Design and Implementation of a Computerized Balancing System

Jaafar Kh. Ali, Ass. Lecturer, email: jaafarkhalaf@yahoo.com
Saleh I. Najem, professor, email: prosismael@yahoo.com
Dept. of Mech. Engineering-College of Engineering-University of Basrah

Abstract

In this work, a new computerized measurement system for multi-plane flexible rotor balancing has been designed and implemented. This system can be used to modernize and enhance conventional low-speed balancing machines or for field balancing applications. This system adds very important features to balancing machines such as multi-plane flexible rotor balancing, high accuracy, stability, and high dynamic range. Also, the proposed flexible rotor balancing technique permits accurate balancing of high-speed rotors utilizing low-speed balancing machines or field balancing at speeds lower than the critical speeds. The proposed digital Wattmetric technique in conjugation with advanced measurement circuitry have led to significant improvement in balancing accuracy even when the unbalance signal is buried into high level of noise.

تصميم وبناء نظام محوسب للموازنة الديناميكية

جعفر خلف علي، قسم الهندسة الميكانيكية، كلية الهندسة – جامعة البصرة.

أ.د. صالح اسماعيل نجم، قسم الهندسة الميكانيكية، كلية الهندسة – جامعة البصرة.

الخلاصة

تم في هذا العمل تصميم وبناء نظام متكامل محوسب للموازنة الديناميكية متعددة المستويات حيث يمكن استخدامه لتطوير وتحديث مكائن الموازنة التي تعمل بالمنظومات التناظرية او تلك التي تحتاج الى استبدال منظومة القياس لتقادمها او عدم دقتها. كذلك يمكن استخدام النظام في تطبيقـــات الموازنة الحقلية للمعدات الكبيرة التي يصعب تفكيكها لغرض موازنتها في الورشة. إن من أهم مزايا النظام المصنع هي الموازنة متعددة المسـتويات (أكثر من مستويين) هذا بالاضافة الى الدقة العالية واختيار المدى التلقائي والثباتية العالية في القراءات.كذلك فان استخدام التقنية المقترحة في هـــذا العمل للموازنة متعددة المستويات للمحاور المرنة أتاح عملية موازنة المحاور ذات السرع الدورانية العالية (مثل محاور التوربينات الغازية والضاغطات) ياستخدام مكائن موازنة واطئة السرعة او باستخدام الموازنة الحقلية تحت سرع أقل بكثير من السرع الحرجة للمحور لتفـــادي ايــة مشـــاكل. إن استخدام التقنيات الرقمية المتقدمة في معالجة الاشارات بالاضافة الى وسائل القياس المتطورة أتاح قياس اشارات عدم الاتزان حتى لو كانت بسعات قليلة جدا.

Keywords: Dynamic Balancing Systems, Flexible Rotor Balancing, Improved Influence Coefficient Method.

1. Introduction

Unbalance may be defined as the unequal distribution of weight of a rotor about its rotating centerline [1]. It results in vibration of the rotating parts due to centrifugal forces, the case which is undesired or dangerous to the machinery. Regardless of the reason of unbalance, a proper balancing procedure must be applied to reduce unbalance to a reasonable level. This is done by dynamic balancing machines.

In the terminology of dynamic balancing, rigid rotors are those rotors operating at speeds lower than the speeds of flexural modes. Examples are rotors of most electrical motors, pumps, fans and other low and intermediate speed machinery. On the other hand, flexible rotors are rotating at high speeds where flexural modes are dominant. Examples of flexible rotors are rotors of turbo-machinery and some high-speed electrical motors. The influence coefficients method (ICM) is the most utilized approach in the balancing of both rigid and flexible rotors. For rigid rotors, accurate results can be obtained by two-plane balancing procedure utilizing the commercially available balancing machines [2]. Flexible rotors require more than two planes to achieve good balancing results. As a general rule, each successive mode shape requires one additional balancing plane. Modal analysis method can also be used to balance flexible rotors but requires

accurate modeling for rotor-bearings system [2, 3]. The application of ICM in flexible rotor balancing may be tracked back to 1964 when Goodman [4] extended this approach for multi-plane balancing by least squares method using the data from multi-speed operation. Later, Goodman approach was investigated and refined by Tessarzik *et al.* [2].

