Combination of Graph Reduction and Tie-set Techniques for Network Reliability Assessment

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***Abstract*—Problem statement: The reliability of the communication network is used as one of the quality of service factors defining the availability and the resilience of the operating networks. Many classical algorithms exist but most of them fail to be applied for real-time reliability assessment in complex networks where the calculation load is increased in function of the complexity. Approach: In the present work, a new algorithm is proposed to resolve the problem of the reliability evaluation for all kinds of networks resulting an exact solution. The algorithm is based on the combination of two classical reliability evaluation methods giving birth to an efficient hybrid algorithm. Graph transformation method and tie sets method are fused into multi stages algorithm. The network passes through many simplification layers based on series, parallel, and edge factoring graph transformation to transform a complex network into a simple analogical network. The simplified topology is then used as input for a classical algorithm; here the tie sets to find the two terminal reliability. The transformations are selected according to their frequency of occurrence. Results: A simulation is performed on a random complex network to compare the performances of the new algorithm to the existing algorithms. The results of the proposed algorithm are compared to those resulting from the use of tie sets algorithm in term of computing time. Conclusion: The implementation of the algorithms by MATLAB shows a noticeable improvement in the time required for reliability calculation in comparison with classical algorithms. Also, there is no limitation on the size and complexity of the network.**

Keywords— Network Reliability, Graph Reduction, tie-set

I. INTRODUCTION

The reliability is the probability that a system will function properly within a specified period of time under a number of constraints like failure rate, system complexity, and operating conditions [1]. This means reliability is a quality measurement over time, affected by the system performances and its environment. The communication networks reliability is an important Quality of Service (Q o S) factor in the study of the system performances especially those based on heterogeneous networks. It checks how long the infrastructure of a network is operational without interruption providing a functionality capability indication. Many technologies rely on reliability calculations in communication networks to ensure that information is sent and received with good reliability. A reliable network contains redundant path between the source node and destination node, which means that in the case of any link failure, it can be replaced by a backup link giving an alternate source-destination path. Network topological presentation using graph theory is the main factor in the network reliability researches. Communications networks are becoming more and more complex because of the development of modern technologies, applications, and services. Various classical probabilistic methods are used for network reliability estimation such as Graph Reduction Method (GRM) [2], State Space Enumeration Method (SSEM) [3], Tie-Sets [4], Cut-Sets [5], and approximation methods [6, 7]. The reliability assessment for complex networks requires a complicated computing operation due to the complexity and availability of many possible paths between various nodes. Therefore, the classical computational methods fail to evaluate the reliability under multiple operational conditions. New researches based on the use of Artificial Neural Network [8], and clustering technique [9] have been proposed to simplify the computation for complex networks. Hybrid algorithms composed of selected classical methods have been proposed for complex communication reliability assessment. Usually, the algorithm starts by simplifying the original network by clustering or graph transformation before applying a second classical algorithm on the simplified analogous network [10]. The two-terminal reliability evaluation between a source node n_s and a destination node n_d is presented in [11]. The general algorithm is presented as a holistic technique based on two sub-algorithms that provide a simple calculation of the minimum cut-sets for complex networks. Other researches are interested in the evaluation of all-terminal (overall network) reliability. The two-terminal algorithm is modified to consider all terminal pairs such that a set of operational edges provides communication paths between every pair of nodes (n_i, n_j with $n_i \neq n_j$) [12]. The problem of k-terminal reliability, where only part of the network is considered, is presented in [13]. The k-terminal reliability is the general method varying between two-terminal to all-terminal methods. The proposed algorithm is implemented using three simplification stages based on graph reduction technique followed by the application of tie set algorithm. The article is organized as follows: the theoretical backgrounds of network modeling, tie-set method and GRM are presented in Section II; the proposed algorithms are presented in Section III; a case study of simple network is implemented, and simulation results from the developed algorithm is compared to those from classical algorithm in Section IV; Section V concludes the article.

II. THEORITICAL BACKGROUND

A. Network Topology Modeling

A Communication network is modeled by a graph G = (N, L), where N is the set of communication nodes while L is the set of undirected/ bi-directed links connecting the network.

