



# Generating a new chaotic system using two chaotic Rossler-Chua coupling systems

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## Abstract

This paper presents a new chaotic system by resulting from the coupling two different chaotic systems such as the Rössler system and the Chua system, where the  $x_2$  dynamic of the Rössler system was coupled with  $z_1$  dynamic of the Chua system. Some of the basic dynamics behaviors were examined for new chaotic system by using the Matlab program. In this work was noticed a difference in the time series of the Chua system and this in turn led to a difference in the attractor, as the attractor of the Chua system changed from double scroll to single scroll. This new behavior led to change of the bandwidth of the Chua system that mean the Rössler system affected on the Chua system and this led to an increase in the possibility of using this system in different applications.

**Keywords** Chaos · Nonlinear dynamic · Chua system · Rössler system · Coupling system · Attractor · Time series · FFT

## 1 Introduction

Chaos in dynamics is one of the scientific revolutions of the twentieth century that deepened our understanding of the nature of unpredictability. Chaos Theory is the theory that deals with nonlinear dynamic systems, which exhibits behavior as if it were random, and that behavior results either by the inability to determine the initial conditions or by the potential physical nature of quantum mechanics. So, Chaos is a characteristic which we stick it to the systems that have unexpected long-term future (Marsden et al. 2003) and (Bechtold et. al. 2006). Any dynamic system (variable over time) is highly sensitive to initial conditions; it means that the future of this system will be very dependent and greatly affected by its current state and condition. The small changes in current circumstances can lead to completely different results in the distant future, so the shape of the system is changing completely and be very difficult to predict. The simple change in the values of the primary system variables has a far greater effect than that initial change in the end result, due to other factors and variables in the system, these systems are very sensitive to any change in the initial values or variables whatever the change values are small (Wiggins et al.

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2003) and (Theories 1983). The science of nonlinear dynamics and chaos theory has interested many researchers to develop mathematical models that simulate the fields of nonlinear chaotic physical systems. Nonlinear phenomena arise in all fields of engineering, physics, chemistry, biology, economics, and sociology. Examples of nonlinear chaotic systems include planetary climate prediction models, neural network models, data compression, turbulence, nonlinear dynamical economics, information processing, preventing the collapse of power systems, high-performance circuits and devices, and liquid mixing with low power consumption (Chen et. al 1998), (Cuomo et. al. 1993), and (Lorenz 1963). In the current research, the effect of coupling the variable  $x$  of the Rössler system with the variable  $z$  of the Chua system and studying the behavior of the chaotic system resulting from this coupling was studied.

## 2 A New chaotic system and its analysis

The first chaotic system that was observed in the laboratory, confirmed by computer simulation and mathematically proven was Chua's dynamical system "double scroll" (Chua et. al. 1986), (Matsumoto et. al. 1985), and (Matsumoto 1988). Originally created as an electric circuit, the Chua system also called Chua circuit (Analyzing the circuit using Kirchhoff's circuit laws, the dynamics of Chua's circuit can be accurately modeled by means of a system of three nonlinear ordinary differential equations in the variables  $x_1$ ,  $y_1$ , and  $z_1$ , which represent the voltages across the capacitors C1 and C2 and the electric current in the inductor L1 respectively) has the following three dimensionless equation representations (Stephen Lynch 2004):

$$\begin{cases} x_1' = \alpha[y_1 - x_1 - g(V)] \\ y_1' = x_1 - y_1 + z_1 \\ z_1' = -\beta y_1 \\ g(V) = c_1 x_1 + \frac{1}{2}(d - c_1)[|x_1 + 1| - |x_1 - 1|] \end{cases} \quad (1)$$

where the parameters  $\alpha$ ,  $\beta$ ,  $c_1$ , and  $d$  were 15.05, 25.58, -0.7142857, -1.142 respectively, while the initial condition  $[x_1, y_1, \text{ and } z_1]$  were  $[1.6, 0, 1.6]$  respectively.