While the ICM is an experimental procedure, modal balancing approach, on the other hand, is based on mathematical modeling and theoretical estimation of rotor response. Saito and Azuma [5] developed an approach to the complex modal method which was further refined by Meacham *et al.* [6] by including the contribution of residual bow effects. Darlow [7] presented a comprehensive review on the theory and methods of rotor balancing. In the paper presented by Kang *et al.* [8], a formulation of influence coefficients matrix was derived from motion equation of asymmetrical rotors using complex coordinate. They proved that ICM is not an art but a science. Another work done by Kang *et al.* [9] in which they proposed a modified approach based on the influence coefficient method to balance crank-shafts using soft-pedestal machines. In the past ten years, many researchers focused their works on flexible rotor balancing using rotordynamics theory, formulated by Finite Element Method (FEM), in combination with influence

coefficients method. Sinha *et al.* [10] presented and investigated a robust method based on the measurement of pedestal vibration from a single run-down test along with a priori models of the rotor and fluid-film bearings.

In paper [11], Zang *et al.* proposed a method of weighted least squares method for multi-plane balancing using the measured response from a single transducer at different rotor speeds. The objective was to minimize the unbalance effect at certain speeds by multiplying the influence coefficients matrix by some weighting matrix. Another approach proposed by Séve *et al.* [12], based on rotordynamics theory coupled with FEM and influence coefficients method, aimed to reduce vibration at some target planes on the rotor and stator. The theoretical responses of target planes are employed instead of the actual measured responses by adopting accurate rotor-stator-foundation model. In the paper of Zhou *et al.* [13], an active balancing scheme was presented to balance rotors during acceleration period based on extended influence coefficients method. Yu [14] presented a generalized influence coefficients method (GICM) which can solve calculation tasks of influence coefficients when a group of trial masses are attached in each single run.

In this work, an approach is proposed for flexible rotor balancing suitable for field or in-housing balancing and based on excerpting influence coefficients at higher speeds from their counterparts at low speeds by employing theory of rotordynamics.

2. Theoretical Background

Consider a multi-disk flexible rotor shown in Fig. 1 with selected balancing (correction) planes at P1, P2, P3 and P4. If the unbalance distribution coincides exactly with the balancing planes, the rotor can be fully balanced by ICM by choosing four measuring planes at single speed or less than four measuring planes at multiple speeds [2, 12] assuming perfect measurement conditions. However, unbalance distribution is almost random and may not coincide with the chosen correction planes especially for continuous rotors or rotors with large number of distributed disks. Moreover, the number of correction planes is limited in many cases due to design or operational constraints. Whenever correction masses do not coincide with the residual unbalance, vibration cannot be nullified along the rotor and speed range but it can be minimized at certain speeds or certain target planes [12].
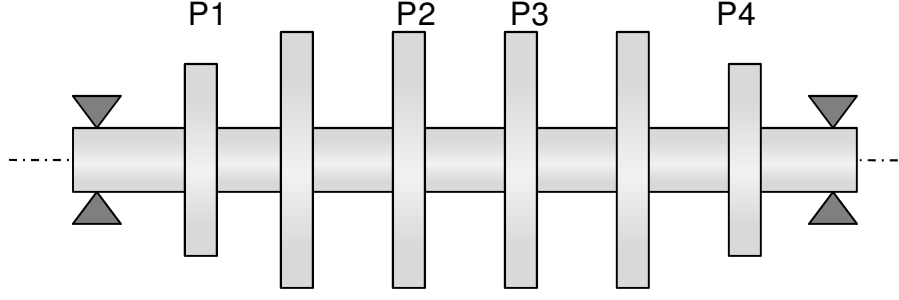
Figure 1 Multi-disk Rotor

Assuming there are P balancing planes, Q measuring planes (target planes) and M balancing speeds, the influence coefficients (IC) matrix A will have ($Q \times M$) rows and P columns. The response vector V is related to the residual unbalance U by eq. (1). Where V_i^m is the response in measuring plane i at balancing speed m, A_{ij}^m is the IC relating the response of plane i with the unbalance force at correction plane j at speed m, and U_p is the residual unbalance at correction plane p. The coefficients, given in eq. (2), can be determined by the standard trial mass procedure repeated at each balancing plane and speed. To obtain the initial unbalance, the IC matrix must be inverted and multiplied by the initial response. If $P = Q \times M$, the IC matrix will be square matrix and can easily be inverted. However, if $P < Q \times M$ then some rows can be merely deleted to get square matrix [2] or alternatively the least square method can be employed to obtain an optimized solution [11, 12].

