Gosper's Algorithm and Rational Solutions of First Order Recurrences

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Abstract

In this paper we give two approaches for Gosper's algorithm. Also we give analysis to the degree of rational solutions of certain first order difference equation. Furthermore, we present an approach to find rational solutions of first order recurrences.

Keywords: Gosper's algorithm, hypergeometric solution, rational solution.

1. Notations

Let $\mathbb{N}$ be the set of natural numbers, K be the field of characteristic zero, $K(n)$ be the field of rational functions over K, $K[n]$ be the ring of polynomials over K, deg(p) denotes the polynomial degree (in n) of any $p\in K[n]$, $p\neq 0$. We define $\deg(0)=-1$. We assume the result of any gcd (greatest common divisor) computation in $K[n]$ as being normalized to a monic polynomial p, i.e., the leading coefficient of p being 1. Recall that a non-zero term t_n is called a hypergeometric term over K if there exists a rational function $r(n)\in K(n)$ such that

$$\frac{t_{n+1}}{t_n}=r(n).$$

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Gosper's algorithm has been extensively studied and widely used to prove hypergeometric identities see, for example, [6, 7, 8, 10, 11, 13, 14]. Given a hypergeometric term t_n, Gosper's algorithm is a procedure to find a hypergeometric term z_n satisfying

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