In order to have the simplest attractor with a chaotic behavior without a property of symmetry, a Rössler model was suggested also. The Rössler model is a system includes three non-linear ordinary differential equations, which represent a continuous-time nonlinear system that shows a chaotic behavior associated with the fractal characteristics of the attractor. The Rossler system can be described by the following three dimensionless differential equations (Stephen Lynch 2004):

$$\begin{cases} x_2' = -(y_2 + z_2) \\ y_2' = x_2 + ay_2 \\ z_2' = b + x_2 z_2 - c_2 z_2 \end{cases} \quad (2)$$

The variables  $x_2$ ,  $y_2$ , and  $z_2$ , are represent the output voltages from amplifiers in Rossler circuit. These differential equations define a continuous-time dynamical system that exhibits chaotic dynamics associated with fractal properties of the attractor. It has seven term,

one quadratic nonlinearity and three parameter, where  $a, b, c_2 \in R$ , and they are dimensionless parameters, and  $x_2, y_2$  and  $z_2$  are the three variables which evolves with continues time. The values of real parameters firstly are studied by Otto. E. Rössler were  $a$  and  $b=0.2$  and  $c_2=5.7$  the system exhibits a chaotic behavior. The first two equations have linear terms that create oscillations in the variable  $x$  and  $y$ . The last equation has only one nonlinear term ( $xz$ ) so the expected chaotic behavior is appeared from the system. The initial condition  $[x_2, y_2, \text{ and } z_2]$  were  $[1, 1, 0]$  respectively.

The Rössler and Chua systems are not identical components, so the signal that drive out from Rössler system and drive in Chua system will be affecting in behavior of the second system. To employ new chaotic scheme, contact two chaotic systems as shown in Fig. 1. The Rössler system consists of three channels can use any channels to couple with Chua circuit. Now when the two circuit (Rössler-Chua circuits) are coupled (i.e. the output of variable  $x_2$  of Rössler system was coupled with variable  $z_1$  of Chua system, as shown in Fig. 1. The dimensionless states equations of Chua circuit with coupled will be (Raied K. Jamal 2017):

$$\begin{cases} x'_1 = \alpha(y_1 - x_1 - g(x_1)) \\ y'_1 = x_1 - y_1 + z_1 \\ z'_1 = -\beta y_1 - (y_2 + z_2) \\ g(V) = c_1 x_1 + \frac{1}{2} [(d - c_1)(|x_1 + 1| - |x_1 - 1|)] \end{cases} \tag{3}$$

So, this new chaotic system has presented a new three-dimensional continuous autonomous chaotic system with fifteen terms. The new system contains four variation parameters.

### 3 Results and discussion

Most researchers developed a new chaotic system depending on one chaotic system like Chua or Rossler systems. The proposed scheme in this paper based on merging two chaotic systems Chua chaotic system and Rossler chaotic system. Therefore will be added two chaotic systems in Eqs. (1) and (2), a new system is shown in Eq. (3). In this paper, the behavior of the new chaotic system was studied numerically by programming the three differential equations of Chua and Rössler (Eqs. 1, 2, and 3) using the Matlab program, where the differential equations were solved using Runge-Kutta integration of the fourth degree. We note after adding the two chaotic systems, it is noticed 1 and 2 that the terms increased

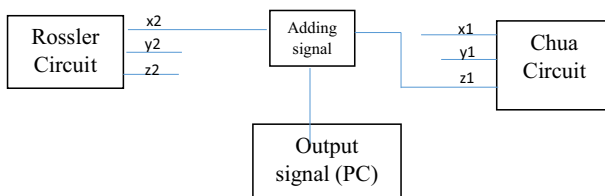
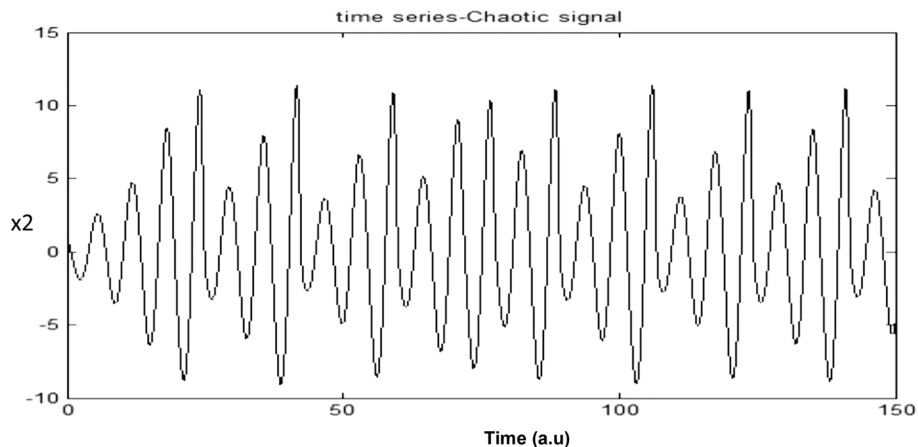
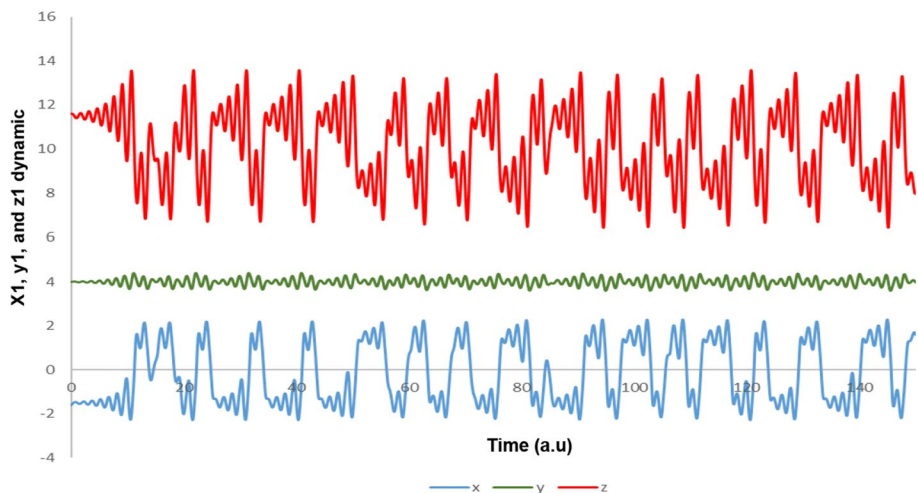