$$
\begin{bmatrix}
V_1^1 \\
V_2^1 \\
\vdots \\
V_Q^1 \\
V_1^2 \\
V_2^2 \\
\vdots \\
V_Q^2 \\
\vdots \\
V_1^M \\
V_2^M \\
\vdots \\
V_Q^M
\end{bmatrix}
=
\begin{bmatrix}
A_{11}^1 & A_{12}^1 & \ldots & A_{1P}^1 \\
A_{21}^1 & A_{22}^1 & \ldots & A_{2P}^1 \\
\vdots & \vdots & \vdots & \vdots \\
A_{Q1}^1 & A_{Q2}^1 & \ldots & A_{QP}^1 \\
A_{11}^2 & A_{12}^2 & \ldots & A_{1P}^2 \\
A_{21}^2 & A_{22}^2 & \cdots & A_{2P}^2 \\
\vdots & \vdots & \vdots & \vdots \\
A_{Q1}^2 & A_{Q2}^2 & \cdots & A_{QP}^2 \\
\vdots & \vdots & \vdots & \vdots \\
A_{11}^M & A_{12}^M & \cdots & A_{1P}^M \\
A_{21}^M & A_{22}^M & \cdots & A_{2P}^M \\
\vdots & \vdots & \vdots & \vdots \\
A_{Q1}^M & A_{Q2}^M & \cdots & A_{QP}^M
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_P \\
U_1 \\
U_2 \\
\vdots \\
U_P \\
\vdots \\
U_1 \\
U_2 \\
\vdots \\
U_P
\end{bmatrix}
$$

…(1)

$$
A_{ij}^m = \frac{V_{i,t}^m - V_{i,0}^m}{M_j} \tag{2}
$$

Where $V_{i,t}^m$ is the response in measuring plane i after attaching trial mass M_j at correction plane j and speed m while $V_{i,0}^m$ is the initial response of the same plane at the

same balancing speed. For example, if $P = 4$ and $Q = 2$, the trial mass procedure must be repeated, at least, twice at well-separated balancing speeds to obtain the sufficient IC matrix. If more than two speeds are employed, an optimized solution can be found by least-squares method. Moreover, the weighted least-squares approach can be used to minimize the effect of the residual unbalance at certain speeds [11].

The steady-state response $X(\omega)$ of a rotating system to a harmonic force $F(\omega)$ is given by :

$$X\left(\omega\right) = \boldsymbol{\alpha}\left(\omega\right) \cdot F\left(\omega\right) \qquad (3)$$

Where

$$\boldsymbol{\alpha}\left(\omega\right) = \left[K - \omega^2 M + i\,\omega C \right]^{-1} \qquad (4)$$

is known as the receptance matrix [15] constituting a response model of the system. The elements of the receptance matrix are given in the following equation:

$$\alpha_{ij}\left(\omega\right) = \sum_{r=1}^{N} \left(\frac{{}_r B_{ij}}{i\,\omega - \lambda_r} + \frac{{}_r B_{ij}^{\,*}}{i\,\omega - \lambda_r^{*}} \right) \qquad (5)$$

Where $\alpha_{ij}\left(\omega\right)$ is relating the response of node i to an excitation at node j, λ_r is the eigenvalue of mode r and ${}_r B_{ij}$ is a modal constant related to eigenvectors (mode shapes) and the star denotes conjugation. It is clear from eq. (5) that the response at any rotation speed ω includes contributions from all dominant mode shapes with different participation ratios [15]. The more the speed is close to a modal speed, the more the ratio of participation of that mode is. For that reason, performing balancing at high speed will ensure more inclusion of higher modes and, hence, better unbalance elimination in the supercritical region. Also, the multi-speed approach requires an advanced measurement system to detect the response amplitude and phase angle accurately due to the fact that noise may overcome and distort some modes having speeds far away from balancing speed. Therefore, balancing speeds must be well-separated and spread over the operating speed range. However, in some cases repeated running of the rotor at high speeds is dangerous and cumbersome process especially for large machines, such as turbo-generators, during trial runs. Some manufacturers put restrictions for the number of run-up coast-down operations in a given period of time. Therefore, it is quite convenient to minimize the number of trial runs especially at higher speeds and this is the main objective of this work.

