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THE HOMOGENEOUS q-DIFFERENCE OPERATOR AND THE RELATED POLYNOMIALS

M. A. ARIF [1], H. L. SAAD[2*], §

ABSTRACT. We create the homogeneous q-difference operator $\widetilde{E}(a,b;\theta)$ as an extension of the exponential operator $E(b\theta)$. A new polynomials $h_n(a,b,x|q^{-1})$ are defined as an extension of the q^{-1}-Rogers-Szegö polynomial $h_n(a,b|q^{-1})$. We provide an operator proof of the generating function and its extension, Rogers formula and the invers linearization formula, and Mehler's formula for the polynomials $h_n(a,b|q^{-1})$. The generating function and its extension, Rogers formula and the invers linearization formula, and Mehler's formula for the polynomials $h_n(a,b|q^{-1})$ are deduced by giving special values to parameters of a new polynomial $h_n(a,b,x|q^{-1})$.

Keywords: the homogeneous q-difference operator, the q^{-1}-Rogers-Szegö polynomial, the generating function, the Rogers formula, the invers linearization formula, the Mehler's formula.

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1. Introduction

The notations in [8] will be utilized throughout this paper. We assume that $|q| < 1$. The q-shifted factorial is defined as

$$(a;q)_k = \begin{cases} 1, & \text{if } k=0, \\ (1-a)(1-aq)\cdots(1-aq^{k-1}), & \text{if } k=1,2,3,\cdots. \end{cases}$$

We also define

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

For multiple q-shifted factorials, we'll use the following notation:

$$(a_1,a_2,\ldots,a_m;q)_n = (a_1;q)_n(a_2;q)_n\cdots(a_m;q)_n,$$
$$(a_1,a_2,\ldots,a_m;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty\cdots(a_m;q)_\infty.$$

[1] Department of Mathematics, College of Science, Basrah University, Iraq.
e-mail: hus6274@hotmail.com; ORCID: https://orcid.org/0000-0001-8923-4759 .
[2] Department of Mathematics, College of Education for Pure Sciences, Basrah University, Iraq.
e-mail: mahmoodabdalwahid@yahoo.com; ORCID: https://orcid.org/0000-0001-6201-326x.
[*] Corresponding author.
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