Fig. 1 New chaotic dynamic scheme



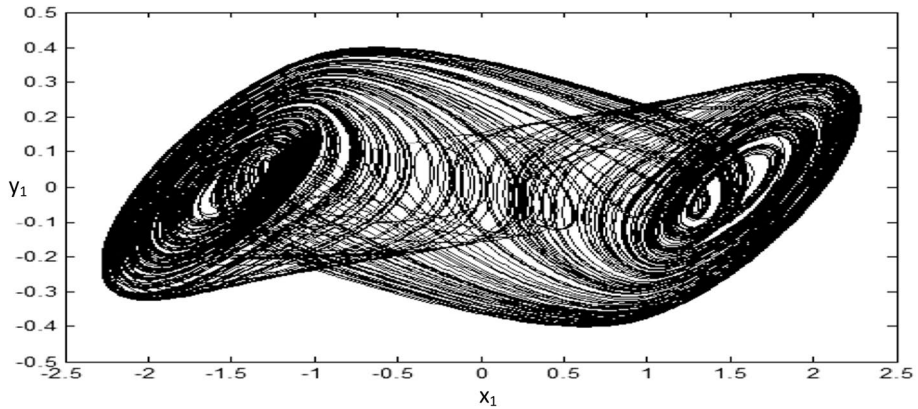
**Fig. 2** Time series of  $x_2$ -dynamics of Rössler system, where the parameters  $a$  and  $b = 0.2$  and  $c_2 = 5.7$ , while the initial condition  $[x_2, y_2, \text{ and } z_2]$  were  $[1, 1, 0]$  respectively



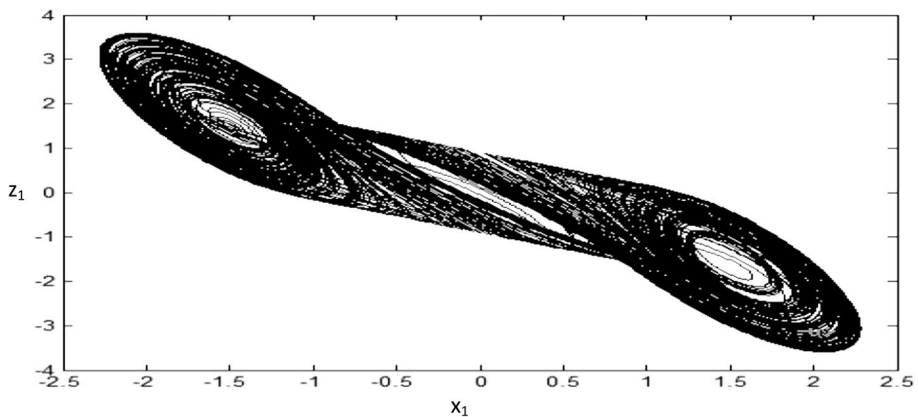
**Fig. 3** Time series of  $x_1$ (*blue line*),  $y_1$ (*green line*), and  $z_1$ (*red line*)- dynamics of Chua system, where the parameters  $\alpha$ ,  $\beta$ ,  $c_1$ , and  $d$  were  $15.05$ ,  $25.58$ ,  $-0.7142857$ ,  $-1.142$  respectively, while the initial condition  $[x_1, y_1, \text{ and } z_1]$  were  $[1.6, 0, 1.6]$

from thirteen to fifteen but to check the new system is suitable for achieving the chaotic requirements, by plot phase plane for a new system.