3. The Proposed Approach

Most, if not all, turbo-machines pass through one or more critical speed during

run-up operation. This may cause some deterioration for the mechanical parts, such as bearings and seals, besides it is time consuming process. Some machines require 30 minutes or more to get it running at its designated speed. This constitutes big restriction for multi-speed field balancing procedures during trial runs. Fortunately, the initial response can be obtained while the machine is running at its operating speed before shutting it down for maintenance and balancing. If the IC matrix elements related to the high speeds can be obtained somehow without trial runs, then the initial unbalance can be obtained straightforwardly. In fact, by adopting accurate modeling for the system, the IC at high speed can be excerpted, to certain accuracy, from their counterpart at low speed that are obtained experimentally by trial runs. This approach is motivated by the accurate measurement system which was built in this work. It is a kind of combined or unified approach which accounts for the sensors and measurement system transfer functions but require prior knowledge of the rotor-bearing system characteristics. The model can be built using FE formulation considering the effects of rotating components on the stiffness and damping matrices. Some available rotordynamics codes may be utilized to achieve this goal. The steps of the proposed approach can be listed as follows:

1. While the machine is running at its operating speed, measure the initial response at the selected measuring planes. Reduce the speed to any other required speed and measure the response. Repeat at each balancing speed.

2. Select an appropriate speed for trial runs which is below the dangerous critical speed

3. Evaluate the IC for the selected speed by the standard trial runs procedure.

4. Excerpt the IC at other speeds from the following equation:

$$A_{ij}^m = \frac{X_{ij}^m}{X_{ij}^1} A_{ij}^1 \qquad (6)$$

Where X_{ij}^m is the calculated response in measuring plane i due to attaching trial mass M_j only at correction plane j at speed m, 1 denotes the first employed balancing speed.

5. Apply IC matrix inversion or least-squares method and multiply by the initial responses obtained in step (1) to find the initial unbalance.

Note that the measured response V is related to the displacement vector X by the transfer function of the vibration sensors and measurement system. It is assumed that the dynamic characteristics of the sensors and measurement system are not affected

too much along the speed range and this can easily be achieved by the advanced hardware and software capabilities.

4. The Proposed Measurement System

The measurement system for rotor balancing is completely designed and built in this work. Its function is to measure vibration signals, applying signal conditioning, amplification and filtration and then converting into digital data and pass them to the computer for analysis. Featuring fully two functional input channels plus one reference signal input channel, high dynamic and frequency ranges. Fig. 2 shows a picture for the analog section while Fig. 3 shows the block diagram for the proposed system. The device consists of three main parts, USB module, analog module and power supply. The USB module is a general-purpose data acquisition module manufactured by National Instrument (NI USB6008) featuring 4-differential 12-bit ADC input channels and 10 ksample/s sampling rate with trigger input. Three analog input channels are used to digitize the signals of left and right vibration probes and the reference sensor. Each channel is sampled at 3300 sample/s which can is sufficiently high for the commonly used balancing machines.

Figure 2 The Analog Section

The measured signals are conditioned, amplified (in two stages) and filtered in this module. 4-pole programmable low pass filter in line with 2-pole fixed low pass filter are implemented to eliminate undesired noise and high frequency vibration. Also, two programmable amplification stages were implemented to increase dynamic range and accuracy. The first amplification stage ranges from 1 to 100 and it is constructed utilizing LTC6912 IC from Linear Technology, while the second one has four amplification settings 1, 10, 100 and 1000 and it utilizes PGA-204 from Burr Brown.

