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Analytical Simulation of Natural Convection Between Two Concentric Horizontal Circular Cylinders: A Hybrid Fourier Transform-Homotopy Perturbation Approach

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ABSTRACT

In this study, a hybrid method combining the homotopy perturbation method (HPM) and Fourier transform (FT) is developed and denoted as FT-HPM. This novel algorithm leverages the properties of convolution theory to facilitate calculations and is applied to obtain approximate analytical solutions for the two-dimensional natural convection between two concentric horizontal circular cylinders maintained at various uniform temperatures. The effects of Rayleigh number, Prandtl number, and radius variation on the fluid flow (air) and heat transfer are investigated. Furthermore, velocity distributions are examined and discussed, while the Nusselt number is calculated to represent local and general heat transfer rates through the relevant Nusselt numbers. The convergence of the FT-HPM method is discussed theoretically, with the formulation of theorems that are applied to the results of the obtained solutions. Tables and graphs of the analytical solutions demonstrate the feasibility and potential usefulness of the proposed algorithm for addressing various nonlinear problems, particularly natural convection problems. This research contributes to the understanding of natural convection in complex geometries and provides a foundation for future studies in this field.

1. INTRODUCTION

In recent years, the study of heat transfer theory due to natural convection has garnered significant interest among scientists and engineers, owing to its crucial applications in various scientific and technological fields. These applications encompass heat storage systems, cooling of electronic components, solar collectors, nuclear reactors, and aircraft cabin insulation, among others [1-3]. One particularly important problem in natural convection, which has intrigued numerous researchers, involves the phenomenon between two concentric horizontal circular cylinders maintained at varying uniform temperatures.

Early contributions to this area include the experimental solutions provided by Crawford and Lemlich [4], who conducted a numerical study to approximate the steady-state differential equations with suitable difference equations. Their work explored the effect of diameter ratios at 2, 8, and 57, in the case of a Prandtl number of 0.7. Subsequent researchers have built upon this foundation, developing numerical and analytical solutions to the problem. For instance, Mack and Bishop [5] performed an analytical investigation of natural convection between two horizontal circular concentric cylinders with small temperature differences. Their approach utilized Rayleigh number power series to solve the governing mathematical equations, and they discussed the characteristics of their solutions, such as local and global heat transfer rates, streamline formation, velocity, and temperature distributions. Additionally, they analyzed the effects of radius ratio, Prandtl number, and Rayleigh number.

Kuehn and Goldstein [1] employed the finite difference method in a numerical study to obtain experimental solutions for natural convection between two horizontal circular concentric cylinders. Tsui and Tremblay [6] conducted a theoretical-numerical study for Grashof numbers ranging from 7×102 to 9×104, with a fixed Prandtl number of 0.7, and diameter ratio differences of 1.2, 1.5, and 2. Pop et al. [7] derived analytical solutions for transient natural convection between two concentric horizontal circular porous cylinders using the method of matched asymptotic expansions. Their approach divided the problem into three regions (inner boundary layer, core region, and outer boundary layer) and determined the analytical solutions for each region separately. They observed that their solutions markedly differed from steady-state solutions.

Sano and Kuribayashi [8] studied transient natural convection around a horizontal circular cylinder using the method of matched asymptotic expansions, resulting in new analytical solutions. Their research aimed to fill a gap in previous work by considering the displacement effect. Hassan and Al-Lateef [9] presented a numerical study to find the solution for two-dimensional transient natural convection heat transfer from isothermal horizontal cylindrical annuli. They employed the alternating direction implicit (ADI) method to solve both the vorticity and energy equations, while the stream function equation was solved using the successive over relaxation (SOR) method. Their results were summarized by Nussult number versus Grashof number curves, with diameter ratios and Prandtl number as parameters. Touzani et al. [10] numerically investigated natural convection in horizontal annulus with two heating blocks, noting that heat transfer was more significant in the upper region of the annulus and that the presence of the block improved overall heat transfer. Al-Saif and Al-Griffi [11] proposed a new algorithm that combined