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A numerical method for solving quadratic fractional optimal control problems

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ABSTRACT

The objective of this article is to present a novel algorithm that can efficiently address fractional quadratic optimal control problems (FQOCPs) through the application of the generalized differential transform method, in conjunction with a Vandermonde matrix. The algorithm's performance, in terms of solution accuracy, reliability, and efficiency, is exemplified by a range of illustrative examples. This paper introduces an innovative methodology for the numerical resolution of FQOCPs, demonstrating its inherent capabilities and efficacy.

1. Introduction

The theory of optimal control is a mathematical branch that focuses on minimizing or maximizing a given cost function in specific dynamical systems. It has gained considerable attention from scientists due to its effectiveness in designing and analyzing real-life models, such as spacecraft [1], engineering [2], physical devices [3–5], biological systems [6–9], economics [10], and others. On the other hand, fractional calculus is employed to model real-world systems, offering enhanced accuracy, efficiency, and precision in capturing their dynamic behavior. This motivated O. P. Agarwal to apply classical control theory within a fractional framework, leading to the development of optimal fractional control theory [11,12]. Consequently, numerous researchers have focused on finding optimal solutions for dynamical systems described by fractional derivatives [13–32].

Fractional derivatives can be defined in various ways, including Caputo, Riemann–Liouville, Grünwald–Letnikov, and others. Consequently, many studies have been conducted on fractional optimal control systems described by Caputo or Riemann–Liouville fractional derivatives. As the demand for the application of fractional optimal control problems (FOCPs) grows, the need for numerical methods to solve the resulting equations has emerged as a rapidly expanding area of research. Two main approaches, direct and indirect methods, are commonly employed in numerical schemes for this purpose. Indirect methods involve solving the Pontryagin's system using suitable numerical techniques, which is complex due to the involvement of both left and right fractional derivatives in the Pontryagin's equations [11,12,33–39]. In contrast, direct methods approximate the FOCP without considering the necessary optimality conditions [40–47]. Inspired by Taylor series expansion, Zhou [48] introduced the differential transform method (DTM), which is a powerful semi-numerical technique. DTM is an appealing option for researchers to solve both linear and nonlinear problems due to its lack of necessity for linearization or domain discretization. In fact, DTM distinguishes itself from the Taylor series method by offering a simplified approach while yielding equivalent outcomes. Notably, DTM exhibits reduced computational time for higher orders, making it advantageous for handling complex calculations. In recent times, there has been an increasing scholarly focus on addressing fractional differential equations (FDEs), resulting in the emergence of dedicated methodologies tailored to this objective. Arikoglu and Ozkol [49], Odibat et al. [50], and Khdair et al. [51] introduced the fractional differential transform method (FDTM), generalized differential transform method (GDTM), and restricted differential transform

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