From previous discussion, the amplitude and phase of the measured signals must be accurately detected. Most balancing devices rely on analog signal processing to separate the unbalance signal and detect its amplitude and phase. Analog techniques include synchronous filtering, Wattmetric filtering and narrow band-pass filtering.
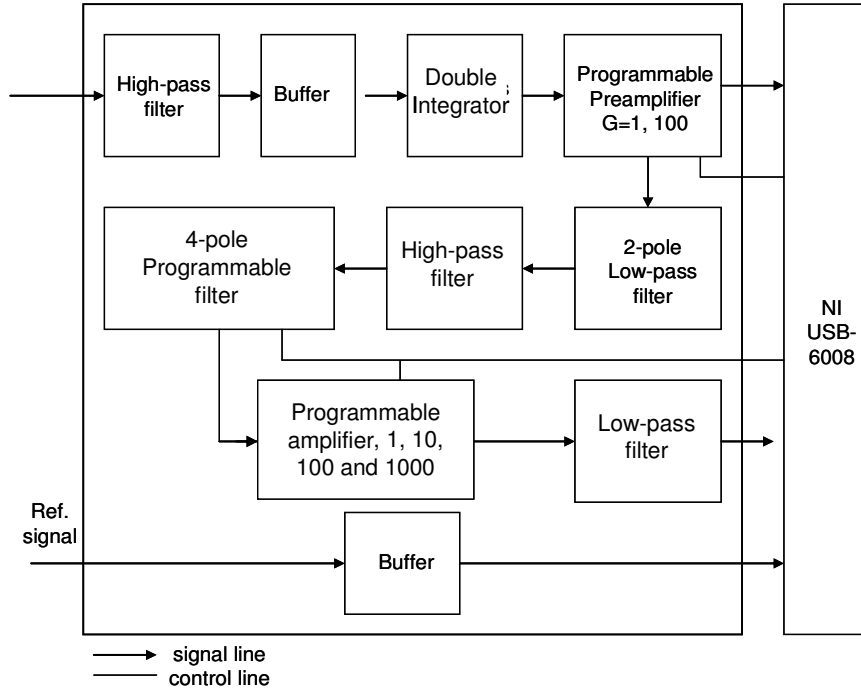
Figure 3 Block Diagram of the Analog Module

The proposed technique is based on digital Wattmetric detection method. According to Fourier series theorem, any signal can be written as a sum of harmonically related sine and cosine components, or in other words, imaginary and real parts. The amplitude and phase of each frequency component can be obtained from these two parts. Let a_n, b_n be the real and imaginary parts, respectively. Then the amplitude A_n and phase angle θ_n can be found as follows [16]:

$$a_n = \frac{1}{p} \int_0^{2p} f(t) \cos \omega_n t \, dt \ ,$$

$$b_n = \frac{1}{p} \int_0^{2p} f(t) \sin \omega_n t \, dt$$

$$A_n = \sqrt{a_n^2 + b_n^2} \ , \quad \theta_n = \tan^{-1} \frac{b_n}{a_n} \qquad (7)$$

Where p is half period, $f(t)$ is the periodic time signal, ω_n is the frequency of the n component. By setting ω_n to the rotational speed (1xRPM), the amplitude and phase of the unbalance frequency component can be found from the discrete version of eq. (7) and this is the basis of digital Wattmetric technique. However, this requires that the reference speed be accurately measured. To achieve better certainty in the measured amplitude and phase and minimize noise disturbance, averaging over a number of cycles can be employed.

5. Results and Discussion

To verify the validity of the proposed technique, a multi-disk rotor was prepared and tested using vibration simulator shown in Fig. 4. The apparatus is equipped with a