The time series of the Rössler system shown in Fig. 2, where represent the output voltage in  $x_2$ -dynamics, where the output signal in a range  $+10$  to  $-10$ , while the time range is  $0$  to  $150$  a.u. with interval time of  $0.01$  a.u. While Fig. 3, is representing the time series of three state variables (dynamics)  $x_1$ (*blue line*),  $y_1$ (*green line*) and  $z_1$  (*red line*) of Chua's system. Through the time series of the two systems, we notice that these time series are completely different in behavior, one from the other, and this is evident through the locations and amplitudes of the peaks.



**Fig. 4** Strange attractor in  $(y_1x_1)$  dynamics of Chua system

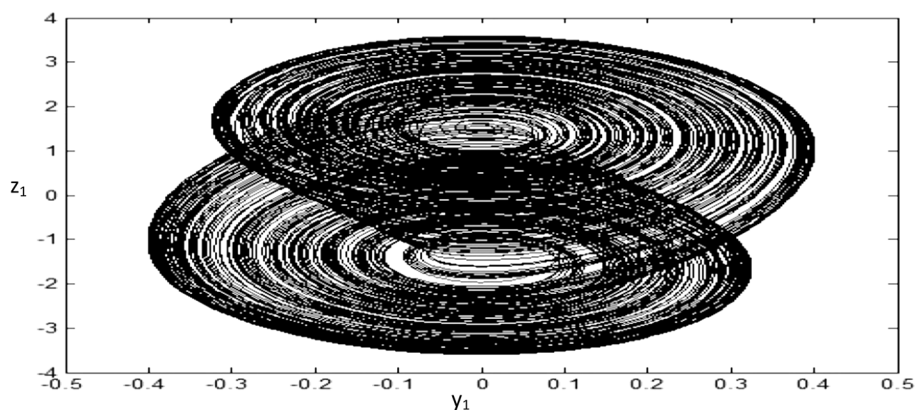


**Fig. 5** Strange attractor in  $(z_1x_1)$  dynamics of Chua system

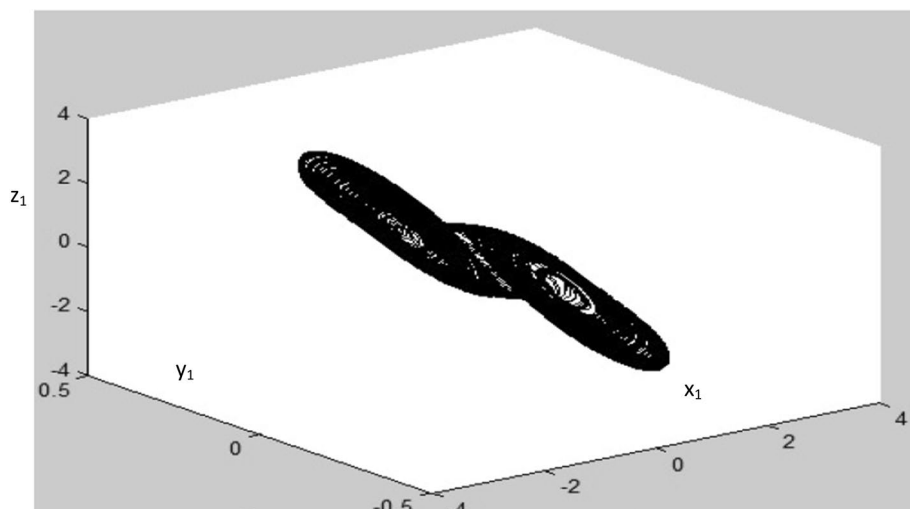
The Figs. 4, 5, 6 that shows double-scroll strange attractor mode that represents the strange attractors of pairs  $(y_1x_1)$ ,  $(z_1x_1)$ , and  $(z_1y_1)$  respectively while Fig. 7 represent the strange attractor in three dimensions 3D  $(z_1y_1x_1)$  of Chua system. When the dynamic  $x_2$  of the Rössler system is coupled with variable  $z_1$  for a Chua system, the time series of Chua circuit will change to new behavior and become as shown in Fig. 8, where it is completely different in first state.