Node and link individual reliabilities obey to the binomial distribution law where each element has two possible states, either success (working) with probability p or failure with the complement probability $q = 1 - p$. The event-state of a network component (working or failed) is considered independent from the other components event-state. A 3-dimention matrix $M[N, N, K]$ describing the network topology where K is the maximum number of parallel links between two nodes in the network. The matrix element $M[i, j, x]$ represents the probability of the x^{ieme} unidirectional link connecting node-pair (n_i, n_j). M is a multi-layer matrix where the first layer shows the basic network connectivity and the x^{ieme} layer describes the network topology composed of the connectivity between node-pair having x links parallel between then with $1 \leq x \leq K$. If there is no link between two nodes, the corresponding element in M is set to zero.

B. Graph Transformation Technique

The Complex networks can be simplified into a simpler analogous network by a series of graph transformations (graph reduction). Series/Parallel [1], delta-to-star [14], and edge factoring [15] are considered among the most popular transformations. Series transformation is the most widely used due to its frequent presence in the functional networks. If three nodes are connected in series as shown in fig. 1-a, the node 2 can be removed, and the new link reliability is calculated by:

$$R_{1\,3} = \mathrm{P}_{12} \times \mathrm{P}_{23}, \tag{1}$$

Parallel simplification is also a frequent simplification when there are two or more links in parallels between two nodes as in fig. 1-b. These links are replaced by one link with reliability equal to:

$$R_{12} = 1 - \prod_{i=1}^{n}(1 - p_i) = 1 - \prod_{i=1}^{n} q_i \tag{2}$$

Where $q_i = 1 - p_i$

The edge factoring is more complex than the previous ones since it demands to consider two cases then summing them in the final equation to evaluate the analogous network reliability. This graph partitioned into two subgraphs as shown in fig. 2 according to the state of the link between the nodes i_1 and i_2. The application of the law of probability twice for the A-graph and B-graph and integrating the solutions in one equation gives the final solution. If the link i_1 -i_2 is up ($p_5 = 1$), then the topology is reduced to A-graph:

$$P(A) = (1 - q_1 q_2)(1 - q_3 q_4) \tag{3}$$

Considering link *i_1 -i_2* is down ($p_5 = 0$), B-graph gives:

$$P(B) = 1 - (1 - p_1 p_3)(1 - p_2 p_4) \tag{4}$$

Using probability theory and combining (3) and (4) the equivalent reliability between nodes $x_1 - x_2$ becomes:

$$R_{x_1 - x_2} = p_5 \; P(A) + q_5 \, P(B) \tag{5}$$

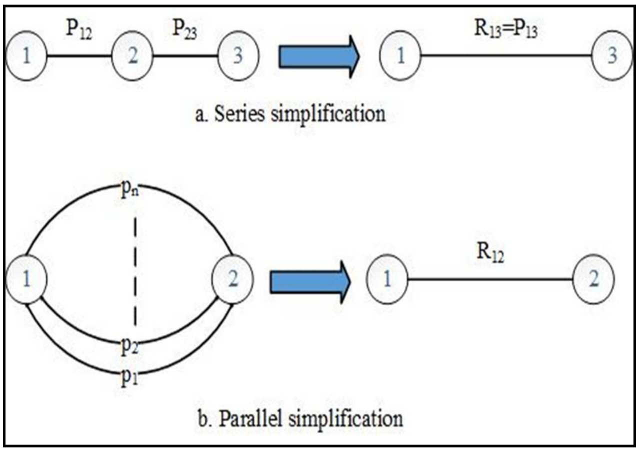

Fig. 1 Series-parallel simplification

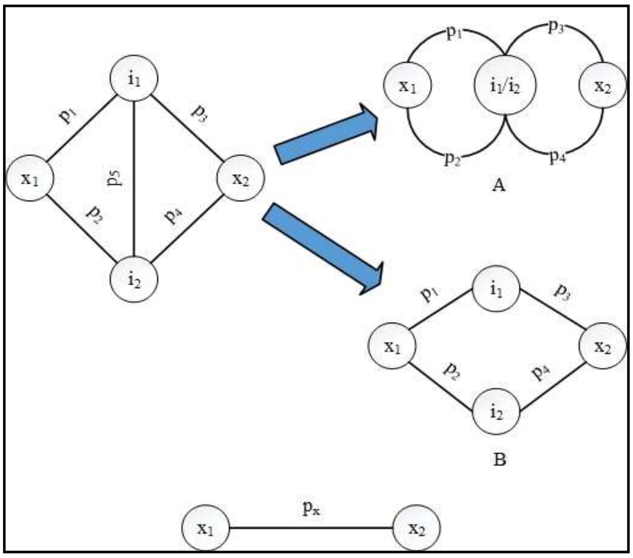

Fig 2. Edge factoring simplification

C. Tie-set Technique

The tie-set technique is based on enumerating all the links groups forming a loop-free path between source nodes $n_s$and the destination node n_d called tie sets. Therefore, this method is applied only to simple and medium networks because the enumeration process increases exponentially with the network complexity. Tie-set algorithm is an exact solution used to evaluate two-terminal, K-terminal, and all-terminal network reliability. It starts by finding all tie sets ($T_1, T_2, \ldots, T_i$) between a source-destination pair and applying the expansion equation (Poincare):

$$R_{sd} = P(\, T_1 + \, T_2 + \cdots + \, T_i) \tag{6}$$

Where R_{sd} is the reliability between the source node and destination node.

Tie sets ($T_1, T_2, \ldots, T_i$) are not disjoint link groups because they have common links. This require the expansion of (6) into:

$$R_{sd} = P[(\,T_1) + P(\,T_2) + \cdots P(\,T_i\,)] - [P(T_1\,T_2\,) + P(T_1\,T_3\,) + \cdots P(T_r\,T_k)_{r \neq k}\,] + [P(T_1\,T_2\,T_3\,) + P(T_1\,T_2\,T_4) + P(\,T_r\,T_k T_j\,)_{r \neq k \neq j} + \cdots + (-1)^{i-1}\,[P(T_1\,T_2\,T_3 \ldots\,T_i)] \tag{7}$$

III. Proposed Algorithm

This developed algorithm consists of two stages: the first is the simplification using GRM algorithms, and the second stage is the application of the tie-set algorithm. The application of the algorithm starts by the input network topology presented by the matrix (M). GRM stage starts by the application of the parallel simplification, then the series reduction, and finishes by applying the edge factoring sub-algorithm. The last stage is the evaluate the reliability evaluation using tie-set algorithm.

A. Initialization

Define The network topology is presented as 3D M-matrix describing the connectivity state of the network. The elements of M are the working probability of an edge or a vertex defined by their corresponding reliability:

$$M_{ijk} = \begin{cases} M[i,j,k]\ \ link\ between\ i-j \\ \ \ 0\ \ \ \ No\ link\ between\ i-j \\ M[i,i]\ Probability\ of\ node\ (i) \end{cases} \tag{8}$$

Where the third dimension defining the link in parallel between *i-j* nodes.

B. Parallel Reduction Algorithm

The developed algorithm starts by removing links in parallel as presented in fig 3. The step begins with a consideration of all links in parallel by checking the values of the element $M[i,j,k]$ for $k > 1$ only. After localization of parallel form, (2) is applied for simplification. The simplification continues until all links in parallel are removed. M becomes with all elements null for $k > 1$, but it is kept as 3D matrix for future simplification (new born parallel links after series simplification).

C. Series Reduction Algorithm

Series node reduction begins with the recognition of nodes in series by considering cases where a node is connected to only two other nodes as depicted in fig. 4. This is carried out by check node connectivity from M, and then the application of (1) for removing the redundant nodes. This process is applied to all nodes except n_s and n_d which are unreducible.