8

variable speed DC motor capable of rotating at 12000 RPM. The rotor consists of a steel shaft of 7mm diameter and length of 900mm with four attached disks of 75mm diameter and 9mm thickness placed at as shown in Fig. 5. The rotor is supported by two swivel (zero-moment) ball bearings on the ends characterized by horizontal and vertical stiffness of K_{xx}= 20x10^6 and K_{yy}= 30x10^6 N/m respectively. The FE model of the rotor-bearing system is defined under the Matlab Toolbox developed by Friswell *et al.* [17] with seven nodes, six Timoshenko beam elements as shown in Fig. 5. This Toolbox considers the gyroscopic effect and internal damping of the rotating components. Two accelerometers are used to measure the response and convert it to displacement by double integration while a photoelectric sensor is used to measure phase angle and rotation speed. The signals are sampled at 3300 sample/sec for each channel and a record length of 4096 samples is considered. The amplitude and phase are extracted using the digital Wattmetric technique. Visual and interactive PC software is written using Microsoft Visual Studio 6.0 to guide the user through balancing procedure. Unbalances are introduced in the attached disks by fixing small bolts of known weights at given angles at the outer race of each disk.
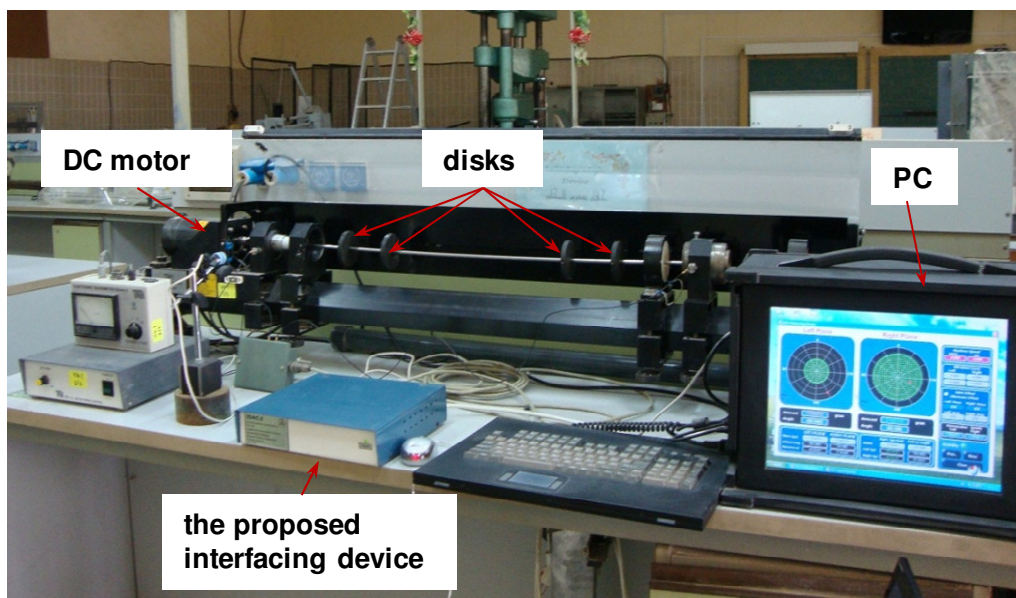
Figure 4 The Experimental Setup
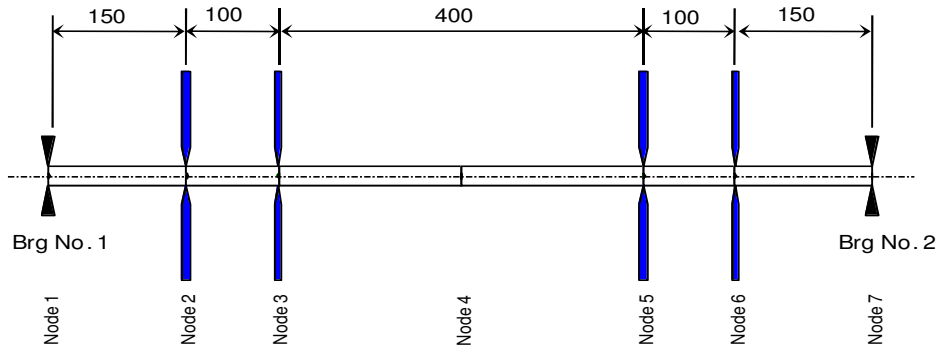
Figure 5 The FE Model

The correction planes are selected to be the disks themselves. The multi-speed algorithm is employed to find the IC matrix at two speeds 800 and 3000 RPM and calculate the unbalance. The first balancing speed lies between the first and second critical speeds at 490 and 1430 RPM respectively, while the second speed is well above the second critical but less than the third critical speed at 4650 RPM. Trial masses are attached individually at each balancing plane (disk). The influence coefficients found at 800 RPM are used to excerpt their counterpart coefficients at 3000 RPM according to eq. (6) from the theoretical model. Two tests are executed with different initial unbalance and trial masses. Vibration data for the initial and trial runs at both balancing speeds are listed in Table 1 for the two executed tests. The calculated IC at balancing speed of 3000 RPM from trial runs and the excerpted IC according to the proposed scheme are shown in Table 2.