To check that the new system has a chaotic behavior or not, no definition of the term chaos has been universally accepted yet but most researchers agree on the three ingredients used in following definition “Chaos is aperiodic long term behavior in a deterministic system that exhibits dependence on initial condition”. Even though the definition of chaos has not been agreed upon by mathematicians, two properties that are generally agreed to characterize it are sensitivity to initial conditions and the presence of period-doubling cycles leading to chaos.

The range values of output voltage are changing as result this coupled. Moreover the attractor of Chua system is showing “double scroll” pattern this behavior convert to single scroll chaotic pattern by coupled  $x_2$ -dynamic of Rössler system with  $z_1$ -dynamic



**Fig. 6** Strange attractor in  $(z_1, y_1)$  dynamics of Chua system



**Fig. 7** Strange attractor in 3D spaces  $(z_1, y_1, x_1)$  dynamics of Chua system

of Chua system, where Figs. 9, 10, and 11 as shows strange attractors in pairs  $(y_1, x_1)$ ,  $(z_1, x_1)$ , and  $(z_1, y_1)$  dynamics of Chua system with new scheme respectively. Figure 12 shows the strange attractors in three dimension  $(z_1, y_1, x_1)$  dynamics of Chua system with new scheme. The new system equations have one equilibrium point. This point which satisfies this requirement is found by setting  $x, y, z = 1.6, 0, 1.6$  in Eq. 3, and solving for  $x, y$  and  $z$ .

Table 1 represent  $x_1, y_1$ , and  $z_1$  dynamics ranges before and after coupling, where the dynamics of Chua system will increase after coupling.

Through Figs. 13 and 14, the value of ranges for chaotic dynamic systems is observed, which plays an important role in communication applications especially confidential and secure ones and this is evident during the process of performing fast Fourier transforms (FFT) of chaotic dynamics before and after coupling. It is believed that the chaotic systems with higher dimensional attractors have much wider applications. There are many previous

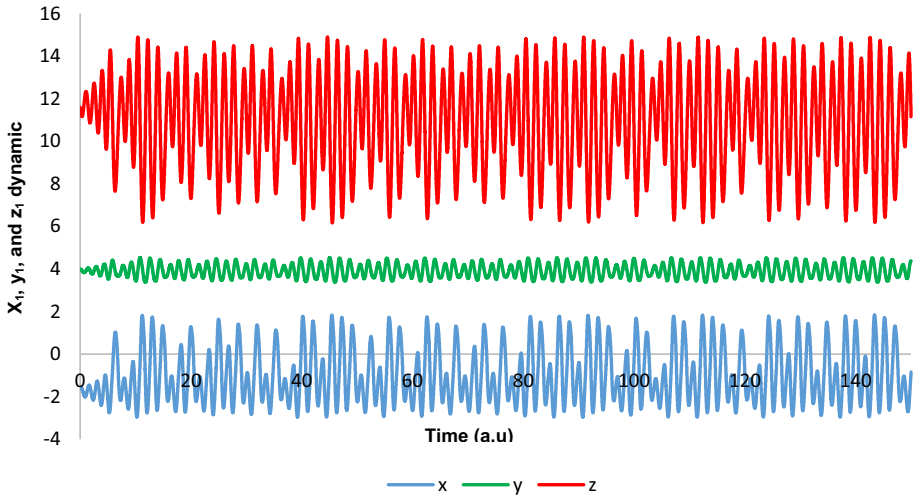


Fig. 8 Time series in  $x_1, y_1$  and  $z_1$ -dynamics of Chua system with new scheme