D. Edge Factoring Algorithm

As in the parallel and series sub-algorithm, the edge factoring algorithm starts by the recognition of the topological form and then applying (5) for simplification as presented in fig. 5. Nodes n_s, and n_d, are excluded from this simplification as for series simplification. The recognition of the edge form is accomplished through many steps by verifying the connectivity as in M. After confirmation of the topology-shape searched for, the adjoining nodes are subject to node simplification procedures by removing two mid node n_i, updating the M elements values and setting the diagonal elements M_{ij1} to zero.

E. Algorithm Application

The application of the GRM is performed by step starting from parallel reduction, series, checking again for possible newly born parallel form, edge factoring. The process is repeated till a simplified analogous network topology is reached (fixed by program as links or nodes number). The Matrix M is reduced in dimensions beginning from its original 3D form with size $(N \times N \times K)$ to a 2D matrix with size $(R \times R)$ with $R \ll N$. The original M is transferred by preserving its main ingredients which make the two matrices exactly analogous but given in two forms. The well-known tie-set algorithm is then applied for two-terminal reliability calculation. All loop-free paths between n_s and n_d are found which represent the tie set link-group, and (7) is applied to determine the exact value of the probability.

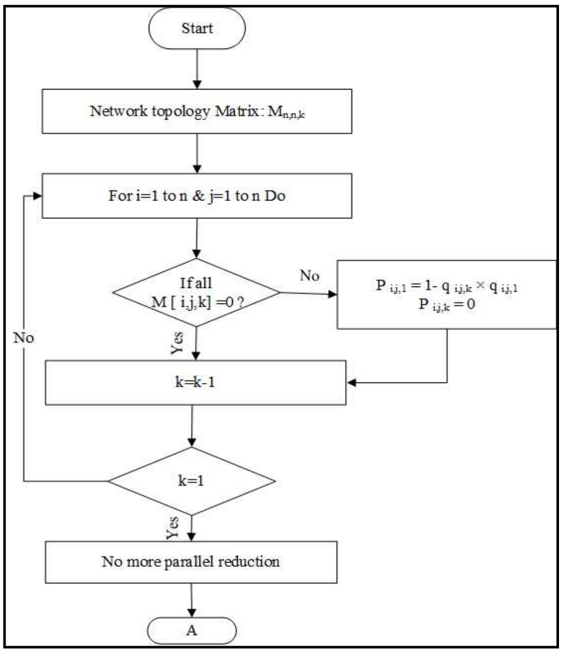

Fig. 3 Parallel reduction algorithm

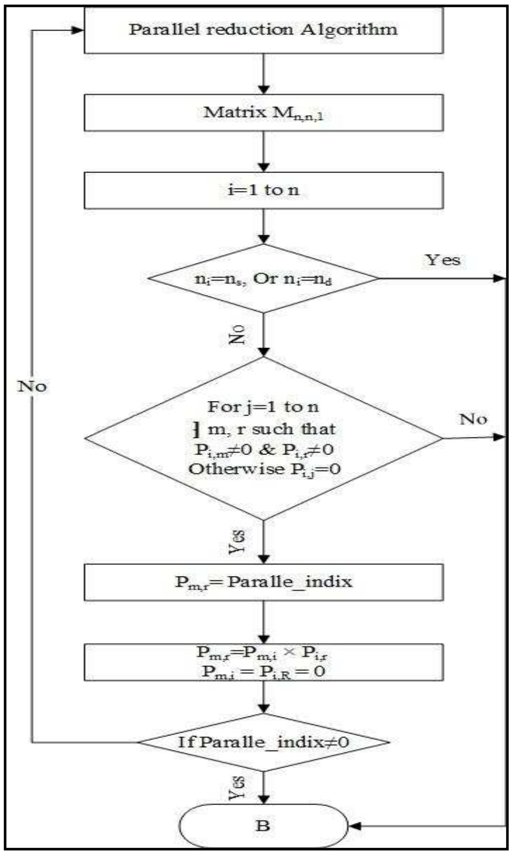

Fig. 4 Series reduction algorithm

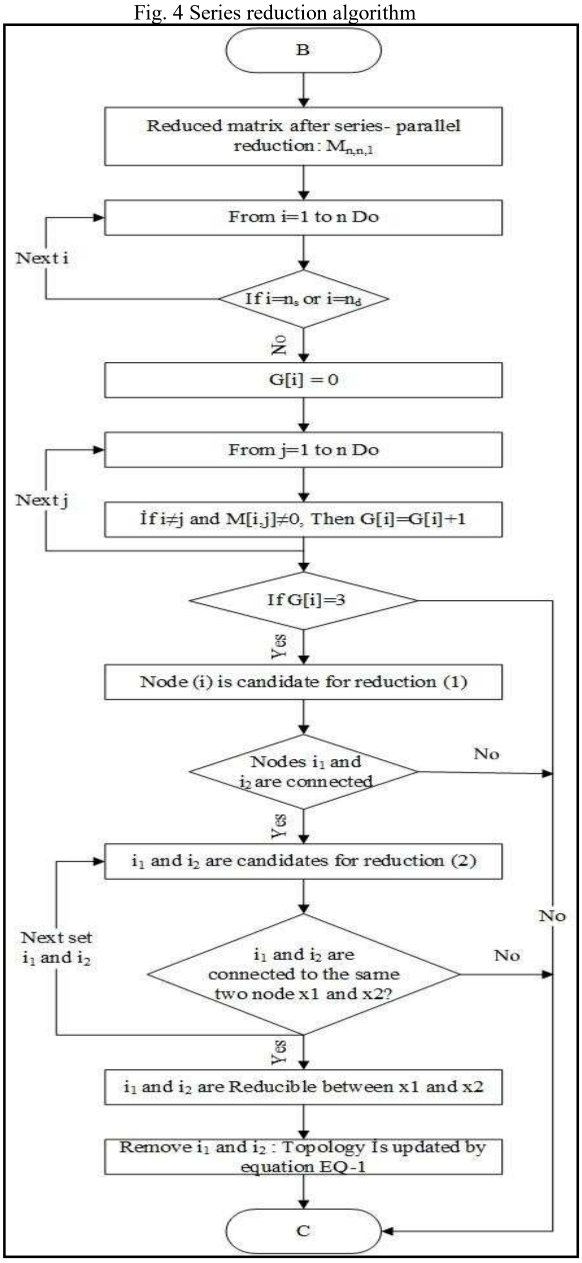

Figure 5 Edge factoring algorithm

IV. Results and Discussion

To validate the new algorithm, the 11-node network in the fig. 6 is implemented by MATLAB. Nodes are considered as perfects (reliability= 100%), and all links are bidirectional with probability ($P_{ik} = 0.9$).

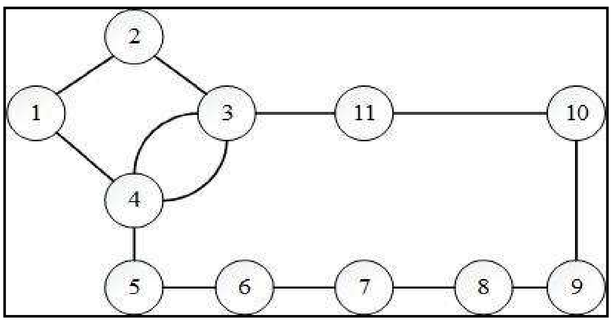

Fig 6. 11-node network

The two-terminal reliability is calculated by both the classical tie-set and the developed algorithm. Results in the table 1 shows that the reliability values, taken as output, are the same for both algorithms which verify the exactness of the new algorithm. The computing time shows a noticeable improvement of the new algorithm compared to the classical tie-set algorithm.