Table-1 Trial Masses and Vibration Data at 800/3000 RPM Balancing Speeds

Test	Trial Masses			Vibration of Bearing-1		Vibration of Bearing-2	
	Trial[*]	Mass (gm)	Angle°	Amp. (μm)	Phase°	Amp. (μm)	Phase°
Test-1	Initial	-	-	3.37/16.0	230/150	1.02/10.3	95/170
	Trial-1	12.0	90	2.43/24.7	205/126	1.26/16.0	266/130
	Trial-2	12.0	45	3.51/26.2	229/187	2.62/9.0	207/147
	Trial-3	12.0	0	5.90/13.3	206/141	1.05/26.7	102/175
	Trial-4	12.0	225	1.18/21.8	238/177	1.23/19.6	185/198
Test-2	Initial	-	-	3.42/16.3	230/149	1.02/10.5	95/169
	Trial-1	23.0	0	2.73/11.2	287/46	4.52/9.33	167/118
	Trial-2	23.0	90	3.57/26.5	231/238	4.95/14.4	268/137
	Trial-3	23.0	180	4.57/23.4	325/160	1.02/20.6	83/5
	Trial-4	23.0	45	7.71/21.8	227/91	3.74/18.3	57/73

*: Trial No.*x* refer to attaching trial mass *x* at plane No.*x*, planes are numbered from left to right.

Table-2 Calculated and Excerpted IC at 3000 RPM Balancing Speeds

Test	Correction Plane	Measuring Plane			
		Bearing-1		Bearing-2	
		Calculated	Excerpted	Calculated	Excerpted
Test-1	Disk-1[*]	9.06∠42	7.69∠49	8.23∠41	6.93∠44
	Disk-2	12.9∠226	27.7∠227	3.66∠38	3.50∠44
	Disk-3	3.64∠43	3.73∠47	12.8∠225	15.2∠204
	Disk-4	8.12∠48	8.21∠44	9.2∠47	9.77∠44
Test-2	Disk-1	9.70∠44	7.98∠46	8.60∠44	6.71∠45
	Disk-2	13.6∠224	9.51∠238	3.38∠41	3.37∠44
	Disk-3	3.45∠45	3.38∠45	13.6∠224	13.4∠219
	Disk-4	8.23∠45	8.22∠44	9.64∠44	9.41∠44

*: disks are numbered from left to right

It is clear from Table 1 that vibration data reflects the effect of trial mass on the response amplitude and phase. In the second test, the relatively large trial masses have caused higher vibration in general as compared to the first test. Also, not only the vibration amplitude is changed when changing the balancing speed but also the phase angle. From Table 2, the calculated and excerpted IC show good matching in general except for some coefficients. Also, the ICs obtained in the second test are comparable to those obtained in the first test indicating accurate measurement system and analysis technique. The detected unbalance using the multi-speed balancing and the proposed approaches are tabulated in Table 3 inline with the introduced initial unbalance.

Table-3 Initial and Detected Unbalance Masses Results

Test	Plane	Initial Unbalance		Detected from Multi-speed Scheme		Detected from the Proposed Scheme	
		Mass (gm)	Angle°	Mass (gm)	Angle°	Mass (gm)	Angle°
Test-1	Disk-1	8.0	180	8.8	177	6.9	175
	Disk-2	6.0	0	6.2	356	5.7	352
	Disk-3	6.0	45	5.5	44	6.1	50
	Disk-4	8.0	90	6.9	94	7.2	81
Test-2	Disk-1	12.0	180	12.2	181	13.1	186
	Disk-2	8.0	0	8.1	2	8.7	5
	Disk-3	8.0	45	8.4	46	7.0	41
	Disk-4	12.0	90	12.4	91	11.0	94

Inspecting Table 3, it can be found that the estimated unbalance masses for both algorithms are more close to the introduced masses in test No.2. The reason behind this observation is that the initial unbalance is high enough to introduce relatively large unbalance signal with less noise effect. Also, the trial masses employed in the second test are approximately as twice as the trial masses in the first test (23 gm vs. 12 gm) which make the change in vibration signal more noticeable and measurable. The inaccuracies in the proposed scheme may be reasoned to the inaccurate bearing model especially the effect of rotational stiffening normally introduced in roller and ball bearings [17]. Also, the spin stiffness of the bearing is actually different from zero especially at higher speeds. More accurate results are expected when these effects are included but this is beyond the scope of the current work.

