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THE GENERALIZED q-OPERATOR ${}_r\Phi_s$ AND ITS APPLICATIONS IN q-IDENTITIES

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Abstract. Based on the basic hypergeometric series ${}_r\phi_s$, we construct a new generalized q-operator ${}_r\Phi_s\left(\begin{array}{c} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{array};q,-c\theta\right)$ and obtain some of its identities. Using these identities, we generalize several well-known q-identities, such as the q-Gauss sum, the q-Chu-Vandermonde sum, and the q-Pffaf-Saalschütz sum.

Keywords: The q-operator, q-Gauss sum, q-Chu-Vandermonde sum, q-Pffaf-Saalschütz sum.

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1. Introduction and Notations

In this paper, we will follow the notations that were used in [5]. We assume that $|q|<1$. Let a be a complex variable. The q-shifted factorial is defined by [5]

$$(a;q)_0=1,\qquad (a;q)_n=\prod_{k=0}^{n-1}(1-aq^k),\qquad (a;q)_\infty=\prod_{k=0}^{\infty}(1-aq^k).$$

We adopt the following compact notation for the multiple q-shifted factorial:

$$(a_1,\ldots,a_r;q)_n=(a_1;q)_n\ldots(a_r;q)_n,$$

where n is an integer or ∞.

The basic hypergeometric series ${}_r\phi_s$ is defined by:

$${}_r\phi_s\left(\begin{array}{c} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{array};q,x\right)=\sum_{k=0}^{\infty}\frac{(a_1;q)_k\cdots(a_r;q)_k}{(q;q)_k\,(b_1;q)_k\cdots(b_s;q)_k}\left[(-1)^kq^{\binom{k}{2}}\right]^{1+s-r}x^k,$$

where $r,s\in\mathbb{N}$; $a_1,\ldots,a_r,b_1,\ldots,b_s\in\mathbb{C}$; and none of the denominator factors evaluate to zero.

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