TABLE I. Simulation Results

Source/ Destination		New Algorithm		Classical tie-set algorithm	
n_s	n_d	R_{sd}	*T (sec)*	R_{sd}	*T(sec)*
1	2	0.9877	0.278000	0.9877	0.386711
1	3	0.9876	0.360643	0.9876	0.337407
1	4	0.9877	0.070094	0.9877	0.312645
1	5	0.9582	0.165634	0.9582	0.395695
1	6	0.9462	0.171653	0.9462	0.355118
1	7	0.9460	0.157285	0.9460	0.339759
1	8	0.9531	0.163187	0.9531	0.340648
1	9	0.9547	0.152618	0.9547	0.365134
1	10	0.9810	0.141991	0.9810	0.341206
1	11	0.9747	0.152074	0.9747	0.360114
2	3	0.9957	0.054248	0.9957	0.301098
2	4	0.9952	0.053762	0.9952	0.272399
2	5	0.9657	0.161163	0.9657	0.295380
2	6	0.9539	0.143847	0.9539	0.304391
2	7	0.9537	0.136448	0.9537	0.291275
2	8	0.9612	0.174797	0.9612	0.299286
2	9	0.9629	0.134323	0.9629	0.287770
2	10	0.9897	0.059673	0.9897	0.277919
2	11	0.9851	0.069975	0.9851	0.270368
3	4	0.9992	0.054306	0.9992	0.332738
3	5	0.9693	0.139933	0.9693	0.357934
3	6	0.9573	0.135766	0.9573	0.317370
3	7	0.9571	0.134853	0.9571	0.334702
3	8	0.9643	0.139210	0.9643	0.330254
3	9	0.9659	0.158596	0.9659	0.333649
3	10	0.9926	0.058455	0.9926	0.333547
3	11	0.9844	0.132820	0.9844	0.347534
4	5	0.9698	0.072435	0.9698	0.281501
4	6	0.9574	0.084149	0.9574	0.290915
4	7	0.9572	0.067408	0.9572	0.284159
4	8	0.9641	0.071134	0.9641	0.293724
4	9	0.9657	0.073020	0.9657	0.327185
4	10	0.9920	0.051816	0.9920	0.283865
4	11	0.9838	0.147329	0.9838	0.290767
5	6	0.9698	0.206428	0.9698	0.236969
5	7	0.9679	0.085324	0.9679	0.234377
5	8	0.9572	0.088975	0.9572	0.228494
5	9	0.9571	0.110107	0.9571	0.223072
5	10	0.9657	0.213340	0.9657	0.226845
5	11	0.9562	0.234195	0.9562	0.246202
6	7	0.9900	0.063522	0.9963	0.202574
6	8	0.9679	0.085406	0.9679	0.197952
6	9	0.9661	0.089928	0.9661	0.209847
6	10	0.9571	0.067579	0.9571	0.200371
6	11	0.9462	0.161715	0.9462	0.241970
7	8	0.9698	0.082619	0.9698	0.192564
7	9	0.9679	0.084962	0.9679	0.201259
7	10	0.9572	0.065704	0.9572	0.200632
7	11	0.9461	0.160539	0.9461	0.221014
8	9	0.9900	0.060175	0.9963	0.204609
8	10	0.9679	0.069408	0.9679	0.214387
8	11	0.9551	0.140828	0.9551	0.230198
9	10	0.9698	0.065825	0.9698	0.234026
9	11	0.9569	0.142071	0.9569	0.255761
10	11	0.9851	0.070863	0.9851	0.312101

This can be explained by the topology simplification performed by the developed GRM before the application of tie-set. The superiority of the developed algorithm is expected to be more clear for complicated networks. Also by the GRM simplification, the new algorithm can be applied to all network topology while the classical tie-set can be applied only for simple networks with $N \leq 20$ nodes.

V. Conclusion

The difficulty of network reliability evaluation depends directly on the its complexity. Classical evaluation algorithms like tie-set give exact solution but can require long computing time for complex network. In this work, a hybrid technique for reliability evaluation is proposed. It is based on a combined successive procedures using GRM and tie-set algorithms. GRM starts by simplifying the topology by a structured application of parallel, series, and edge factoring techniques. The topology is transformed to simple one regardless of complexity of the original network. Finally, an exact solution is found by the application of the tie-set algorithm to the simplified topology. As a final stage of calculation, any classical exact method can be used as part of this hybrid algorithm at the place of tie-set algorithm. A case study of a 11-node random network is used to validate the developed algorithm. By reducing the complexity of the network, the same reliability evaluation value is obtained with a much better computing time.

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