6. Conclusions

A reliable and accurate system for dynamic balancing has been built. The proposed digital Wattmetric technique in combination with the programmable amplifiers and low pass filter, are proved to give very accurate results. The system offer very successful option to upgrade and enhance already exist balancing machines whether they are small or large sized. The multi-speed balancing technique is proved to be sufficiently accurate. The proposed IC calculation scheme reduces the time required for flexible rotor balancing by eliminating the trial runs at some high speeds. The results of the proposed scheme are comparable with that obtained by multi-speed flexible rotor balancing scheme.

References

[1] "ISO 1925 Balancing Vocabulary", International Standards, Switzerland, 1990.

[2] J. M. Tessarzik, R. H. Badgley And W. J. Anderson, "Flexible Rotor Balancing by the Exact Point-Speed Influence Coefficient Method", *ASME J. of Eng. for Industry*, 94(1), 1972, pp. 148-158.

[3] A.G. Parkinson, "The balancing of flexible rotors", *Proceedings of the IUTAM Symposium on Dynamics of Rotors*, Lyngby 1974, pp. 413–435.

[4] T. P. Goodman, "A least-squares method for computing balance corrections", *Trans. American Society of Mechanical Engineers, J. of Eng. for Industry,* 86(3), 1964, pp. 273-279.

[5] S. Saito and T. Azuma, " Balancing of flexible rotors by the complex modal method", *Trans. American Society of Mechanical Engineers, J. of Vibration, Acoustics, Stress, and Reliability in Design,1983,* 105, pp. 94-100.

[6] W. L. Meacham, P. B. Talbert, H. D. Nelson and N. K. Cooperrider, " Complex modal balancing of flexible rotors including residual bow", *Journal of Propulsion,* 1988, 4, pp. 245-251.

[7] M.S. Darlow, "Balancing of High Speed Machinery: Theory, Methods & Experimental Results", *J. of MSSP* **1**(1), 1987, pp. 105-134.

[8] Y. Kang, C. P. Liu and G. J. Sheen, "A modified influence coefficients method for balancing unsymmetrical rotor-bearing systems", *Journal of Sound and Vibration*, (1996) 194(2), pp. 199-218.

[9] Y. Kang, Y.-P. Chang, M.-H. Tseng, P.-H. Tang And Y.-F. Chang, "A Modified Approach Based on Influence Coefficient Method for Balancing Crank-Shafts", *J. of Sound and Vibration* (2000) 234(2), 277-296.

[10] J. K. Sinha, M. I. Friswell, A. W. Lees and R. K. Sinha, "Robust Method for the Rotor Unbalance Estimation", Proceedings of VETOMAC-2, 16-18 December, 2002.

[11] C. Zang , A. W. Lees and M. I. Friswell, "Multi plane balancing of a rotating machine using a single transducer", *Proceeding of the Sixth International Conference on Rotor Dynamics*, Sydney, Australia, Sept. 30 to October 4, 2002.

[12] F. Seve, M.A. Andrianoely, A. Berlioz, R. Dufour and M. Charreyron, "Balancing of machinery with a flexible variable-speed rotor", *Journal of Sound and Vibration*, 264 (2003), pp. 287–302.

[13] S. Zhou, W. Stephen K. Dyer, J. Shin and J. Ni, "Extended Influence Coefficient Method for Rotor Active Balancing During Acceleration", *ASME J. of Dynamic Systems, Measurement, and Control*, March 2004, Vol. 126, pp. 219-223.

[14] X. Yu, "General Influence Coefficient Algorithm in Balancing of Rotating Machinery", *International J. of Rotating Machinery*, Vol. 10, 2004, pp. 85–90.

[15] E. S. Gutierrez-Wing, "Modal Analysis of Rotating Machinery Structures", *PhD. Thesis, Department of Mechanical Engineering, Imperial College London*, Sept. 2003, Ch. 3-4.

[16] Randall, R. B., "Frequency Analysis", Brüel & Kjær, Denmark, 1987.

[17] M.I. Friswell, J.E.T. Penny, S.D. Garvey and A.W. Lees, "Dynamics of Rotating Machines", *Cambridge University Press*, 2010, **ISBN** 9780521850162.