are the complexifications of $X(M)$ and J respectively. The corresponding frame of this basis is $\{p; \varepsilon_1, \dots, \varepsilon_n, \hat{\varepsilon}_1, \dots, \hat{\varepsilon}_n\}$. Suppose that the indexes i, j, k and l are in the range $1,2, \dots, 2n$ and the indexes a, b, c, d and f are in the range $1,2, \dots, n$. And $\hat{a} = a + n$.

The G-structure space is the principal fiber bundle of all complex frames of manifold M with structure group is the unitary group $U(n)$. This space is called an adjoined G-structure space.

In the adjoined G-structure space, the components matrices of complex structure J and Riemannian metric g are given by the following:

$$\left(J_j^i\right) = \begin{pmatrix} \sqrt{-1} I_n & 0 \\ 0 & -\sqrt{-1} I_n \end{pmatrix}, \left(g_{ij}\right) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} \quad (2.1)$$

where I_n is the identity matrix of order n.

* Corresponding author:
iraqsafwan2006@gmail.com (Habeeb M. Abood)
Published online at http://journal.sapub.org/ajms
Copyright © 2017 Scientific & Academic Publishing. All Rights Reserved