Partial Differential Equations in Applied Mathematics 8 (2023) 100536

Contents lists available at ScienceDirect

Partial Differential Equations in Applied Mathematics

journal homepage: www.elsevier.com/locate/padiff

Applications of q-difference equation and homogeneous q-shift operator ${}_r\Phi_s(D_{xy})$ in q-polynomials

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ARTICLE INFO

MSC:
05A30
33D45

Keywords:
Generating function
Mehler's formula
Rogers formula
A transformational identity

ABSTRACT

In this paper, the generalized homogeneous q-shift operator is constructed. The q-difference equation is then utilized to construct numerous polynomial q-identities, such as the generating function and its extension, Rogers' formula and its extension, and Mehler's formula and its extension for the generalized q-hypergeometric polynomials. Also demonstrated is a transformational identity involving generating functions for the generalized q-hypergeometric polynomials.

1. Introduction

Basic (or q-) polynomials, q-series, and q-hypergeometric polynomials are fundamental to many branches of mathematical and physical disciplines. The most applications are included in statistics, mechanical engineering, combinatorial analysis, the theory of heat conduction, cosmology, non-linear electric circuit theory, quantum mechanics, Lie theory, and finite vector spaces (see[1–4]). Precisely, we concern with the technique of q-difference equations as one of the fundamental concepts of q-calculus. It is basically related to use a function f that should satisfy the q-difference equation. However, this may difficult to be proved in some aspects as this happened in some studies (see[2,3,5,6]). In this study, we will generalize homogeneous q-shift operator which could help to prove satisfying the q-difference equations by the f function we have. We will concern with using the notations and definitions of q-series concepts in[7] which is practically assumed that $0 < q < 1$.

The q-shifted factorial is defined for $a \in \mathbb{C}$ as:

$$(a;q)_0 = 1, \quad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), \quad (a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

The multiple q-shifted factorials is given by[7]:

$$(a_1,a_2,\ldots,a_r;q)_m = (a_1;q)_m(a_2;q)_m\cdots(a_r;q)_m,$$

where $m \in \mathbb{Z}$ or ∞.

The basic hypergeometric series ${}_r\phi_s$ is presented as follows[7]:

$${}_r\phi_s\left(\begin{matrix}\alpha_1,\ldots,\alpha_r\\ \beta_1,\ldots,\beta_s\end{matrix};q,x\right) = \sum_{n=0}^{\infty}\frac{(\alpha_1,\ldots,\alpha_r;q)_n}{(q,\beta_1,\ldots,\beta_s;q)_n}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-r}x^n,$$

where $q \neq 0$ when $r > s+1$. Note that

$${}_{r+1}\phi_r\left(\begin{matrix}\alpha_1,\ldots,\alpha_{r+1}\\ \beta_1,\ldots,\beta_r\end{matrix};q,x\right) = \sum_{n=0}^{\infty}\frac{(\alpha_1,\ldots,\alpha_{r+1};q)_n}{(q,\beta_1,\ldots,\beta_r;q)_n}x^n.$$

The q-binomial coefficient is defined as[7]:

$$\begin{bmatrix}n\\k\end{bmatrix} = \frac{(q;q)_n}{(q;q)_k\,(q;q)_{n-k}} \quad for \quad 0 \leqslant k \leqslant n,$$

where $n,k \in \mathbb{N}$.

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https://doi.org/10.1016/j.padiff.2023.100536
Received 1 May 2023; Received in revised form 26 June 2023; Accepted 3 July 2023