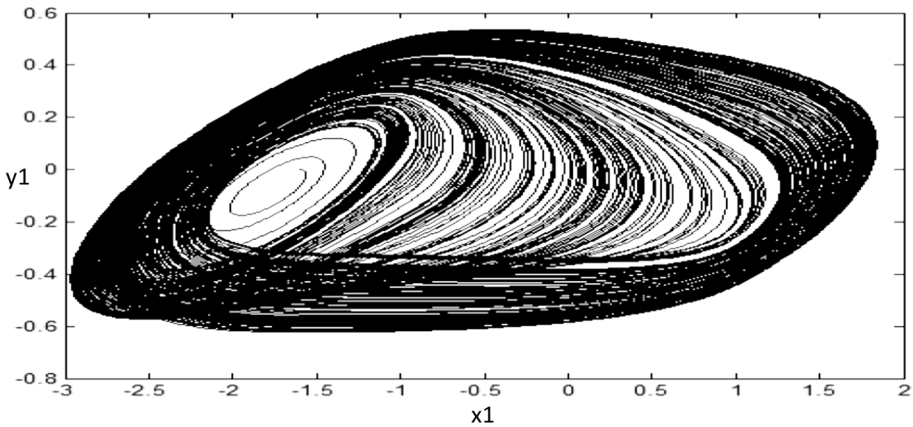


Fig. 9 Strange attractor in  $(y_1-x_1)$  dynamics of Chua system with new scheme

researches that agree with the results obtained, but using different chaotic systems that differ from the one dealt with in this research (Jamal 2019) and (Jamal 2021).

### 4 Conclusions

This work explains how to create a new and powerful chaotic system by using other chaotic systems and merging them. The change of the Chua system attractor, which was shown in the form of double scroll then converted to homoclinic chaotic by coupled  $x_2$ -dynamic of Rössler circuit with  $z_1$ -dynamic of Chua’s circuit. In the FFT figures, it is noticed that the dynamic bandwidth  $z_1$ -coupled has become more broad than it is in the case of

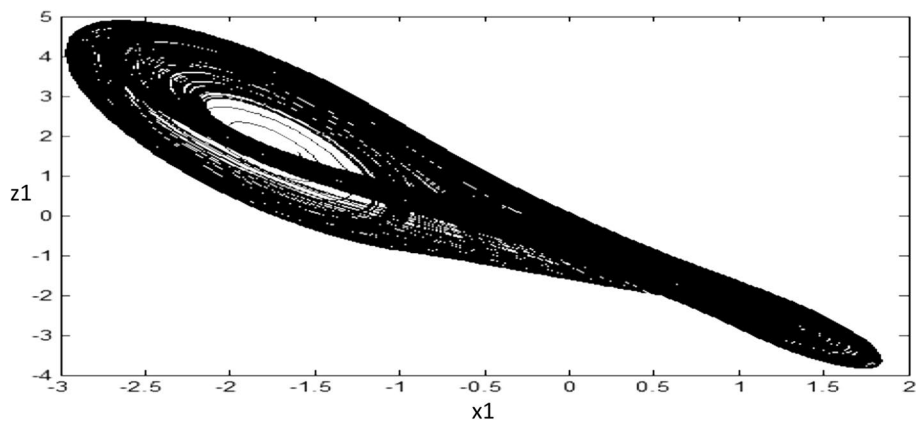


Fig. 10 Strange attractor in  $(z_1-x_1)$  dynamics of Chua system with new scheme

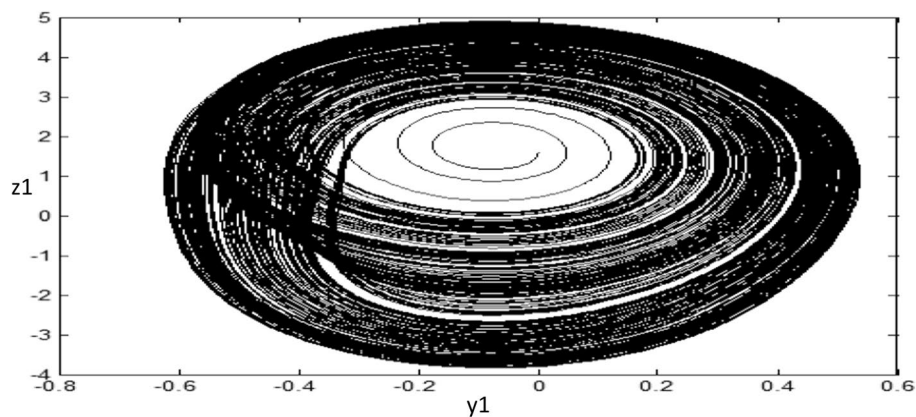


Fig. 11 Strange attractor in  $(z_1-y_1)$  dynamics of Chua system with new scheme

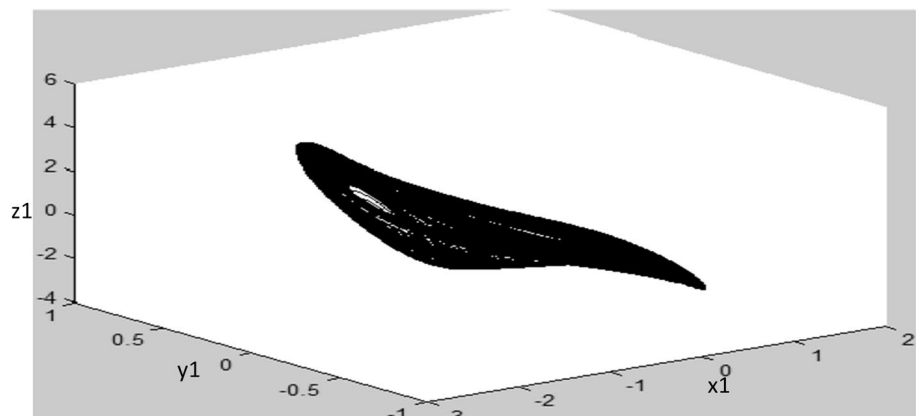
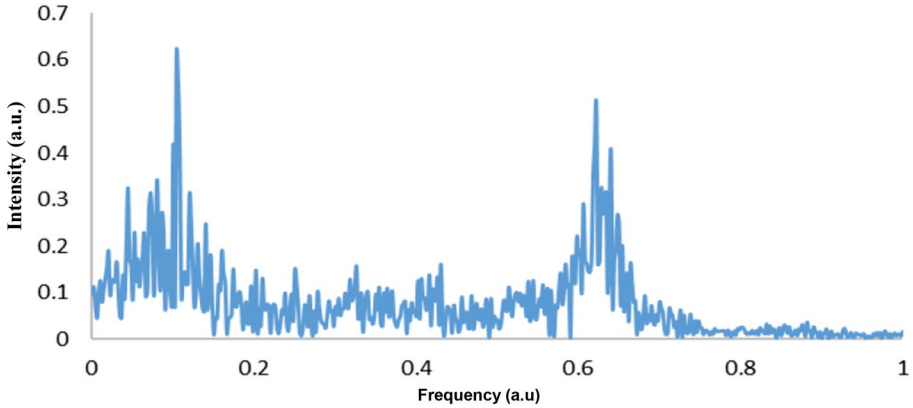


Fig. 12 Strange attractor in  $(z_1-y_1-x_1)$  dynamics of Chua system with new scheme

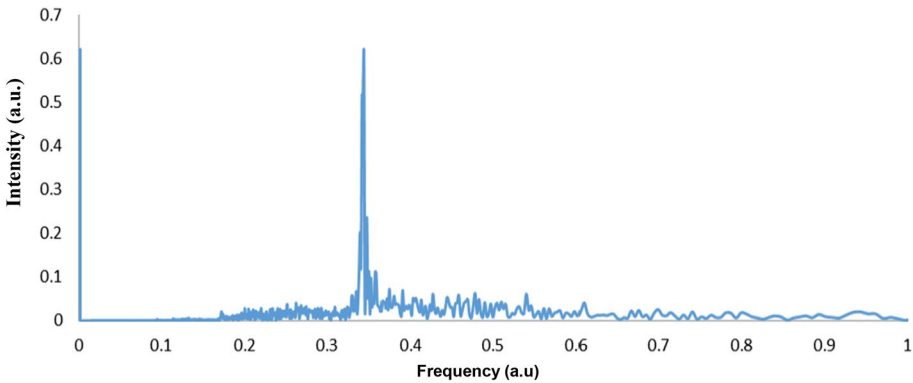


**Table 1**  $x_1, y_1,$  and  $z_1$  dynamics ranges before and after coupling

Dynamics	$x_1$	$y_1$	$z_1$
Before coupling	-2.4:2.4	-0.4:0.4	-3.5:3.5
After coupling	-3:2	-0.6:0.6	-4:5



**Fig. 13** FFT spectrum of  $z$ -dynamics of Chua system



**Fig. 14** FFT spectrum of  $z$ -dynamics of Chua system with new scheme

non-coupling, and this point is very important in the subject of secret communications. Thus, the characteristic frequency of the Chua system has completely disappeared within the wide range formed as a result of the coupling, so the dynamic  $z$ -dynamic is more important than the other dynamics due to its discontinuous exponential dicey distribution.

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